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3-D Visualization and Optimization of Input-Output Relation for Linear Systems Using Parametrization of Two-Stage Compensator Design

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***Abstract*—In this paper, we consider the two-stage compensator designs. As an investigation of the characteristics of the two-stage compensator designs, which is not well investigated yet, we implement three dimensional visualization systems of input-output relation and optimization system of the parametrization of stabilizing controllers based on the two-stage compensator design.**

***Index Terms*—Linear systems, Feedback stabilization, Visualization, Two-Stage Compensator Design, Mathematica**

I. Introduction

In this paper, we consider the two-stage compensator designs in the framework of the factorization approach. In the design, during the first stage, a new closed loop system selects stabilizing compensator for the plant. In the second stage, a stabilizing controller is selected for the new closed-loop system that also achieves some other design objectives such as decoupling and sensitive minimization.

Recently, we have given a parametrization of stabilizing controllers of the two stage compensator design based only on the factorization approach, which is in the form of the Youla-Kučera-parametrization[1], [12], [13], [14].

The factorization approach to control systems has the advantage that embraces, within a single framework, numerous linear systems such as continuous-time as well as discrete-time systems, lumped as well as distributed systems, one-dimensional as well as multidimensional systems, etc[1], [2], [3]. Hence the result given in this paper will be able to a number of models in addition to the multidimensional systems. In the factorization approach, when problems such as feedback stabilization are studied, one can focus on the key aspects of the problem under study rather than be distracted by the special features of a particular class of linear systems. This approach leads to conceptually simple and computationally tractable solutions to many important and interesting problems[4]. A transfer matrix of this approach is considered as the ratio of two stable causal transfer matrices.

In some design problems, one uses a so-called *two-stage compensator design* for selecting an appropriate stabilizing compensator. One of examples of two-stage compensator design is earthquake-resistant dumpers for a building shown in Figures 1. Another example of two-stage compensator design is earthquake-resistant dumpers for a bridge shown in Figures 1. By attaching resistant dumpers to these building and bridge, these building and bridge become strong against earthquake.

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Fig. 1. Earthquake-Resistant Dumpers for building.

Fig. 2. Earthquake-Resistant Dumpers for bridge.

The problem of the two-stage compensator design is that the relationship between inputs and outputs are not theoretically clarified yet. Thus, we consider to make a software to present the relationship. Thus, the objective of this paper is to make systems to visualize the input-output relationship based on the two-stage compensator design. The systems are implemented on Mathematica[9], one of the most common computer algebra systems. By using visualization technique and Golden Section Method[15], we also consider the optimization of the system based on the two-stage compensator design.

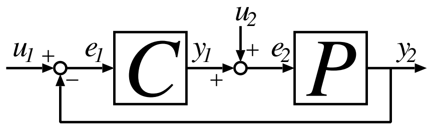

Fig. 3. Feedback system Σ.

To achieve this, we have implemented visualization systems of the parametrization of stabilizing controllers based on the two-stage compensator designs [16], [17], [18] and also implemented system, which present norms of output signals and optimize the system based on the two-stage compensator design. We call these system Visualization system and Optimization system, respectively.

In Visualization system, output signals can be visualized as 3D graphs. Because we use Mathematica, we can overlook output signal with all parameter by using some implemented functions of 3D graph system such as we can rotate 3D graph by dragging the mouse inside the graphic. In Optimization systems, norms of output signals can be visualized 3D graphs and minimum norm of output signals can be found by Golden Section Method[15].

In this paper, we consider the SISO and MIMO discrete-time LTI systems as a model of the factorization approach.

II. Preliminary

The stabilization problem considered in this paper follows the papers [6], [7], in which the feedback system Σ [4] is as in Figure 3. For further details the reader is referred to the literatures[4], [6], [7], and [8].

