An Efficient Scheme for Two-Phase Flow in Porous Media Including Dynamic Capillary Pressure

Mohamed F. El-Amin, Shuyu Sun, and Amgad Salama

***Abstract*—In this paper, The IMplicit Pressure Explicit Saturation (IMPES) scheme has been developed to treating the model of the two-phase flow in a porous medium including a dynamic capillary pressure. The pressure equation is treated implicitly with the saturation equation. Stability condition of the scheme is determined. The results show that the IMPES scheme with dynamic capillary pressure is more stable than it with static capillary pressure. Moreover, unlike the case of static capillary pressure, the IMPES scheme with dynamic capillary pressure is stable with larger time steps.**

***Index Terms*—dynamic capillary pressure, two-phase flow, porous media, implicit method.**

I. Introduction

ALL standard empirical relationships between capillary pressure and saturation (namely, the static capillary pressure) were correlated from laboratory experiments under equilibrium conditions. These formulas of the static capillary pressure (see e.g. [1], [2]) have been used in most of the mathematical models of immiscible two–phase flow in porous media. However, it was found experimentally and theoretically that under non–equilibrium conditions the capillary pressure does not correspond to the static capillary pressure. The dynamic capillary pressure–saturation relationship has been obtained in the framework of a macroscopic theory of porous media flow by Hassanizadeh and Gray [3]. Moreover, many of experiments reported in the literature include evidence of dynamic effects, such as Stauffer [4]. So, no longer to use the static capillary pressure-saturation relationship in the modeling of capillarity when the fluid content is in motion. An alternative model of the capillary pressure-saturation relationship was proposed and referred to as the dynamic capillary pressure is more suitable. When gradients of fluids pressure and fluids velocities are large, non-equilibrium effects in capillary pressure can be significant such as flow in some industrial porous media, such as paper pulp drying process (Lewalle et al. [5]). Recently, many experimental studies on dynamics capillary pressure have been introduced (see e.g. [6], [7], [8], [9], [10], [11], [12], [13]. A comprehensive review by [14] surveyed the experimental works in which non–equilibrium effects have been observed. Moreover, other computational studies on dynamics capillary pressure using Darcy-scale models have been also done (see e.g. [15], [16]). Pore-scale models are also considered (see e.g. [17], [18], [19]).

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The model of two-phase fluid flow in porous media is a coupled system of nonlinear time-dependent partial differential equations. Two different types of time discretization schemes are often used to solve this coupled system. The first one is the fully implicit scheme [23], [24], [25], [26], [27] that implicitly treats with all terms including capillary pressure. This scheme results in a system of nonlinear equations and has unconditional stability and maintains the inherent coupling of two-phase flow model. The second scheme is the IMplicit-EXplicit (IMEX) [28], [29], [30], [31], [32] which generally treats the linear terms implicitly and evaluates the others explicitly, and consequently. This scheme is conditionally stable, however, it has advantage that is to eliminate the nonlinearity of original equations. The IMplicit Pressure Explicit Saturation (IMPES) approach is viewed as an IMEX method, solves the pressure equation implicitly and updates the saturation explicitly. The IMPES method is conditionally stable, and hence it must take very small time step size, especially for highly heterogeneous permeable media where the capillary pressure affects substantially on the path of fluid flow. The instability of the IMPES method [33] results from the decoupling between the pressure equation and the saturation equation as well as the explicit treatment of the capillary pressure. The IMPES for two-phase flow has been improved in several versions (e.g. [34], [35], [36]). Iterative IMPES splits the equation system into a pressure and a saturation equation that are solved sequentially as IMPES [37], [38], [39]. As an iterative method, the computational cost and memory required by iterative IMPES method is smaller than the fully coupled approach at each iterative step, which is more pronounced for very large size computational problems. The main disadvantage of iterative IMPES method is the decoupling of pressure and saturation equations, which results from the explicit treatment for capillary pressure. A linear approximation of capillary function is introduced to couple the implicit saturation equation into pressure equation [40]. Kou and Sun [22] presented an iterative version of their previous scheme proposed in [40]. Unlike iterative IMPES, capillary pressure is not computed by the saturations at the previous iteration, but the linear approximation of capillary function at the current iteration is used, which is constructed by the saturations at the current and previous iterations.

