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Linear Stability of Poiseuille Flow in a Vertical Pipe Filled with Porous Medium

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***Abstract*—The present paper reports a numerical investigation on the stability of of non-isothermal poiseuille flow in a vertical pipe filled with porous medium. The flow is induced by external pressure gradient and buoyancy force. The non-Darcy Brinkman-extended model has been considered. Here, it is assumed that the buoyancy force acting in the opposite direction of the forced flow. To study the instability mechanism of the poiseuille flow, the linear theory of stability analysis has been used for three different values 0.7, 7.0, and 70 of Prandtl number (Pr). To this end, coupled ordinary differential equations obtained from linear theory of stability analysis, have been solved numerically using Spectral collocation method. Four different values 10^{-1}, 10^{-2}, 10^{-3}, and 10^{-4} of Darcy number (Da) have been considered to study the impact of permeability of the medium on the flow stability. The numerical experiments on the stability show that, the first azimuthal mode is always least stable mode. Second, effect of azimuthal numbers die out on decreasing permeability of the media. Third, the instability boundary curves in zero azimuthal mode for Da equal to 10^{-1} and 10^{-2} show anticipated results for air and water. Further, the influence of media permeability on base flow stability shows that the stability of the flow increases on decreasing of Da.**

***Index Terms*—Porous media, mixed convection, linear stability, spectral methods.**

I. Introduction

An instability mechanism of isothermal/non-isothermal is an attractive problem in the research area of fluid dynamics, especially transition of poiseuille flow is still a field of active and ongoing research. It is well known that the linear theory of stability analysis fails to capture the instability boundary of isothermal pipe, channel, and annulus. Recently, Kerstin Avila and his group [1] have used the linear theory of stability to capture the instability boundary points of poiseuille flow in isothermal pipe assuming that the surface of the pipe is not smooth. There are some other works experimental and theoretical related to the non-isothermal pipe for viscous fluid flow only. But, if the pipe is filled with porous media (isotropic/anisotropic) then how the physics of the stability characteristics of poiseuille flow will change still was not clear.

Many previously published results indicate that non-isothermal flow instability and transition differ substantially from those of an isothermal flow. The earliest work on the stability of non-isothermal pipe flow was conducted by, among others, Hanratty, Rosen and Kabel [2] and Scheele and Hanratty [3]. Based on fully-developed and laminar parallel flow approximations, Hanratty et al. [2] obtained analytical solutions for mixed convection flow in a vertical pipe and observed experimentally that the flow is stable in the entry region but highly unstable after the flow is fully developed. Scheele and Hanratty ([2]-[4]) have reported that in the case of buoyancy assisted flow (heated upward flow), the development of points of inflection in velocity profiles is cause of the instability. The flow is super-critically stable and the transition to turbulence is gradual. For the case of buoyancy opposed flow (heated downward flow) the instability is associated with separation from the wall and the flow is sub-critically unstable. Transition is sudden and occurs shortly after the flow becomes unstable. They have also observed in their experiment that the flow stability depends primarily on the shape of the velocity profile, which is modified by heating, and the dependence on the Reynolds number is only secondary. In fluid environment, the theoretical work on the stability of poiseuille flow in a vertical pipe has been presented by Yao ([5], [6]). Yao has investigated the linear stability analysis for an upward fully-developed flow in a heated vertical pipe (with the consideration that fluid is water i.e Pr=7.0) and found that the fully-developed non-isothermal flow is super-critically unstable. The flow can become unstable when Rayleigh number > 75 and Reynolds number > 40 and the most unstable flow pattern is double spiral i.e. the most unstable azimuthal wave number is unity. Yao and his group also have reported important pioneering work on identifying linear thermal instability in vertical annulus ([7] - [11]) and a vertical pipe [12]. Instability in pipe flow also reported recently by Cotrell et. al. [13] and their results shows that the long-puzzling, unphysical result that linear stability analyzes lead to no transition in pipe flow, even at infinite Reynolds number, is ascribed to the use of stick boundary conditions, because they ignore the amplitude variations associated with the roughness of the wall. Once that length scale is introduced (here, crudely, through a corrugated pipe), linear stability analyzes lead to stable vortex formation at low Reynolds number above a finite amplitude of the corrugation and unsteady flow at a higher Reynolds number, where indications are that the vortex dislodges.