We consider that the set of stable causal transfer functions is an integral domain, denoted by $\mathcal{A}$. The total ring of fractions of $\mathcal{A}$ is denoted by $\mathcal{F}$; that is, $\mathcal{F} = \{n/d \,|\, n,\ d \in \mathcal{A},\ d \neq 0\}$. This $\mathcal{F}$ is considered as the set of all possible transfer functions, which is given as ratio of two stable causal transfer functions. Matrices over $\mathcal{F}$ are transfer matrices. Let $\mathcal{Z}$ be a prime ideal of $\mathcal{A}$ with $\mathcal{Z} \neq \mathcal{A}$. Define the subsets $\mathcal{P}$ and $\mathcal{P}_{\rm s}$ of $\mathcal{F}$ as follows:

$$\begin{aligned} \mathcal{P} &= \{a/b \in \mathcal{F} \,|\, a \in \mathcal{A},\ b \in \mathcal{A}\backslash\mathcal{Z}\}, \\ \mathcal{P}_{\rm s} &= \{a/b \in \mathcal{F} \,|\, a \in \mathcal{Z},\ b \in \mathcal{A}\backslash\mathcal{Z}\}. \end{aligned}$$

Then, every transfer function in $\mathcal{P}$ ($\mathcal{P}_{\rm s}$) is called *causal* (*strictly causal*). Analogously, if every entry of a transfer matrix is in $\mathcal{P}$ ($\mathcal{P}_{\rm s}$), the transfer matrix is called *causal* (*strictly causal*).

In this paper, we consider the discrete-time LTI system, then

$$\begin{aligned} \mathcal{A} &= \{\frac{u}{v} \,|\, u,\ v \in \mathbb{R}[d],\ \text{all roots } r \text{ of } v \text{ are with } |r| > 1\}, \\ \mathcal{Z} &= (d), \end{aligned}$$

d is the unit delay operator.

Throughout the paper, the plant can be either SISO or MIMO, and its transfer function, which is also called a *plant* itself simply, is denoted by P and belongs to $\mathcal{P}^{n\times m}$, which means that the plant has m inputs and n outputs. We can always represent P in the form of a fraction $P = ND^{-1}$, where $N \in \mathcal{A}^{n\times m}$ and $D \in \mathcal{A}^{m\times m}$ with nonsingular.

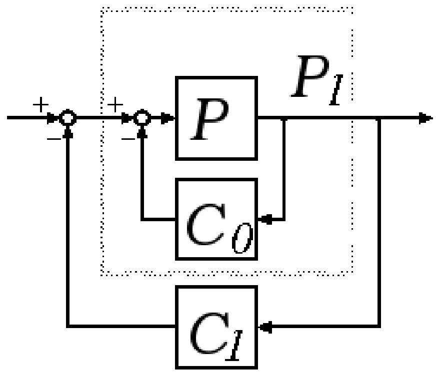

Fig. 4. Two-Stage Compensator Design (y_2 to u_2).

For $P \in \mathcal{F}^{n\times m}$ and $C \in \mathcal{F}^{m\times n}$, a matrix $H(P,C) \in \mathcal{F}^{(m+n)\times(m+n)}$ is defined as

$$H(P,C) := \left[\begin{array}{cc} (I_n + PC)^{-1} & -P(I_m + PC)^{-1} \\ C(I_n + PC)^{-1} & (I_m + PC)^{-1} \end{array} \right] \tag{1}$$

provided that $I_n + PC$ is a nonzero of $\mathcal{A}$. This $H(P,C)$ is the transfer matrix from $\left[\begin{array}{cc} u_1^t & u_2^t \end{array} \right]^t$ to $\left[\begin{array}{cc} e_1^t & e_2^t \end{array} \right]^t$ of the feedback system Σ. If $I_n + PC$ is a nonzero of $\mathcal{A}$ and $H(P,C) \in \mathcal{A}^{(m+n)\times(m+n)}$, then we say that the plant P is *stabilizable*, P is *stabilized* by C, and C is a *stabilizing controller* of P. In the definition above, we do not mention the causality of the stabilizing controller. However, it is known that if a causal plant is stabilizable, there is always a causal stabilizing controller of the plant [7].

We will denote by $\mathcal{S}(P)$ the set of stabilizing controllers of P.

The following is well known Youla-Kučera-parametrization(Theorem 1)to provide the set of all stabilizing controllers.