In this work, we develop the IMPES scheme introduced in [22], [40] to solve the flow equation of the model with dynamic capillary pressure.

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II. Modeling and Mathematical Formulation

A. Two-phase flow Model

In this section, a mathematical model is developed to describe the two-phase flow in porous media including dynamic capillary pressure. Consider two-phase immiscible incompressible flow in a porous medium domain that governed by the Darcys law and the equations of mass conservation for each phase as,

$$\frac{\partial(\phi S_\alpha)}{\partial t} + \nabla \cdot \mathbf{u}_\alpha = q_\alpha, \quad \alpha = w, n. \tag{1}$$

$$\mathbf{u}_\alpha = -\frac{k_{r\alpha}}{\mu_\alpha}\mathbf{K}\nabla p_\alpha, \quad \alpha = w, n. \tag{2}$$

where S_α is the saturation, $\mathbf{u}_\alpha$ is the velocity of the phase α. w stands for the wetting phase, and n stands for the nonwetting phase. ϕ is the porosity of the medium, q_α is the external mass flow rate. $\mathbf{K}$ is the absolute permeability tensor is chosen as $\mathbf{K} = k\mathbf{I}$, where $\mathbf{I}$ is the identity matrix and k is a positive real number. $k_{r\alpha}$ is the relative permeability, ρ_α is the density, and p_α is the pressure of the phase α. μ_α is the viscosity and $k_\alpha = k_{r\alpha}\mathbf{K}$ is the effective permeability. The fluid saturations for the wetting and non-wetting are interrelated by,

$$S_w + S_n = 1. \tag{3}$$

Now, we describe the governing equations used in [20], [21] and [22] as,

$$\nabla \cdot (\mathbf{u}_a + \mathbf{u}_c) \equiv -\nabla \cdot \lambda_t \mathbf{K}\nabla p_w - \nabla \cdot \lambda_n \mathbf{K}\nabla p_c = q_w + q_n. \tag{4}$$

and

$$\frac{\partial(\phi S_\alpha)}{\partial t} - q_w = -\nabla \cdot (f_w \mathbf{u}_a) \equiv -\nabla \cdot \lambda_t \mathbf{K}\nabla p_w. \tag{5}$$

where $f_w = \lambda_w/\lambda_t$ is the flow fraction, $\lambda_\alpha = k_{r\alpha}/\mu_\alpha$ is the mobility, p_w is the wetting fluid pressure, and p_c is the capillary pressure. The total velocity $\mathbf{u}_t = \mathbf{u}_w + \mathbf{u}_n = \mathbf{u}_a + \mathbf{u}_c$ is defined as the sum of the two velocity variables $\mathbf{u}_a = -\lambda_t \mathbf{K}\nabla p_w$ and $\mathbf{u}_c = -\lambda_n \mathbf{K}\nabla p_c$. The wetting-phase velocity may be expressed by, $\mathbf{u}_w = f_w \mathbf{u}_a$. The two-phase capillary pressure is discussed below.

B. Dynamic Capillary Pressure

The classical capillary pressure–saturation relationship based on the thermodynamic equilibrium assumption, is commonly written as,

$$p_n - p_w = p_s(S_w). \tag{6}$$

One of the major assumptions is that fluids pressure difference is equal to capillary pressure under all conditions at all times. However, according to Entov [41], capillary pressure–saturation relationship is not unique and, even though it is obtained under equilibrium conditions, it is a function of the history of fluids movements. Moreover, it was theoretically established fact that $p_n - p_w$ is equal to capillary pressure but only under equilibrium conditions (see Hassanizadeh et al. [14]). For non-equilibrium conditions, the following equation for the fluids pressure difference as been suggested (Hassanizadeh and Gray [3]),

$$p_n - p_w = p_s(S_w) - \tau(S_w)\frac{\partial S_w}{\partial t}. \tag{7}$$

where τ is a non-equilibrium capillarity coefficient (material property) that may be a function of saturation and other fluid–fluid properties.