More recently in fluid zone, Su and Chung [14] have presented the numerical study on the linear stability of mixed convective flow in a vertical pipe in both the cases: (i) buoyancy assisted and opposed and found that the most unstable flow pattern is double spiral i.e. the most unstable azimuthal wave number is unity. They have given the main emphasis on the instability mechanism and the effect of Prandtl number. They have selected three Prandtl numbers, 0.0248, 7.0 and 100 to simulate the stability characteristics of liquid mercury, water and oil respectively. The effect of Prandtl number plays a integral role in buoyancy assisted flow instability but in the buoyancy opposed flow, the effect of Prandtl number is less significant since the flow is unstably stratified. In porous media, we have studied the least stable mode of convective flow, induced by external pressure gradient and

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buoyancy force in the vertical pipe filled with porous medium ([15], [16]). From the above reviews, most works on mixed convection through vertical pipe is pertained in the purely fluid environment, whereas, in porous media, it introduces a new research area. It is essential to investigate the stability of buoyancy opposed poiseuille flow in vertical pipe filled with porous media under different controlling parameters.

A. Mathematical Formulation

We consider a fully developed mixed convection flow caused by an external pressure gradient and a buoyancy force in a vertical pipe filled with porous medium. The wall temperature is linearly varying with z as $T_w = T_0 + C_1 R_0 z^*$, where C_1 is a constant and T_0 is upstream reference wall temperature and R_0 is radius of the pipe. The gravitational force is aligned in the negative z^*-direction. As shown schematically in figure 1.
The thermo-physical properties of the fluid are assumed to be constant except for density dependency of the buoyancy term in the momentum equations. The porous medium is saturated with a fluid that is in local thermodynamic equilibrium with the solid matrix. The medium is assumed to be isotropic in permeability. In expressing the equations for the flow in the porous medium, it should be noted that the Darcy model presents a linear relationship between velocity of discharge and the pressure gradient. As the Darcy model does not hold when the flow velocity is not sufficiently small or when the permeability is high, extensions to this model known as Brinkman-extended or Forchheimer-extended models exist. In short, the Brinkman term is found to be needed for satisfying a no-slip boundary condition at solid walls, whereas the Forchheimer term accounts for the form drag. Also in analogy with the Navier-Stokes equations, the Darcy model has been extended by including the material derivative. The necessity of the simultaneous inclusion of all or some of these extensions has been discussed in the literature [17]. Therefore, in order to cover extreme values of input parameters i.e., high permeability, high velocity and low thermal diffusivity; the governing equations for the flow and heat transfer in cylindrical polar coordinate system using following non-dimensional quantities:

$$r = \frac{r^*}{{R_0}^*}, \quad u = \frac{u^*}{{W_c}^*}, \quad v = \frac{v^*}{{W_c}^*}, \quad w = \frac{w^*}{{W_c}^*}, \quad z = \frac{z^*}{{R_0}^*},$$
$$p = \frac{p^*}{\rho_f {W_c^*}^2}, \quad t = \frac{t^* {W_c}^*}{{R_0}^*}, \quad \theta = \frac{{T_w}^* - T^*}{C_1 {R_0}^* RePr}$$

are written as:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \psi} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\left[\frac{1}{\epsilon}\frac{\partial u}{\partial t} + \frac{1}{\epsilon^2}\left(\mathbf{J}u - \frac{v^2}{r}\right)\right] = -\frac{\partial p}{\partial r} + \frac{\Lambda}{Re}\left[\mathbf{D}^2 u - \frac{2}{r^2}\frac{\partial v}{\partial \psi} - \frac{u}{r^2}\right] - \frac{\Lambda}{ReDa}u \tag{2}$$

$$\left[\frac{1}{\epsilon}\frac{\partial v}{\partial t} + \frac{1}{\epsilon^2}\left(\mathbf{J}v - \frac{uv}{r}\right)\right] = -\frac{1}{r}\frac{\partial p}{\partial \psi} + \frac{\Lambda}{Re}\left[\mathbf{D}^2 v - \frac{v}{r^2} + \frac{2}{r^2}\frac{\partial u}{\partial \psi}\right] - \frac{\Lambda}{ReDa}v \tag{3}$$