Theorem 1: ([1], [12], [13], [14]) Let P denote a causal plant of $\mathcal{P}^{n\times m}$. Let $P = ND^{-1} = \tilde{D}^{-1}\tilde{N}$. Select $\tilde{X}$,$\tilde{Y}$,X and Y such that

$$\tilde{Y}N + \tilde{X}D = I_m, \quad \tilde{N}Y + \tilde{D}X = I_n. \tag{2}$$

Then the S(P) is given by

$$\begin{aligned} &\mathcal{S}(P) \\ &= \{(\tilde{X} - R\tilde{N})^{-1}(\tilde{Y} + R\tilde{D}) \,|\, R \in \mathcal{A}^{m\times n},\ |\tilde{X} - R\tilde{N}| \neq 0\} \\ &= \{(Y + RD)(X - NR)^{-1} \,|\, R \in \mathcal{A}^{m\times n},\ |X - NR| \neq 0\}, \end{aligned}$$

where R is a parameter matrix.

III. Two-Stage Compensator Design

The two-stage compensator design is for selecting an appropriate stabilizing compensator[4]. Given a plant P, the first stage consists of selecting a stabilizing compensator for P. Let $C_0 \in \mathcal{S}(P)$ denote a compensator of P (that is, an arbitrary but fixed compensator of P) and define $P_1 = P(I_m + C_0P)^{-1}$. The second stage consists of selecting a stabilizing controller for P_1 that also achieves some other design objectives such as decoupling, sensitivity minimization, etc. The resulting configuration with its inner and outer loops is shown in Figure 4.

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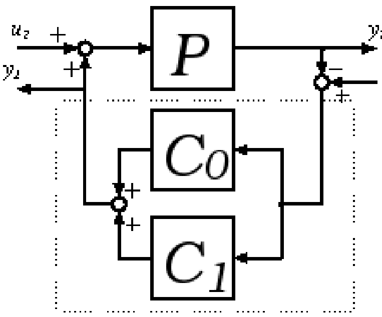

Fig. 5. Composite Stabilized Feedback with c_0 and c_1.

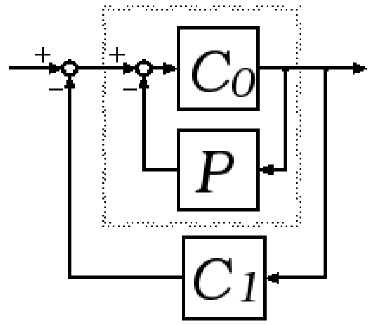

Fig. 6. Feedback from y_1 to u_1.

Theorem 2 is same as Theorem 5.3.10 of [4]. Theorem 3 is a generalized version of Theorem 2 with coprime factorizability. We will employ following theorems and corollary to achieve two stage compensator design.

Theorem 2: ([5]) Let P denote a causal plant of $\mathcal{P}^{m\times n}$ and C_0 a causal stabilizing controller of P ($C_0 \in \mathcal{P}^{m\times n}$). Further let P_1 be $P(I_m + C_0P)^{-1}$. Denote by $C_0 + \mathcal{S}(P_1)$ the following set:

$$\{C_0 + C_1 \,|\, C_1 \in \mathcal{S}(P_1)\}.$$

Then

$$C_0 + \mathcal{S}(P_1) \subset \mathcal{S}(P), \tag{3}$$

with equality holding if and only if $C_0 \in \mathcal{A}^{m\times n}$.

Figure 4 cannot implement all controllers as the stabilizing controllers in general.

Theorem 3: ([5]) Let P, C_0, P_1 be as in Theorem 2.

Let N, D, $\tilde{N}$, $\tilde{D}$, Y, X, $\tilde{Y}$, $\tilde{X}$ be matrices over A such that

$$\begin{cases} P = ND^{-1} = \tilde{D}^{-1}\tilde{N}, \quad C_0 = YX^{-1} = \tilde{X}^{-1}\tilde{Y}, \\ \tilde{Y}N + \tilde{X}D = I_m, \quad \tilde{N}Y + \tilde{D}X = I_n. \end{cases}$$

Then,

$$\begin{aligned} & C_0 + \mathcal{S}(P_1) \\ &= \{(\tilde{X} - R\tilde{N})^{-1}(\tilde{Y} + R\tilde{D}) \,|\, R = \tilde{X}R_1X, R_1 \in \mathcal{A}^{m\times n}\} \\ &= \{(Y + RD)(X - DR)^{-1} \,|\, R = \tilde{X}R_1X, R_1 \in \mathcal{A}^{m\times n}\}. \end{aligned}$$

We can obtain Theorem 3 by replacing parameter R of Theorem 1 with $\tilde{X}R_1X$.