On the other hand, in the model of Barenblatt [46], the capillary pressure relationship under non–equilibrium conditions is determined using future water saturation that is may be a lower water saturation on drainage or a larger water saturation on imbibition for a given capillary pressure. The difference between the current and future saturations is a function of the desaturation rate and is related to a redistribution time. Juanes [47] has presented a relationship of the dynamic coefficient and a characteristic redistribution time as,

$$\tau(S_w) = \tau_B(S_w)\frac{dp_s}{dS_w}. \tag{8}$$

where τ_B is the redistribution (or relaxation) time. There is a great deal of uncertainty regarding the appropriate magnitude and functional form of the phenomenological coefficients [14].

From now on we will write the dynamic capillary pressure as $p_d(S_w, \dot{S}_w) = p_n - p_w$, $\dot{S}_w = \frac{\partial S_w}{\partial t}$, $p_s' = \frac{dp_s}{dS_w}$. Substitute from (8) into (7), one can get,

$$p_d(S_w, \dot{S}_w) = p_s(S_w) - \tau_B(S_w)\, p_s' \frac{\partial S_w}{\partial t}. \tag{9}$$

The relationship of dynamic capillary pressure suggest that the dynamic capillary pressure defined as the capillary pressure measured under non-equilibrium conditions, is larger than the static capillary pressure measured under equilibrium conditions, on water drainage and smaller on water imbibition.

C. Initial and Boundary Conditions

The saturation of the wetting phase in the computational domain Ω at the beginning of the flow displacing process is initially defined by,

$$S_w = S_w^0 \quad \text{in} \quad \Omega \qquad \text{at} \quad t = 0. \tag{10}$$

The boundary $\partial\Omega$ of the computational domain Ω is subjected to both Dirichlet and Neumann conditions such that $\partial\Omega = \Gamma_D \cup \Gamma_N$ and $\Gamma_D \cap \Gamma_N = \emptyset$, where Γ_D is the Dirichlet boundary and Γ_N is the Neumann boundary. The boundary conditions considered in this study are summarized as follow,

$$p_w \,(\text{or } p_n) = p^D \quad \text{on} \quad \Gamma_D, \tag{11}$$

$$\mathbf{u}_t \cdot \mathbf{n} = q^N \quad \text{on} \quad \Gamma_N, \tag{12}$$

where $\mathbf{n}$ is the outward unit normal vector to $\partial\Omega$, p^D is the pressure on Γ_D and q^N the imposed inflow rate on Γ_N, respectively. The saturations on the boundary are subject to,

$$S_w \,(\text{or } S_n) = S^N \quad \text{on} \quad \Gamma_N, \tag{13}$$

III. Solution Method

Define the time step length $\Delta t^n = t^{n+1} - t^n$, the total time interval $[0, T]$ may be divided into N_T time steps as $0 = t^0 < t^1 < \cdots < t^{N_T} = T$. The current time step is represented by the superscript $n+1$, while the current time step is represented by the superscript n. The backward Euler time discretization is used for the equations of both pressure

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and saturation. The discretized governing equations can be given as,

$$-\nabla\cdot\lambda_t\left(S_w^n\right)\mathbf{K}\nabla p_w^{n+1}-\nabla\cdot\lambda_n\left(S_w^n\right)\mathbf{K}\nabla p_d\left(S_w^{n+1}\right)=q_t^{n+1} \tag{14}$$

$$p_d\left(S_w^{n+1}\right)=p_s\left(S_w^n\right)-\tau_B\left(S_w^n\right)p_s'\left(S_w^n\right)\frac{S_w^{n+1}-S_w^n}{\Delta t^n} \tag{15}$$

$$\phi\frac{S_w^{n+1}-S_w^n}{\Delta t^n}=q_w^{n+1}-\nabla\cdot\lambda_t\left(S_w^n\right)\mathbf{K}\nabla p_w^{n+1}, \tag{16}$$

where $q_t^{n+1}=q_w^{n+1}+q_n^{n+1}$. The function $\tau_B\left(S_w^n\right)$ may rewritten as $\tau_B\cdot F\left(S_w^n\right)$, where τ_B is constant and $F\left(S_w^n\right)$ is a function of saturation. The saturation time derivative which appears in the dynamic capillary pressure is calculated as difference between the saturation at current time step S_w^{n+1} and saturation at the previous time step S_w^n. It is worth mentioning that the above equation of saturation (15) is coupled with the pressure equation to be solved implicitly together, however it is not used to update the saturation. In order to update the saturation, we use the following explicit scheme of the saturation equation,

$$\phi^n\frac{S_w^{n+1}-S_w^n}{\Delta t^n}+\nabla\cdot\left(f_w^n\mathbf{u}_a^{n+1}\right)=q_w^{n+1}, \tag{17}$$