$$\left[\frac{1}{\epsilon}\frac{\partial w}{\partial t} + \frac{1}{\epsilon^2}(\mathbf{J}w)\right] = -\frac{\partial p}{\partial z} + \frac{\Lambda}{Re}\left[\mathbf{D}^2 w\right] - \frac{\Lambda}{ReDa}w - \frac{Ra}{Re}\theta \tag{4}$$

$$\left[\sigma\frac{\partial \theta}{\partial t} + (\mathbf{J}\theta)\right] = \frac{1}{RePr}w + \frac{1}{RePr}\left[\mathbf{D}^2\theta\right] \tag{5}$$

where,

$$\mathbf{J} = u\frac{\partial}{\partial r} + \frac{v}{r}\frac{\partial}{\partial \psi} + w\frac{\partial}{\partial z}, \text{and } \mathbf{D}^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \psi^2} + \frac{\partial^2}{\partial z^2}$$

In the above equations the dimensionless parameters are the Rayleigh number, $Ra = \frac{\mathbf{g}\beta_T {C_1}^* {R_0^*}^4}{\tilde{\nu}\alpha}$, the Prandtl number, $Pr = \frac{\tilde{\nu}}{\alpha}$, and $\Lambda = \frac{\mu_f}{\tilde{\mu}}$.

B. Disturbance Equations of Basic Flow

The basic flow is a fully developed and steady flow that depends on r only. Using these assumptions, the governing equations (1)–(5) are reduced to

$$\frac{d^2 W_0}{dr^2} + \frac{1}{r}\frac{dW_0}{dr} - \frac{\Lambda}{Da}W_0 - Ra\Theta_0 - Re\frac{dp}{dz} = 0 \tag{6}$$

$$\frac{d^2\Theta_0}{dr^2} + \frac{1}{r}\frac{d\Theta_0}{dr} = -W_0. \tag{7}$$

The corresponding boundary conditions are given by:

$$\frac{dW_0}{dr} = \frac{d\Theta_0}{dr} = 0 \quad \text{at} \quad r = 0, \tag{8}$$

$$W_0 = \Theta_0 = 0 \quad \text{at} \quad r = 1, \tag{9}$$

with W_0 and Θ_0 being the basic velocity and temperature, respectively. The analytic as well as numerical solutions of basic flow are given by Ashok Kumar et. al. [15]. The temperature may be eliminated from the above equations (6)-(7) to give the following expression for the velocity field.

$$\left[(\nabla^2 - \frac{1}{2Da})^2 + \lambda^2\right] W_0 = 0 \tag{10}$$

where, $\lambda^2 = Ra - \frac{1}{4Da^2}$ and $\nabla^2 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}$. The axial pressure gradient can be determined by requirement of global mass conservation:

$$\int_0^1 rW_0(r)dr = \frac{1}{2} \tag{11}$$

The solution of Eq. (10), for the two different cases: (i) $\lambda^2 > 0$, (ii) $\lambda^2 < 0$, is given by

$$W_0(r) = a_1 J_0({P_1}^{1/2} r) + a_2 I_0({P_2}^{1/2} r) \tag{12}$$

where,

$$a_1 = \frac{\left[0.25 {P_1}^{1/2} {P_2}^{1/2} I_0({P_2}^{1/2})\right]}{\left[{P_2}^{1/2} J_1({P_1}^{1/2}) I_0({P_2}^{1/2}) - {P_1}^{1/2} J_0({P_1}^{1/2}) I_1({P_2}^{1/2})\right]},$$

$$a_2 = -a_1 \frac{J_0({P_1}^{1/2})}{I_0({P_2}^{1/2})},$$

$$P_1 = \begin{cases} i\left(\sqrt{Ra - \frac{1}{4Da^2}}\right) - \frac{1}{2Da} & \text{for } \lambda^2 > 0 \\ \left(\sqrt{\frac{1}{4Da^2} - Ra}\right) - \frac{1}{2Da} & \text{for } \lambda^2 < 0, \end{cases}$$

and

$$P_2 = \begin{cases} i\left(\sqrt{Ra - \frac{1}{4Da^2}}\right) + \frac{1}{2Da} & \text{for } \lambda^2 > 0 \\ \left(\sqrt{\frac{1}{4Da^2} - Ra}\right) + \frac{1}{2Da} & \text{for } \lambda^2 < 0. \end{cases}$$