By Theorem 2, we see that the sum of C_0 and a stabilizing controller of P_1, say C_1, is again a stabilizing controller of P. This sum, a stabilizing controller of P, is the parallel allocation of C_0 and C_1, as shown in Figure 5.

The theorems were based on the feedback from y_2 to u_2 (cf. Figures 3 and 4). Even so, we note that, from Figure 3, we have two inputs u_1 and u_2 and two outputs y_1 and y_2. Thus we can consider alternative two-stage compensator design based on other input(s) and other output(s). Let us consider the two-stage compensator design based on the feedback from y_1 to u_1. In this case, the feedback system is as in Figure 6.

The configuration is as in Figure 7.

Based on this feedback, the following result has also been given in [5].

Corollary 1: ([5]) Let P,C_0,P_1 be as in Theorem 2. Let N,D,$\tilde{N}$,$\tilde{D}$,Y,X,$\tilde{Y}$,$\tilde{X}$ be as in Theorem 3.

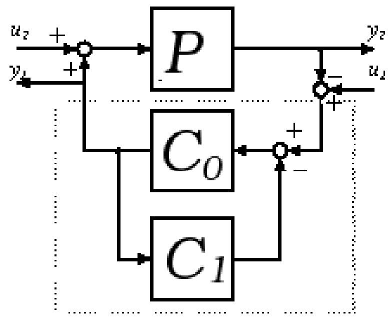

Fig. 7. Composite Stabilized Feedback with c_0 and c_1 based on Feedback from y_1 to u_1.

Then we have,

$$\begin{aligned} & C_0 + \mathcal{S}(P_1) \\ = \; & \{(\tilde{X} - R\tilde{N})^{-1}(\tilde{Y} + R\tilde{D}) \,| \\ & \quad R = -\tilde{Y}R_2Y, R_2 \in \mathcal{A}^{n\times m}, |\tilde{X} - R\tilde{N}| \neq 0\} \\ = \; & \{(Y + RD)(X - NR)^{-1} \,| \\ & \quad R = -\tilde{Y}R_2Y, R_2 \in \mathcal{A}^{n\times m}, |X - NR| \neq 0\}. \end{aligned}$$

We can obtain Corollary 1 by replacing parameter R of Theorem 1 with $\tilde{Y}R_2Y$ where R_2 of Corollary 1 is equal to R of Theorem 1.

IV. Demonstration

Due to the space limitation, we present here the SISO plant case only. As an example, we consider P as follows:

$$P = \frac{d^2 + 1}{d^2 - \frac{1}{2}d + \frac{1}{4}},$$

and the inputs u_1 and u_2 be 0 and 1, respectively, where d denotes the delay operator. In this case, the coprime factorization is given as

$$N = \widetilde{N} = \frac{16}{25}(d^2 + 1), \; D = \widetilde{D} = \frac{4}{25}(2d + 1)^2,$$
$$Y = \widetilde{Y} = \frac{7}{4} + d, \; X = \widetilde{X} = -\frac{3}{4} - d.$$

We consider two constants a and b and the form $a + bd$ as a parameter.

Let $-20 \leq a, b \leq 20$, $R_1 = a + bd$, $u_1 = 0$, and $u_2 = 1$. Then outputs y_1 and y_2 based on Theorem 3 are visualized as Figures 8 and 9, respectively. Also outputs y_1 and y_2 based on Corollary 1 are visualized as Figures 10 and 11, respectively.

