Irreversibility Analysis of a Radiative MHD Poiseuille Flow through Porous Medium with Slip Condition

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***Abstract*—In this article, irreversibility analysis of thermal radiation with slip condition on MHD Poiseuille flow through porous medium is investigated. The upper and lower walls are kept constant with the same temperature. The radiative heat flux in the energy equation is assumed to follow Roseland approximation. Semi-analytical solutions of the non-linear boundary value problems obtained from the governing equations is constructed using Adomian decomposition method, and the effects of some fluid parameters on fluid motion, temperature, entropy generation and Bejan number are presented.**

***Index Terms*— Irreversibility, radiation, MHD, Poiseuille flow, slip condition**

I. Introduction

Recent researches reveal that more attention has been devoted to the preservation of scarce resources. This has led to the investigation of the causes of irreversibility in various flow systems; some of these are found in Refs. [1-5]. In addition, Arikoglu [6] submitted that, all energy producing, converting and consuming systems must be re-examined carefully and possible available-work destruction mechanisms be removed.

Available research works show that the effect of velocity slip on entropy generation of plane Poiseuille flow has not been fully addressed. Few investigations on this subject are [7-9]. Motivated by [8, 9], this article examines the entropy generation due to thermal radiation and velocity slip on MHD Poiseuille flow through porous medium.

Numerous semi-analytical methods for solving boundary value problems are found in literature, most of these techniques have difficulties in relation to the size of computational work and convergence. However the technique of Adomian Decomposition Method (ADM) [10-, 12] applied in this article is easy to apply with high accuracy and rapid convergence.

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II. Mathematical Formulation

The assumptions made include:
The flow is steady, electrically conducting and incompressible; the fluid is viscous and flow through parallel porous medium; both plates are fixed and maintained at uniform temperature; uniform transverse magnetic field B_0 is applied neglecting the induced magnetic field and the Hall effect; Navier slip boundary condition is assumed at the fluid-solid interface; the fluid is optically thick following Roseland approximation.
The governing equations are given as [8, 9]

$$\frac{d^2u'}{d\eta'^2}-\sigma\frac{B_0^2u'}{\rho}-\frac{bu'}{K}-\frac{dp}{d\eta} \tag{1}$$

$$k\frac{d^2T'}{d\eta'^2}+\mu\left(\frac{du'}{d\eta'}\right)^2+\sigma\frac{B_0^2u'^2}{\rho}+\frac{bu'^2}{K}+\frac{dq_r}{d\eta}=0 \tag{2}$$

$$E_G=\frac{k}{T_0^2}\left(\frac{dT'}{d\eta'}\right)+\frac{\mu}{T_0}\left(\frac{du'}{d\eta'}\right)^2+\frac{\sigma B_0^2u'^2}{T_0}+\frac{\mu u'^2}{T_0K} \tag{3}$$

$$u(0)=\psi_1\frac{du'(0)}{d\eta'},u(h)=\psi_2\frac{du'(0)}{d\eta'};\quad T(0)=T_0,T(h)=T_h \tag{4}$$

The Roseland approximation term for optimally thick fluid is written as

$$q_r=\frac{4\sigma^c}{3k^c}\frac{dT'^4}{d\eta^4} \tag{5}$$

The temperature term (T'^4) in equation (5) can be expressed in term of its linearity function as given by Raptis et al. [13], then the expansion in Taylor series about T_0 gives

$$T'^4=T'^4+4T_0'^3\left(T'-T_0'\right)+6T_0'^2\left(T'-T_0'\right)^2+4T_0'\left(T'-T_0'\right)^3+\left(T'-T_0'\right)^4 \tag{6}$$

Using equations (5) and (6) in equation (2) and neglecting higher order terms, we obtain

$$k\frac{d^2T'}{d\eta'^2}+\mu\left(\frac{du'}{d\eta'}\right)^2+\frac{\sigma B_0^2u'^2}{\rho}+\frac{\mu u'^2}{K}+\frac{16\sigma^c T_0^3 d^2T'}{3k^c d\eta'^2} \quad (7)$$

