A Modified AT Coons Patch with Multiple Parameters

Jin Xie, Xiaoyan Liu, Zhi Liu and Le Zou

Abstract—**In this paper, a set of blending basis functions based on algebraic and trigonometric polynomials are presented, which are called AT Hermite bases. The AT Hermite bases inherit most properties from the standard cubic Hermite bases. A modified AT Coons patch with several parameters is constructed using the AT Hermite bases. Furthermore, sufficient conditions of G^1 continuity for connecting AT Coons patches are provided. When the interpolation conditions are given, the shape of the AT Coons patch can be adjusted by changing the values of the parameters. The examples are offered for accurate reconstruction of a torus or ellipsoid via the AT Coons patch with proper control points and parameters.**

Index Terms—**AT Hermite bases, Coons patch, parametric method, smooth connection, torus, ellipsoid.**

I. Introduction

In 1964, Coons presented a method of curve and surface design, namely Coons surface method. The theory of the method is rigorous, and the technique is applicable by means of the parametric method and subdivision technology. It has broad implementations in the surface design for airplanes, ships, and automobiles. For more applications of the Coons method to other manufacturing industries, please refer to Sobczyk[1], Rogers[2], Farin[3], Farin et al. [4] and Gindis[5]. However, Coons' method is not perfect, specifically in the process of surface design; once the boundary curve is set, it is difficult to adjust the shape of the Coons patch, which can be amended only by changing torsional vectors.

In order to overcome the shortcomings of Coons method, non-parametric methods to generate Coons patches had been investigated by many authors [6-10]. Farouki et al. [6] considered the method of constructing a C^2-Coons patch depending on the given boundary curves satisfying two types of consistency constraints. Farouki, Szafran and Biard [7] constructed the triangular Coons patch by means of a cubically-blended interpolation scheme. Worsey [8] presented an efficient transfinite interpolation scheme over rectangles to generate a modified C^2-Coons patch, demonstrating how it can be modified and simplified. Techniques for minimizing the energy of the surface by using third order non parametric patches were presented, and the problem of allocating the patches in the surface was discussed [9]. Farin [10] discussed a new method based on a blend of variational principles to produce "good" shapes for all boundary curve geometries.

In recent years, new methods by shape parameters to amend the Coons method have been proposed [11-18]. Zhang [11-13] investigated curves in the space spanned by {sin t, cos t, t, 1}, known as C-curves. C-curves include C-Hermite polynomials, C-Bezier curves and C-B-splines and so on. C-curves were applied to construct modified C-Coons patches in much of the literature [14-17]. Li [14] investigated Hermite basis function of fourth-order with shape parameters to improve the Hermite basis function method and supply a class of C-Coons patch with two shape parameters. Wang [15] constructed a bi-cubic C-Coons patch based on the analysis of the properties of C-Hermite polynomials, which can be applied to geometric modeling and image processing. A new-type of Coons patch called C-Coons patch constructed by C-Hermite blending functions was discussed [16], which is an extension of the bi-cubic Coons patch. Further, a class of mixed Coons patches with shape parameters was constructed [17]. These Coons patches not only kept the same advantage of bi-cubic Coons patches but could also adjust the shapes of surfaces by changing the values of parameters. Szilvási-Nagy and Szabó [18] presented two schemes for surface construction. In both cases the shape of the resulting Coons surface was influenced not only by boundary data but also by the shapes of the given surfaces and additional shape Parameters.

The methods mentioned above have a common characteristic: the Coons patches all have one shape parameter for each direction, so its ability to modify the shape of constructed surfaces is relatively weak.

In order to extend the applications of the Coons patch, in this paper, we present a class of AT Coons patch based on algebraic and trigonometric polynomials with two parameters for each direction. The shape of AT Coons patch can be modified by adjusting the parameters and therefore becomes more flexible and more suitable for surface designs.

The rest of this paper is organized as follows. In Section 2, we define AT Hermite basis functions and discuss their properties. A modified AT Coons surface patch with different parameters is provided in Section 3. Section 4 provides sufficient conditions of G^1 continuity for connecting two adjacent AT Coons patches. In Section 5, the

Manuscript received March 15, 2017; revised April 15, 2017. This work was funded by the Key Project Foundation of Scientific Research, Education Department of Anhui Province under Grant No.KJ2014ZD30. and was supported in part by the grant of Anhui Provincial Natural Science Foundation, Nos.1508085QF116.

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exact representations of both ellipsoid and torus by means of AT Coons surface patch are given. We conclude the paper by highlighting the benefits of the AT Coons patch in Section 6.