J_0 and I_0 are zeroth order first and second kind of Bessel functions respectively. The corresponding temperature is given by

$$\Theta_0(r) = \frac{a_1}{P_1}\left[J_0({P_1}^{1/2}r) - J_0({P_1}^{1/2})\right] - \frac{a_2}{P_2}\left[I_0({P_2}^{1/2}r) - I_0({P_2}^{1/2})\right] \quad (13)$$

In linear stability analysis, infinitesimal disturbances are imposed on the base flow. Thus the velocity, pressure and temperature fields can be written as

$$(u, v, w, \theta, p) = \left(\tilde{u}, \tilde{v}, W_0(r) + \tilde{w}, \Theta_0(r) + \tilde{\theta}, P_0(z) + \tilde{p}\right) \quad (14)$$

where the tilde quantities denote the infinitesimal disturbances to the corresponding term. The linear disturbances equations can be obtained by using the relation (14) in equations (1) -(5) and neglecting the small non-linear terms. They are

continuity:

$$\frac{\partial \tilde{u}}{\partial r} + \frac{\tilde{u}}{r} + \frac{1}{r}\frac{\partial \tilde{v}}{\partial \psi} + \frac{\partial \tilde{w}}{\partial z} = 0 \quad (15)$$

r-momentum:

$$\left[\frac{1}{\epsilon}\frac{\partial \tilde{u}}{\partial t} + \frac{W_0}{\epsilon^2}\frac{\partial \tilde{u}}{\partial z}\right] \quad (16)$$

$$= -\frac{\partial \tilde{p}}{\partial r} + \frac{1}{Re}\left[D^2\tilde{u} - \frac{2}{r^2}\frac{\partial \tilde{v}}{\partial \psi} - \frac{\tilde{u}}{r^2}\right] - \frac{\tilde{u}}{ReDa} \quad (17)$$

ψ-momentum:

$$\left[\frac{1}{\epsilon}\frac{\partial \tilde{v}}{\partial t} + \frac{W_0}{\epsilon^2}\frac{\partial \tilde{v}}{\partial z}\right] \quad (18)$$

$$= -\frac{1}{r}\frac{\partial \tilde{p}}{\partial \psi} + \frac{1}{Re}\left[D^2\tilde{v} - \frac{\tilde{v}}{r^2} + \frac{2}{r}\frac{\partial \tilde{u}}{\partial \psi}\right] - \frac{\tilde{v}}{ReDa} \quad (19)$$

z-momentum:

$$\left[\frac{1}{\epsilon}\frac{\partial \tilde{w}}{\partial t} + \frac{1}{\epsilon^2}\left(\frac{dW_0}{dr}\tilde{u} + \frac{\partial \tilde{w}}{\partial z}\right)\right] \quad (20)$$

$$= -\frac{\partial \tilde{p}}{\partial z} + \frac{1}{Re}\left[D^2\tilde{w}\right] - \frac{\tilde{w}}{ReDa} - \frac{Ra}{Re}\tilde{\theta} \quad (21)$$

energy:

$$PrRe\left[\sigma\frac{\partial \tilde{\theta}}{\partial t} + \frac{d\Theta_0}{dr}\tilde{u} + W_0\frac{\partial \tilde{\theta}}{\partial z}\right] = \tilde{w} + \left[D^2\tilde{\theta}\right] \quad (22)$$

where

$$D^2 = \frac{d^2}{dr^2} + \frac{d}{dr} \quad (23)$$

Equations (15)-(22) can be reduced to a set of ordinary differential equations if the disturbance quantities are expressed in the normal-mode form such as