For each current time step $(n+1)$, the variables λ_w, λ_n, λ_t, τ_B, p_s' and f_w is calculated using the saturation from the previous time step n. The pressure equation is solved firstly to obtain the wetting-phase pressure at the current time step and then the Darcys velocity can be calculated. Therefore, the saturation at the current time step is computed explicitly in the current iteration. Finally, the other parameters such as λ_w, λ_n, λ_t, τ_B, p_s' and f_w are updated.

Now, let us apply the CCFD scheme to the system of equations (14) to obtain,

$$\mathbf{A}_a\left(S_w^n\right)\mathbf{P}_w^{n+1}+\mathbf{A}_c\left(S_w^n\right)\mathbf{P}_d\left(S_w^{n+1}\right)=\mathbf{Q}_{ac}^{n+1}. \tag{18}$$

It is noted from above algebraic equations that the matrices $\mathbf{A}_a$, $\mathbf{A}_c$ and $\mathbf{P}_d$ depend on the vector S_w. The vector $\mathbf{P}_d\left(S_w^n,S_w^{n+1}\right)$ is given by discretizing the approximation of capillary pressure defined by (15), which may be rewritten in a matrix-vector form as follows,

$$\mathbf{P}_d\left(S_w^n,S_w^{n+1}\right)=\mathbf{P}_s\left(S_w^n\right)+\tau_B\mathbf{P}_f\left(S_w^n\right)\frac{S_w^{n+1}-S_w^n}{\Delta t^n}. \tag{19}$$

where $\mathbf{P}_s$ depends on the vector S_w. $\mathbf{P}_f$ is resulted from the discretization of the p_s' and F that is a diagonal matrix defined by,

$$\mathbf{P}_f\left(S_w^n\right)=\mathbf{diag}\left(F(S_w^n)\cdot p_s'(S_w^n)\right),\quad h=1,2,...,N_c, \tag{20}$$

where N_c is the total number of all cells. In fact, the derivative of p_s is a function of p_s when the saturation at each spatial point is varies with time. At the same time, the saturation is smoothly changing along with time at each spatial point even if it is discontinuously distributed in space. Moreover, the CCFD discretization of the saturation equation (15) is,

$$\mathbf{M}\frac{\mathbf{S}_w^{n+1}-\mathbf{S}_w^n}{\Delta t^n}+\mathbf{A}_w\left(S_w^n\right)\mathbf{P}_w^{n+1}=\mathbf{Q}_w^{n+1}, \tag{21}$$

where M is a diagonal matrix replaces the porosity.

Now, let us substituting from (19) and (21) into (18) one may obtain the coupled pressure equation in the following form,

$$\mathbf{A}_t\left(S_w^n\right)\mathbf{P}_w^{n+1}=\mathbf{Q}_t\left(S_w^n\right) \tag{22}$$

where

$$\mathbf{A}_t\left(S_w^n\right)=\mathbf{A}_a\left(S_w^n\right)-\tau_B\mathbf{A}_c\left(S_w^n\right)\mathbf{P}_f\left(S_w^n\right)\mathbf{M}^{-1}\mathbf{A}_w\left(S_w^n\right) \tag{23}$$

and,

$$\mathbf{Q}_t=\mathbf{Q}_{ac}^{n+1}-\mathbf{A}_c\left(S_w^n\right)\left[\mathbf{P}_s\left(S_w^n\right)+\tau_B\mathbf{P}_f\left(S_w^n\right)\mathbf{M}^{-1}\mathbf{Q}_w^{n+1}\right] \tag{24}$$