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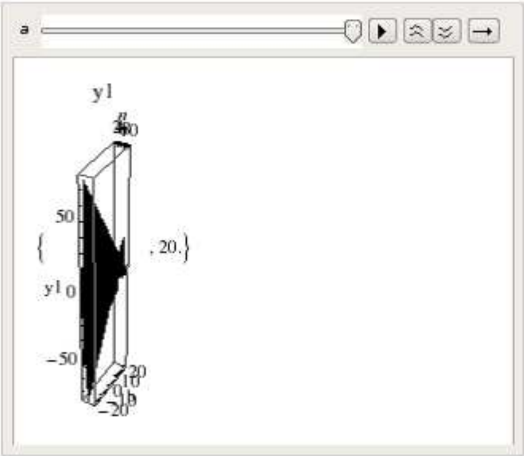

Fig. 8. 3D graph animation of output signal y_1 based on Theorem 3. R_1 form is $a + bd$. a and b range are $-20 \le a, b \le 20$.

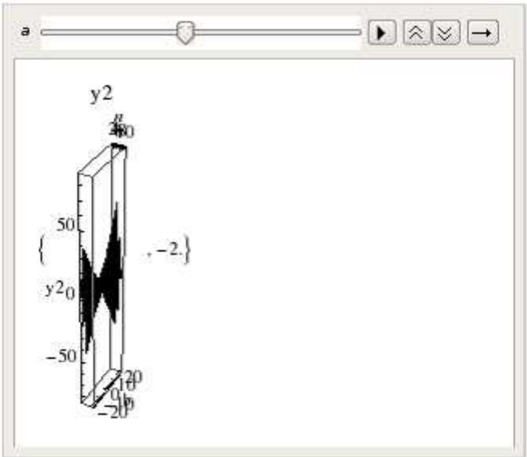

Fig. 9. 3D graph animation of output signal y_2 based on Theorem 3. R_1 form is $a + bd$. a and b range are $-20 \le a, b \le 20$.

In these visualization, the constant a is implemented by Slider Function of Mathematica. On the other hand, the constant b is one of three axes. Thus the figure changed by time (the value of the constant a), so that they are animations on Mathematica.

Next, we consider l_2-norms of signals. Again let $-20 \le a, b \le 20$, $R_1 = a + bd$, $u_1 = 0$, and $u_2 = 1$. Then the norms of y_1 and y_2 based on Theorem 3 are visualized as Figures 12 and 13, respectively. Also the norms of y_1 and y_2 based on Corollary 1 are visualized as Figures 14 and 15, respectively.

Minimum norms based on Golden Section Method[15] are shown in Table I.

Figure	Minimum norm
Figure 12	1.60389
Figure 13	1.29109
Figure 14	1.94602
Figure 15	1.56893

TABLE I
MINIMUM NORM OF FIGURES 12 TO 15.

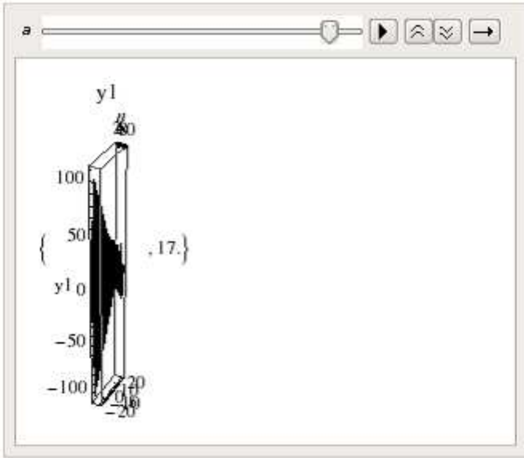

Fig. 10. 3D graph animation of output signal y_1 based on Corollary 1. R_2 form is $a + bd$. a and b range are $-20 \le a, b \le 20$.

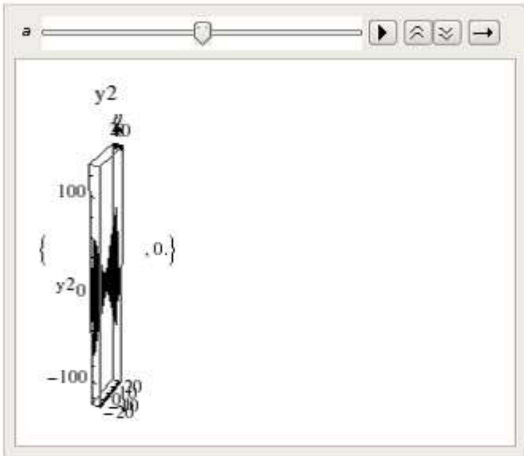

Fig. 11. 3D graph animation of output signal y_2 based on Corollary 1. R_2 form is $a + bd$. a and b range are $-20 \le a, b \le 20$.