The dimensionless expressions for the present problem are:

$$\eta=\frac{\eta'}{h}, u=\frac{u'}{U}, \theta=\frac{T'-T_0}{T_f-T_0}, A=-\frac{h^2}{\mu}\frac{dp}{dx},$$
$$\Pr=\frac{\nu\rho c_p}{k}, Br=\frac{\mu U}{k(T_f-T_0)}, Ns=\frac{T_0^2h^2E_G}{k(T_f-T_0)^2},$$
$$\Omega=\frac{kT_f-T_0}{T_0}, H^2=\frac{\sigma B_0^2h^2}{\mu}, R=\frac{4\sigma^c T_0'^3}{kk^c}, \quad (8)$$
$$\alpha=\frac{h^2}{K}, \beta_1=\frac{\psi_1}{h}, \beta_2=\frac{\psi_2}{h}$$

Applying the above dimensionless variables in equations (1, 3, 4, 7) yields

$$\frac{d^2u}{d\eta^2}-H^2u-\alpha^2u+A=0 \quad (9)$$

$$\left(1+\frac{4}{3}R\right)\frac{d^2\theta}{d\eta^2}-\left(\frac{du}{d\eta}\right)^2-BrH^2u^2-Br\alpha^2u^2=0 \quad (10)$$

$$Ns=\left(1+\frac{4}{3}R\right)\frac{d^2\theta}{d\eta^2}-\frac{Br}{\Omega}\left\{\left(\frac{du}{d\eta}\right)^2-H^2u^2-\alpha^2u^2\right\} \quad (12)$$

$$u(0)=\beta_1\frac{du(0)}{d\eta}, u(1)=\beta_2\frac{du(0)}{d\eta}; \quad \theta(0)=0, \theta(1)=1 \quad (13)$$

Solving equations (9-10) by ADM yields the solution of the boundary value problems.

III. ENTROPY GENERATION

The dimensionless entropy generation expression in equation (11) provides four sources of irreversibility, that is equation (11) is of the form;

$HTI+TRI=\left(1+\frac{4}{3}R\right)\frac{d^2\theta}{d\eta^2}$ heat transfer and thermal radiation irreversibility;

$VDI=\frac{Br}{\Omega}\left(\frac{du}{d\eta}\right)^2$ viscous dissipation irreversibility; $MFI=\frac{BrH^2u^2}{\Omega}$ magnetic field irreversibility and

$PI=\frac{Br\alpha^2u^2}{\Omega}$ porosity irreversibility.

The Bejan number assumes values between 0 and 1. $Be=0$ for (VDI), $Be=1$ for (HTI) and $Be=0.5$ is when both VDI and HTI contribute equally to entropy generation. Then setting

$$Be=\frac{N_1}{Ns}=\frac{1}{1+\Phi}, \Phi=\frac{N_2}{N_1} \quad (14)$$

where

$$N_1=\left(1+\frac{4}{3}R\right)\frac{d^2\theta}{d\eta^2},$$
$$N_2=\frac{Br}{\Omega}\left(\left(\frac{du}{d\eta}\right)^2+H^2u^2+\alpha^2u^2\right) \quad (15)$$

IV. RESULTS AND DISCUSION

In this article, the effect of Navier slip and thermal radiation are investigated on the entropy generation of MHD Poiseuille flow through porous medium. The effects of some parameters on fluid velocity, temperature, entropy generation and Bejan number are presented in this section.

Figs. 1 and 2 depict the effect of slip parameters on fluid velocity. It is observed from Fig. 1 that fluid velocity increases with increase in lower wall slip parameter while the situation is reversed with upper wall slip parameter in Fig. 2. In Fig. 3, we present the effect of radiation parameter on the temperature. It is obvious that fluid temperature is lowered with increased values of radiation parameter. This is caused by the absorption of heat emitted by the absorptivity parameter. Figs. 4 and 5 reveal that fluid temperature is enhanced by increase in slip parameter.

Furthermore, Figs. 6 and 7 depict that entropy generation is retarded at the lower wall while it is enhanced at the upper wall. Also, Fig. 8 is the plot of thermal radiation effect on entropy generation. The Figure shows that entropy generation is significantly increased with increase in radiation parameter (R).