The upwind scheme is used in the advection term of the discretization of the saturation equation (17) which used for updating saturation. Thus, the discretized saturation equation is given by,

$$\mathbf{M}\frac{\mathbf{S}_w^{n+1}-\mathbf{S}_w^n}{\Delta t^n}+\mathbf{A}_s\left(S_w^n,P_w^{n+1}\right)f_w\left(S_w^n\right)=\mathbf{Q}_s^{n+1} \tag{25}$$

IV. Stability Analysis

The dynamic capillarity parameter τ_B, has the main effect on the stability of this scheme method. In the following analysis, the effect of saturation error on the matrices $\mathbf{A}_w$, $\mathbf{A}_a$ and $\mathbf{A}_c$ is neglected and the capillary pressure is concentrated. From (18), (19) and (21), it follows that

$$\begin{aligned}\mathbf{S}_w^{n+1} &= \mathbf{S}_w^n+\Delta t^n\mathbf{M}^{-1}\mathbf{Q}_w^{n+1}-\Delta t^n\mathbf{M}^{-1}\mathbf{A}_w\mathbf{A}_a^{-1}\\ &\quad\left[\mathbf{Q}_{ac}^{n+1}-\mathbf{A}_c\left(\mathbf{P}_s(\mathbf{S}_w^n)+\right.\right.\\ &\quad\left.\left.\frac{\tau_B}{\Delta t^n}\mathbf{P}_f\left(S_w^n\right)\left(\mathbf{S}_w^{n+1}-\mathbf{S}_w^n\right)\right)\right]\end{aligned} \tag{26}$$

Now we need to consider the propagation of numerical errors from time step n to time step $(n+1)$. Local truncation error is not considered here. For the nth step saturation $\mathbf{S}_w^n$ we denote for a perturbed saturation by $\widetilde{\mathbf{S}}_w^n=\mathbf{S}_w^n+\delta\mathbf{S}_w^n$, where $\delta\mathbf{S}_w^n$ represents the error for $\mathbf{S}_w^n$. Similarly, we can obtain an inexact saturation at the $(n+1)$th step that is expressed as $\widetilde{\mathbf{S}}_w^{n+1}=\mathbf{S}_w^{n+1}+\delta\mathbf{S}_w^{n+1}$ and $\delta\mathbf{S}_w^{n+1}$ is the $(n+1)$th step saturation error. In this case, $\widetilde{\mathbf{S}}_w^{n+1}$ satisfies,

$$\begin{aligned}\widetilde{\mathbf{S}}_w^{n+1} &= \widetilde{\mathbf{S}}_w^n+\Delta t^n\mathbf{M}^{-1}\mathbf{Q}_w^{n+1}-\Delta t^n\mathbf{M}^{-1}\mathbf{A}_w\mathbf{A}_a^{-1}\\ &\quad\left[\mathbf{Q}_{ac}^{n+1}-\mathbf{A}_c\left(\mathbf{P}_s(\widetilde{\mathbf{S}}_w^n)\right.\right.\\ &\quad\left.\left.\frac{\tau_B}{\Delta t^n}\mathbf{P}_f(\widetilde{\mathbf{S}}_w^n)\left(\widetilde{\mathbf{S}}_w^{n+1}-\widetilde{\mathbf{S}}_w^n\right)\right)\right]\end{aligned} \tag{27}$$

Subtracting (26) from (27), one gets,

$$\begin{aligned}\delta\mathbf{S}_w^{n+1} &= \delta\mathbf{S}_w^n+\Delta t^n\mathbf{H}\left[\mathbf{P}_s(\widetilde{\mathbf{S}}_w^n)-\mathbf{P}_s(\mathbf{S}_w^n)\right.\\ &\quad+\frac{\tau_B}{\Delta t^n}\mathbf{P}_f(\widetilde{\mathbf{S}}_w^n)\left(\widetilde{\mathbf{S}}_w^{n+1}-\widetilde{\mathbf{S}}_w^n\right)-\\ &\quad\left.\frac{\tau_B}{\Delta t^n}\mathbf{P}_f(\mathbf{S}_w^n)\left(\mathbf{S}_w^{n+1}-\mathbf{S}_w^n\right)\right]\\ &\simeq \delta\mathbf{S}_w^n+\Delta t^n\mathbf{H}\left[\mathbf{P}_s(\widetilde{\mathbf{S}}_w^n)-\mathbf{P}_s(\mathbf{S}_w^n)\right.\\ &\quad\left.+\frac{\tau_B}{\Delta t^n}\mathbf{P}_f(\widetilde{\mathbf{S}}_w^n)\left(\delta\mathbf{S}_w^{n+1}-\delta\mathbf{S}_w^n\right)\right]\end{aligned} \tag{28}$$