$$\left(\tilde{u}, \tilde{v}, \tilde{w}, \tilde{\theta}, \tilde{p}\right) = \left(\hat{u}(r),\ \hat{v}(r),\ \hat{w}(r),\ \hat{\theta}(r),\ \hat{p}(r)\right) e^{[i\alpha(z-ct)+n\psi]} \quad (24)$$

where, α is the wave number, n is integer azimuthal wave number and c = $\hat{c_r} + i\hat{c_i}$ is complex wave speed. The growth and decay of the disturbances depend on $\hat{c_i}$. Depending on whether $\hat{c_i} < 0$, $\hat{c_i}$ = 0 or $\hat{c_i} > 0$, three different possibilities stable, neutrally stable or unstable may be distinguished. The governing linear equations for the infinitesimal disturbances can be written as

$$\frac{d\hat{u}}{dr} + \frac{\hat{u}}{r} + \frac{in\hat{v}}{r} + i\alpha\hat{w} = 0 \quad (25)$$

$$\frac{d^2\hat{u}}{dr^2} + \frac{1}{r}\frac{d\hat{u}}{dr} - \left[\left(\frac{n^2+1}{r^2}\right) + \alpha^2 + \frac{1}{Da} + i\alpha Re\left(\frac{W_0}{\epsilon^2} - \frac{c}{\epsilon}\right)\right]\hat{u} - \frac{2in}{r^2}\hat{v} - Re\frac{d\hat{p}}{dr} = 0 \quad (26)$$

$$\frac{d^2\hat{v}}{dr^2} + \frac{1}{r}\frac{d\hat{v}}{dr} - \left[\left(\frac{n^2+1}{r^2}\right) + \alpha^2 + \frac{1}{Da} + i\alpha Re\left(\frac{W_0}{\epsilon^2} - \frac{c}{\epsilon}\right)\right]\hat{v} + \frac{2in}{r^2}\hat{u} - \frac{inRe}{r}\hat{p} = 0 \quad (27)$$

$$\frac{d^2\hat{w}}{dr^2} + \frac{1}{r}\frac{d\hat{w}}{dr} - \left[\frac{n^2}{r^2} + \alpha^2 + \frac{1}{Da} + i\alpha Re\left(\frac{W_0}{\epsilon^2} - \frac{c}{\epsilon}\right)\right]\hat{w} + \frac{1}{\epsilon^2}\frac{dW_0}{dr}\hat{u} - Ra\hat{\theta} - i\alpha Re\hat{p} = 0 \quad (28)$$

$$\frac{d^2\hat{\theta}}{dr^2} + \frac{1}{r}\frac{d\hat{\theta}}{dr} - \left[\frac{n^2}{r^2} + \alpha^2 + i\alpha RePr\left(W_0 - \sigma c\right)\right]\hat{\theta} + \hat{w} - RePr\frac{d\Theta_0}{dr}\hat{u} = 0 \quad (29)$$

The required boundary conditions accompanying the above equations (25)-(29) are specified at the wall and center. At the wall all disturbances must vanish. This implies

$$\hat{u} = \hat{v} = \hat{w} = \hat{\theta} = \frac{d\hat{p}}{dr} = 0, \quad \text{at} \quad r = 1 \quad (30)$$

At the center of the pipe, the boundary conditions are as follows [18]:

$$\begin{cases} \hat{u} = \hat{v} = \frac{d\hat{w}}{dr} = \frac{d\hat{\theta}}{dr} = \frac{d\hat{p}}{dr} = 0, & r = 0\,, \quad n = 0, \\ \hat{u} + i\hat{v} = 2\frac{d\hat{u}}{dr} + \frac{d\hat{v}}{dr} = \hat{w} = \hat{\theta} = \hat{p} = 0, & r = 0\,, \quad n = 1, \\ \hat{u} = \hat{v} = \hat{w} = \hat{\theta} = \hat{p} = 0, & r = 0\,, \quad n \geq 2. \end{cases} \quad (31)$$

C. Method of Solution and Validation of Code

In order to solve the coupled differential equations (25)–(29) along with the boundary conditions (30) and (31), the Spectral collocation method has been used ([18], [19], [20]). Verification of the code is given in several ways. First, the independence of the solution is examined for variation of order of base polynomial (N). Secondly, it is checked whether or not the total rate of change of kinetic and thermal energy at critical points are zero. Finally, some comparisons are made with published results with our data for specific cases ([15], [16]).