Finally, Figs. 9 and 10 show similar results. In the plots the Bejan number increases at the lower wall while there a reduction in the middle and upper walls of the channel. This is an indication that heat transfer irreversibility dominates entropy generation at the lower wall while viscous dissipation irreversibility is the major contributor to irreversibility at the upper wall. In Fig. 11 a rise in thermal radiation parameter leads to an increase in Bejan number across the channel. This shows that heat transfer irreversibility is the dominant contributor to entropy generation.

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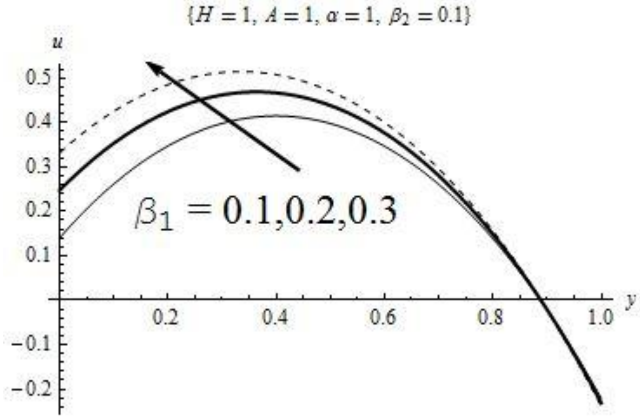