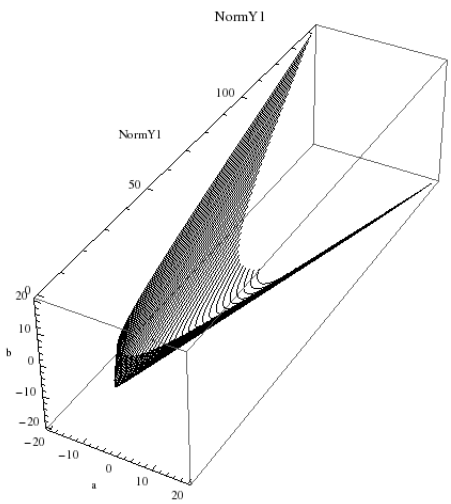

Fig. 12. Norms of output signal y_1 based on Theorem 3. R_1 form is $a+bd$. a and b range are $-20 \le a, b \le 20$.

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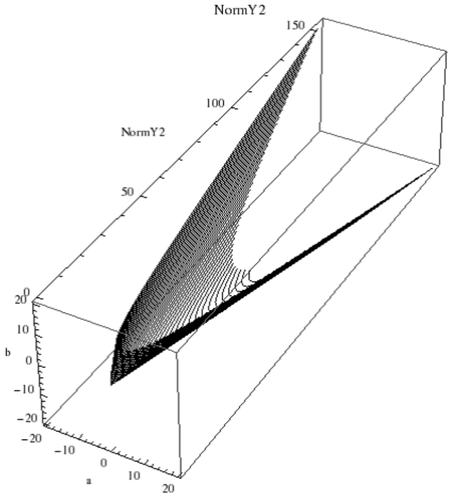

Fig. 13. Norms of output signal y_2 based on Theorem 3. R_1 form is $a+bd$. a and b range are $-20 \leq a, b \leq 20$.

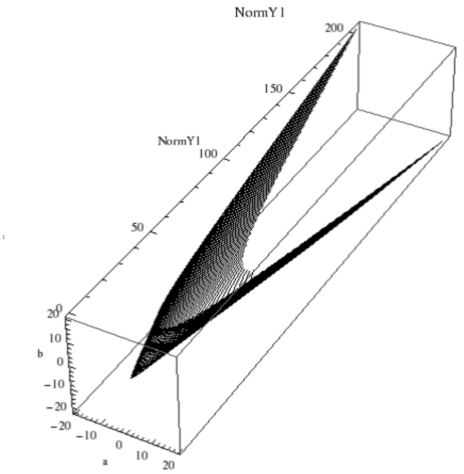

Fig. 14. Norms of output signal y_1 based on Corollary 1. R_2 form is $a + bd$. a and b range are $-20 \leq a, b \leq 20$.

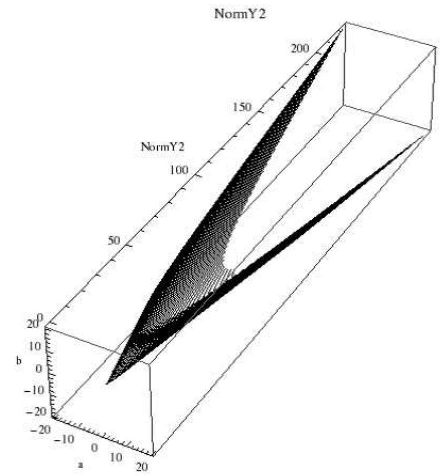

Fig. 15. Norms of output signal y_2 based on Corollary 1. R_2 form is $a + bd$. a and b range are $-20 \leq a, b \leq 20$.

V. Conclusion and Future Works

In this paper, we have visualized the input-output relation for discrete-time LTI systems using parametrization of two-stage compensator design. We also visualize the norms of the outputs and obtained the optimization by using the golden section method.

We consider that the optimization by golden section method is to obtain the minimal or maximal values numerically, which is not theoretical. We will investigate the method to obtain the optimal values by theoretical method.

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