II. Results and Discussion

In the study presented here, the linear theory of stability analysis has been used to answer the question of the condition for the occurrence of the maximum stability of different fluids in the vertical pipe filled with porous medium. The investigation is concentrated only the buoyancy opposed cases. For all numerical results presented here porosity ϵ and heat capacity ratio σ, always take the values 0.9 and 1, respectively. Three different fluids: air, water and heavy oil are mainly considered to simulate the stability characteristics. To give a thorough study in the highly porous media, four different Darcy numbers, i.e., 10^{-1}, 10^{-2}, 10^{-3} and 10^{-4} are used. The selection of the lower limit of Da is based on numerical experiments only. To characterize the effect of azimuthal number on the least stable mode of the buoyancy opposed flow figures 2, 3 and **??** are plotted as a function of Re for air, water and heavy oil respectively. As can be seen from above figures, first, the first azimuthal mode is always least stable mode. Second, effect of azimuthal numbers die out on decreasing permeability of the media. Third, the instability boundary curves in zero azimuthal mode for Da equal to 10^{-1} and 10^{-2} show anticipated results for air and water. Regarding the influence of media permeability on base flow stability, in general it can be seen from the above figures that, similar to assisted flow, stability of the flow increases on decreasing of Da. Since permeability increase implies allowing more flow into the system (equivalent to strengthen the convective flow), consequently, a slight increase of Re enhances the convection in the main flow direction and more fluid can be transported upwards to destabilize the flow shown by fall of critical heating condition. It is expected that enhancement of media permeability reduces flow stability. In order to check the existence of anticipated result at Da = 10^{-2}, the table

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TABLE I
CRITICAL RAYLEIGH NUMBER ($|Ra|$) FOR OPPOSED FLOW AT Re = 500 AND n = 1.

Da	Pr = 0.7	Pr = 7.0	Pr = 70
10^{-1}	221.15286	221.58543	221.20206
10^{-2}	1351.40322	1354.93697	928.41921
10^{-3}	15204.40642	13212.27782	7601.62278
10^{-4}	142972.18284	124748.38083	90369.59374

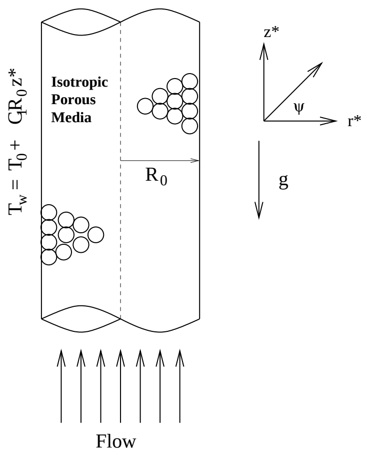

Fig. 1. Physical model and co-ordinate system.

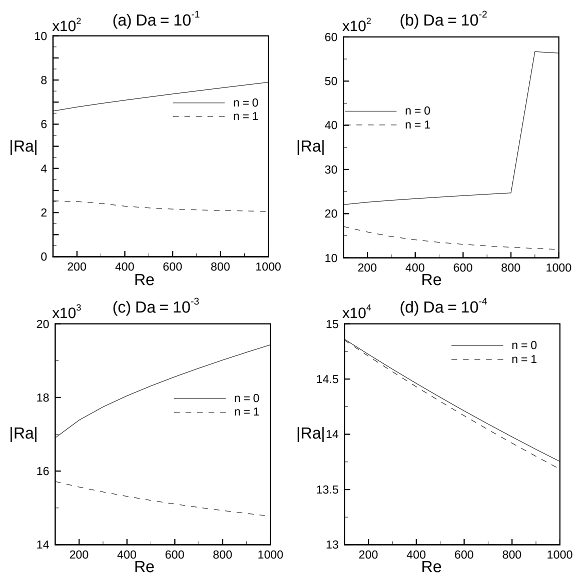

Fig. 2. The stability boundary for (a) Da = 10^{-1}, (b) Da = 10^{-2}, (c) Da = 10^{-3} and (d) Da = 10^{-4} at and Pr= 0.7.

of critical Ra is made at Re equal to 500, Pr = 70. It can be seen from the table I that the results are as usual expected one. From the same table it can be also observed that for Da= 10^{-1} critical value of Ra is almost independent of fluid i.e. Pr, which is not usual.