Fig 1: Velocity versus lower wall slip parameter

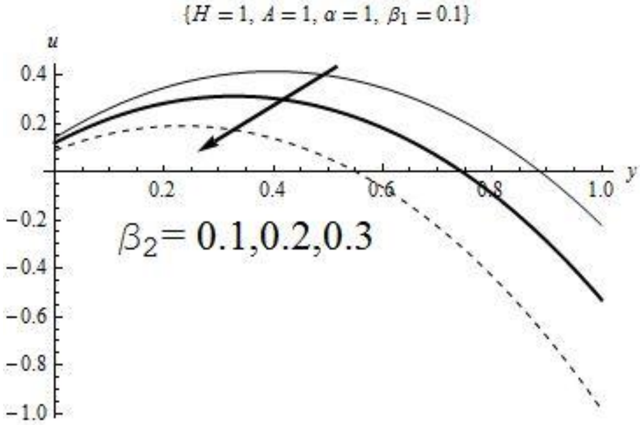

Fig 2: Velocity versus upper wall slip parameter

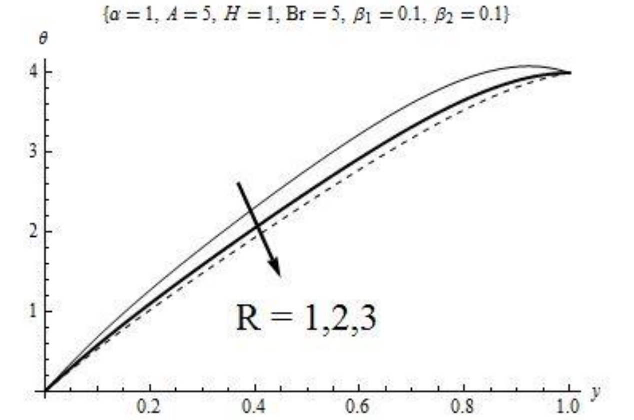

Fig 3: Temperature versus thermal radiation parameter

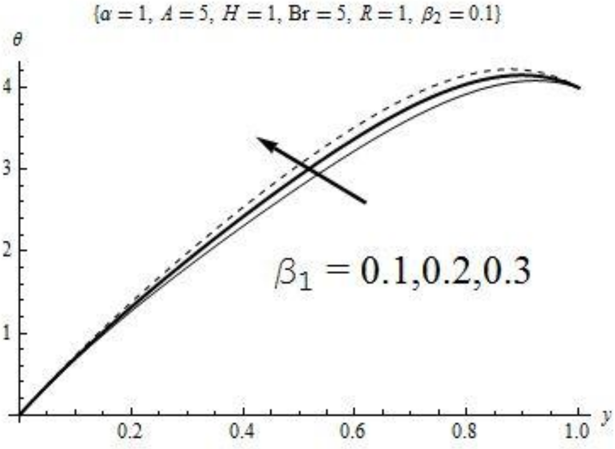

Fig 4: Temperature versus lower wall slip parameter

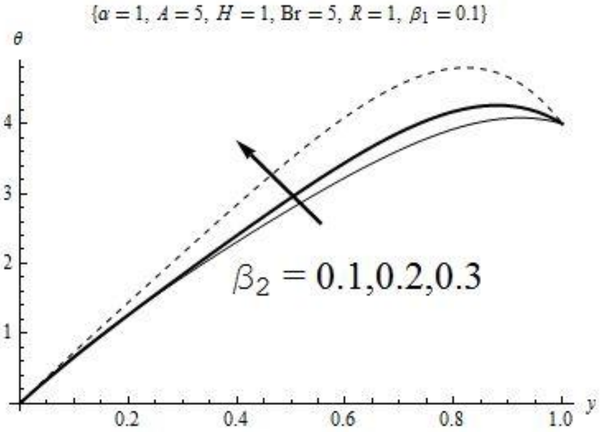

Fig 5: Temperature versus upper wall slip parameter

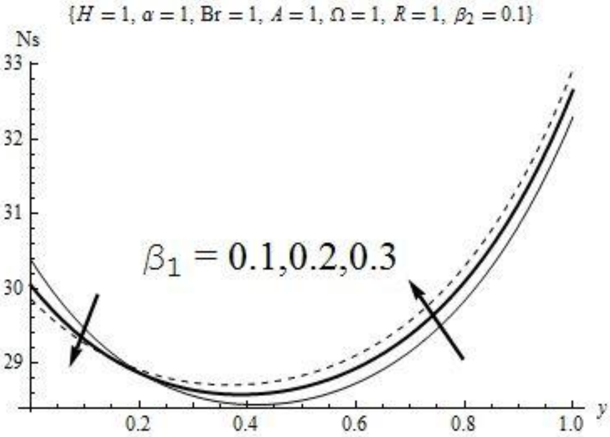

Fig 6: Entropy generation versus lower wall slip parameter

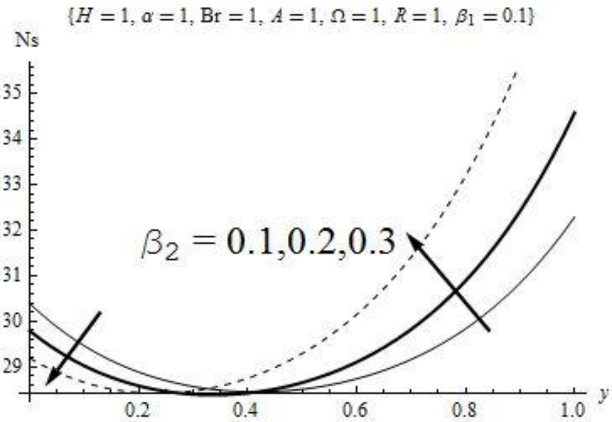

Fig 7: Entropy generation versus upper wall slip parameter

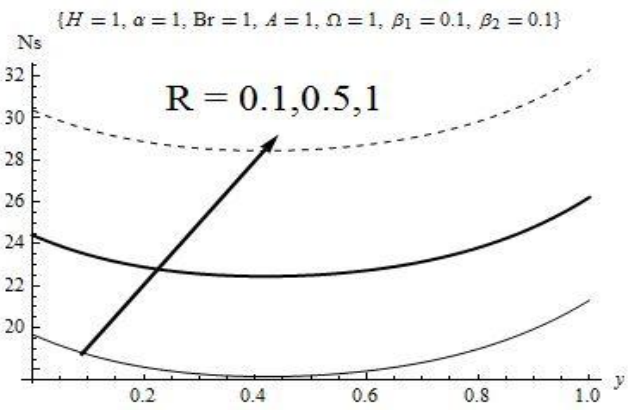

Fig 8: Entropy generation versus Thermal radiation parameter

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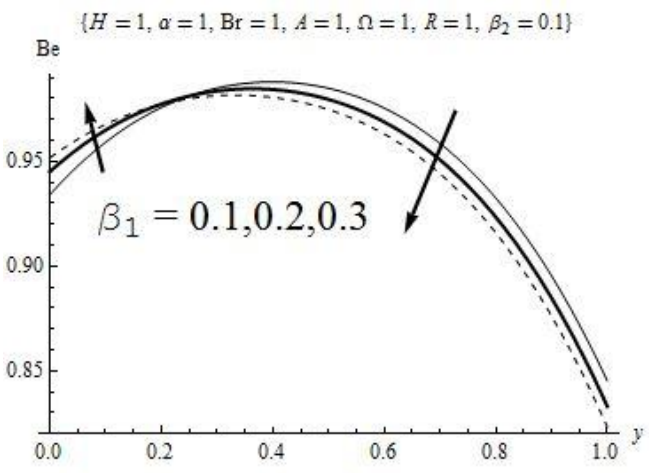

Fig 9: Bejan number versus lower wall parameter

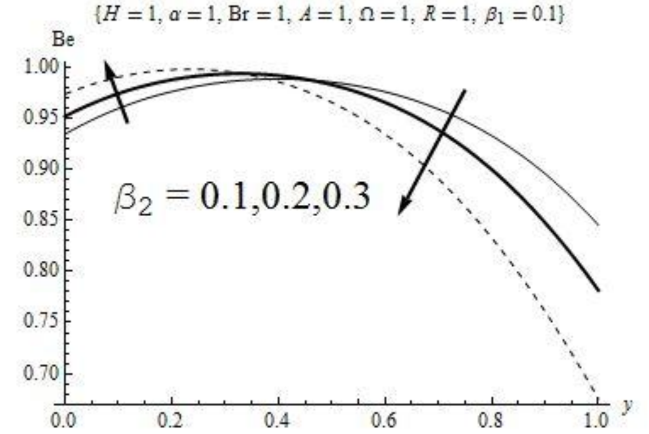

Fig 10: Bejan number versus upper wall parameter

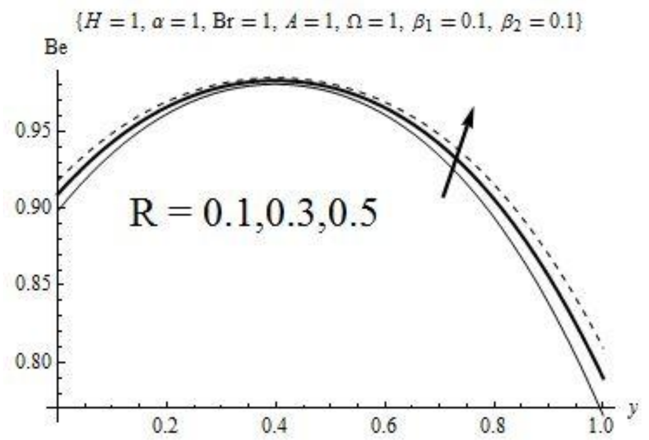

Fig 11: Bejan number versus thermal radiation parameter

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u'	axial velocity
μ	dynamic viscosity
p	fluid pressure
h	channel width
ρ	fluid density
T'	fluid temperature
T_0	initial fluid temperature
T_f	final fluid temperature
k	thermal conductivity of the fluid
c_p	specific heat at constant pressure
σ	electrical conductivity of the fluid Navier slip coefficients
B_0	uniform transverse magnetic field
q_r	radiative heat flux
u	dimensionless velocity
θ	dimensionless temperature
Pr	Prandtl number
Br	Brinkman number
Ω	parameter that measures the temperature difference between the two heat reservoirs
b	empirical constant in the second order (porous inertia resistance)

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H	magnetic field parameter
Be	Bejan number
A	axial pressure gradient
R	thermal radiation parameter
K	porous media permeability
$\beta_{1,2}$	Navier slip parameters respectively
α	porous media shape parameter
E_G	local volumetric entropy generation rate
Ns	dimensionless entropy generation rate
ν	is the kinematic viscosity
σ^c	Stefan-Boltzman constant
k^c	mean absorption coefficient for thermal radiation.

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