where $\mathbf{H}=\mathbf{M}^{-1}\mathbf{A}_w\mathbf{A}_a^{-1}\mathbf{A}_c$. Moreover, since we have,

$$\mathbf{P}_s(\widetilde{\mathbf{S}}_w^n)-\mathbf{P}_s(\mathbf{S}_w^n)\simeq\mathbf{P}_f(\widetilde{\mathbf{S}}_w^n)\delta\mathbf{S}_w^n,$$

one may get,

$$\delta\mathbf{S}_w^{n+1}\simeq\mathbf{C}\delta\mathbf{S}_w^n \tag{29}$$

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where,

$$\mathbf{C} = \left(\mathbf{I} - \tau_B \mathbf{H}\mathbf{P}_f(\widetilde{\mathbf{S}}_w^n)\right)^{-1}\left(\mathbf{I} + (1 - \frac{\tau_B}{\Delta t^n})\Delta t^n \mathbf{H}\mathbf{P}_f(\widetilde{\mathbf{S}}_w^n)\right)$$

and $\mathbf{I}$ is the identity matrix. Consequently, the scheme (I) is stable if the following condition holds,

$$\rho\left(\mathbf{C}\right) < 1, \tag{30}$$

where $\rho(\mathbf{C})$ is the spectral radius of the matrix $\mathbf{C}$, i.e. all of the eigenvalues of this matrix must lie within the unit circle in the complex plane. Theoretically, this condition is computable, but practically it is too expensive to compute for guiding the choice of Δt^n.

Here, we give a simple and typical example to show the effect of relaxation factor on stability. One popular capillary pressure function is given by

$$p_c(S_w) = -B_c \log(S_w), \tag{31}$$

where B_c is a positive parameter. Assume that a square domain is partitioned into one cell. As the results of the discretization of CCFD, all the matrices $\mathbf{M}, \mathbf{A}_w, \mathbf{A}_a$ and $\mathbf{A}_c$ become positive scalar numbers and so is $\mathbf{H}$. The stability condition (30) becomes

$$\frac{\left|1 - (1 - \frac{\tau_B}{\Delta t^n})\Delta t^n \mathbf{H} B_c / \widetilde{S}_w^n\right|}{\left|1 + \tau_B \mathbf{H} B_c / \widetilde{S}_w^n\right|} < 1. \tag{32}$$

This means that the scheme is stable for all $\frac{\tau_B}{\Delta t^n} \geq 0.5$. It is interesting to note that we can say that this scheme is unconditionally stable because naturally $\tau_B >> \Delta t^n$. Moreover, this scheme attains using a large time step, because τ_B is of order $10^5 - 10^7$. For the case of static capillary pressure ($\tau_B = 0$), a small time step size is often required to attain the stability of IMPES.

V. Results

Here we introduce an example to test the performance of the presented scheme. Before presenting the numerical example, we need to define the used physical parameters. Consider the following static capillary pressure formula, $p_s = -B_c \log(S)$, and the normalized wetting phase saturation are given by, $S = \frac{S_w - S_{wr}}{1 - S_{nr} - S_{wr}}, \quad 0 \leq S \leq 1$, where B_c is the static capillary pressure parameter, S_{wr} is the irreducible (minimal) water (wetting phase) saturation, and S_{nr} is the residual (minimal) oil (nonwetting phase) saturation after water flooding. Also, the expressions of the relation between the relative permeabilities and the normalized wetting phase saturation S is given as, $k_{rw} = k_{rw}^0 S^a, k_{rn} = k_{rn}^0 (1 - S)^b$, where a and b are positive real numbers, $k_{rw}^0 = k_{rw}\,(S = 1)$ is the endpoint relative permeability to the wetting phase, and $k_{rn}^0 = k_{rn}\,(S = 0)$ is the endpoint relative permeability to the non-wetting phase.