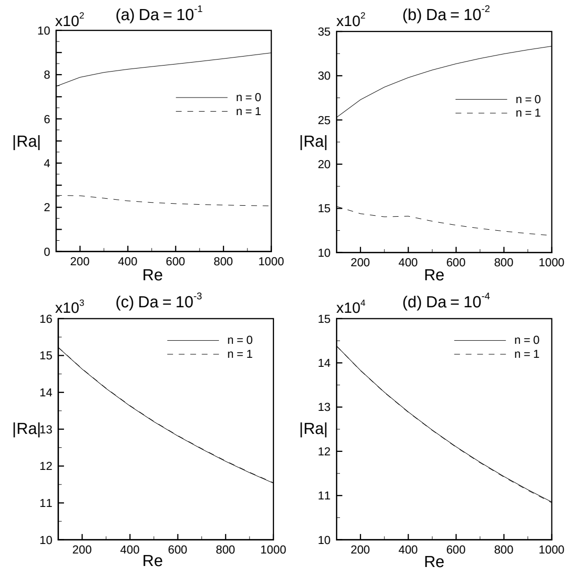

Fig. 3. The stability boundary for (a) Da = 10^{-1}, (b) Da = 10^{-2}, (c) Da = 10^{-3} and (d) Da = 10^{-4} at and Pr=7.

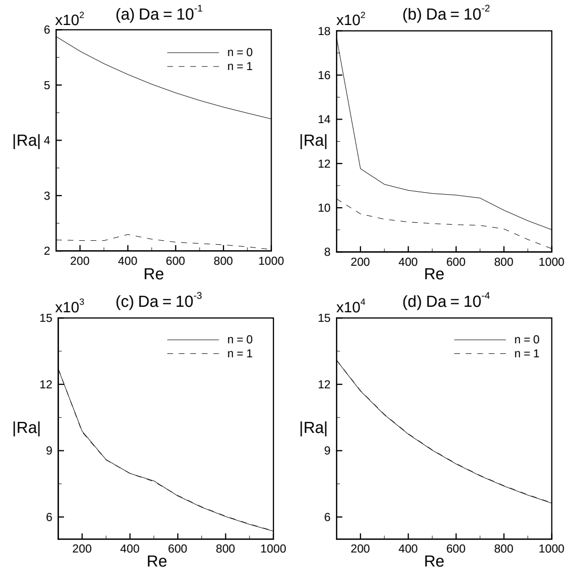

Fig. 4. The stability boundary for (a) Da = 10^{-1}, (b) Da = 10^{-2}, (c) Da = 10^{-3} and (d) Da = 10^{-4} at and Pr=70.

A. Conclusion

We have attempted to gain an understanding of instability of pressure gradient driven buoyancy opposed poiseuille flow in a vertical pipe filled with fluid-saturated porous medium. To this end, we adopted Brinkman-Wooding-extended Darcy model. By means of linear theory, we were able to extract detailed information of transition of basic flow through a porous medium for different fluids. The Spectral Collocation Method is used to solve the set of linear ordinary differential equations. The main objective in this study was to investigate the effect of permeability as well as Prandtl number on the stability of base flow. The following conclusions can be drawn from this study.

- The first azimuthal mode is always least stable mode.
- Increasing the the media permeability, reduces the stability of the basic flow.
- The effect of azimuthal numbers die out on decreasing permeability of the media.
- The instability boundary curves in zero azimuthal mode for Da equal to 10^{-1} and 10^{-2} show anticipated results for air and water.

- The stability analysis indicates that for the same Reynolds number (Re), the fully developed base flow is highly unstable for fluid of high Prandtl number.
- In contrast to a pure viscous fluid, where the effect of Pr is not significant, in isotropic porous medium Prandtl number takes a significant role in characterizing the flow stability in buoyancy opposed case.

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