The capillary pressure function and relative permeabilities are chosen to be zero for the residual saturations of water and oil; that is, $S = S_w$. In computation, we take the minimum of saturation as $S_{w,min} = 10^{-4}$. The viscosities of water and oil are 1 cP and 0.45 cP, respectively. The injection rate is 0.1 PV/year and we continue the calculation until 0.5 PVI, 0.7 PVI and 0.9 PVI that has the time steps, 0.01, 0.014 and 0.018 days, respectively. The relative permeabilities are quadratic, $k_{rw0} = k_{ro0} = 1$, $a = b = 2$, and the static capillary pressure parameter is $B_c = 50$ bar. The porosity is taken as 0.1 while the permeability is 1 md. The domain dimension is taken as 0.3 m $\times$ 0.2 m. The computational domain is divided into 1200 uniform rectangles.

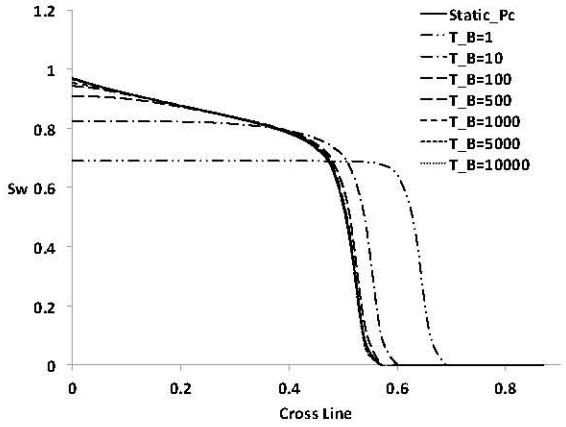

Fig. 1. Saturation plot after five days of injection with different values of τ_B

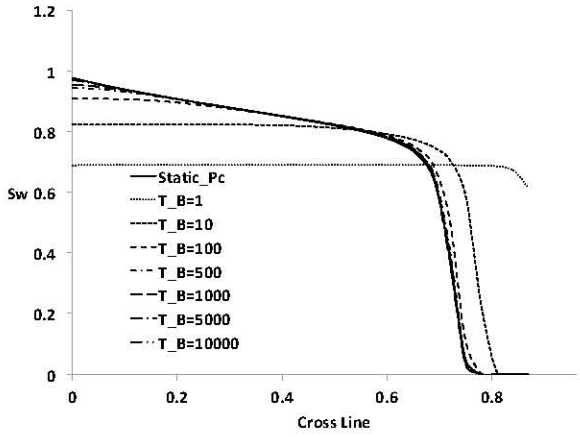

Fig. 2. Saturation plot after seven days of injection with different values of τ_B

In this example, we run the simulation for different values of the dynamic pressure parameter, τ_B =0; 1;10; 100; 500; 1,000; 5,000; 10,000; 100,000. Figs. 1 and 2 represent water saturation after 5 and 7 days, respectively, with various values of τ_B. All these figures show that as the value of τ_B tends to infinity ($\tau_B \to \infty$), the saturation with dynamic capillary pressure tends to the saturation with static capillary pressure, $S_w(p_d) \to S_w(p_s)$. This is very interesting results show that the IMPES scheme with dynamic capillary pressure is more stable than it with static capillary pressure. On the other hand, unlike the case of static capillary pressure, the IMPES scheme with dynamic capillary pressure is stable with large time steps.

VI. Conclusion

In the current work, the problem of two-phase flow in porous media including the dynamic capillary pressure has been studied. The IMPES scheme is considered to solving the problem under consideration. The saturation equation is used in two different locations of the scheme. The first location was when substituting the time derivative of the saturation in the capillary pressure term in the pressure equation. In the second location was the saturation equation which explicitly

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solved with the upwinding advection to update the saturation. Stability of the scheme has been investigated. We found that the IMPES scheme with dynamic capillary pressure is more stable than it with static capillary pressure and it is stable with even larger time steps.

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