II. AT HERMITE BASIS FUNCTIONS AND THEIR PROPERTIES

Definition1□For $t \in [0,1]$, the following four functions

$$\begin{cases} F_0(t,\alpha)=\frac{2-\pi}{4-\pi}-\frac{2\alpha}{\pi}+\left(\frac{\pi}{4-\pi}-\alpha\right)t+\alpha t^2+\left(\frac{2\alpha}{\pi}-\frac{2}{4-\pi}\right)\sin\frac{\pi}{2}t+\left(\frac{2\alpha}{\pi}+\frac{2}{4-\pi}\right)\cos\frac{\pi}{2}t, \\ F_1(t,\alpha)=\frac{2\alpha}{\pi}+\frac{2}{4-\pi}-\left(\frac{\pi}{4-\pi}-\alpha\right)t-\alpha t^2-\left(\frac{2\alpha}{\pi}-\frac{2}{4-\pi}\right)\sin\frac{\pi}{2}t-\left(\frac{2\alpha}{\pi}+\frac{2}{4-\pi}\right)\cos\frac{\pi}{2}t, \\ G_0(t,\beta)=\frac{4}{\pi(4-\pi)}-\frac{2\beta}{\pi}+\left(\frac{2}{4-\pi}-\beta\right)t+\beta t^2+\left(\frac{4-2\pi}{\pi(4-\pi)}+\frac{2\beta}{\pi}\right)\sin\frac{\pi}{2}t+\left(\frac{4}{\pi(4-\pi)}+\frac{2\beta}{\pi}\right)\cos\frac{\pi}{2}t, \\ G_1(t,\beta)=\frac{4-2\pi}{\pi(4-\pi)}+\frac{2\beta}{\pi}+\left(\frac{2}{4-\pi}+\beta\right)t-\beta t^2-\left(\frac{4}{\pi(4-\pi)}+\frac{2\beta}{\pi}\right)\sin\frac{\pi}{2}t-\left(\frac{4-2\pi}{\pi(4-\pi)}+\frac{2\beta}{\pi}\right)\cos\frac{\pi}{2}t. \end{cases} \quad (1)$$

are defined as AT Hermite basis function , or EH bases.

The AT Hermite bases have similar properties to the cubic standard Hermite basis functions.

(1) $F_i(j,\alpha)=G_i'(j,\beta)=\delta_{ij}=\begin{cases}1, & i=j,\\ 0, & i\neq j.\end{cases}$

(2) $F_i'(j,\alpha)=G_i(j,\beta)=0, i,j=0,1$.

(3) $F_0(t,\alpha)+F_1(t,\alpha)=1$.

(4) $G_0(t,\beta)+G_1(1-t,\beta)=0$.

Naturally, the proposed AT Hermite bases (1) are extended forms of basis functions in [19] and [20]. More specifically, $\alpha=\beta=\frac{\lambda}{4-\pi}$, yields the same basis functions in [19]. Furthermore, the basis functions in [20] are the special cases of the bases (1) with $\alpha=\beta=0$.

Figure 1 exhibits four AT Hermite bases, where the value of parameter equals -3 for the dot-dash lines, and the value of parameter is equal to 3 for dashed lines, the solid lines are the standard Hermite bases.

III. THE MODIFIED AT COONS PATCH

A. The construction of the AT Coons patch

Defintion 2 Let u, $v\,(0\le u,v\le 1)$ be the two directions of a given surface, the α_1 and β_1 be parameters of u-direction, and the α_2 and β_2 be parameters of v-direction. Suppose four given corner points are $p(0,0)$, $p(0,1)$, $p(1,0)$ and $p(1,1)$, respectively; the tangent vectors along u-direction are $p_u(0,0)$, $p_u(0,1)$, $p_u(1,0)$ and $p_u(1,1)$, respectively, and the tangent vectors along v-direction are $p_v(0,0)$, $p_v(0,1)$, $p_v(1,0)$ and $p_v(1,1)$ respectively, and the twist vectors are $p_{uv}(0,0)$, $p_{uv}(0,1)$, $p_{uv}(1,0)$ and $p_{uv}(1,1)$, respectively. The following surface

$$p(u,v)=\left(F_0(u,\alpha_1),F_1(u,\alpha_1),G_0(u,\beta_1),G_1(u,\beta_1)\right)C\begin{pmatrix}F_0(v,\alpha_2)\\F_1(v,\alpha_2)\\G_0(v,\beta_2)\\G_1(v,\beta_2)\end{pmatrix}$$

(2)□

is labeled a modified algebraic-trigonometric Coons patch, briefly called AT Coons patch, where

$$C=\begin{bmatrix} p(0,0) & p(0,1) & p_v(0,0) & p_v(0,1)\\ p(1,0) & p(1,1) & p_v(1,0) & p_v(1,1)\\ p_u(0,0) & p_u(0,1) & p_{uv}(0,0) & p_{uv}(0,1)\\ p_u(1,0) & p_u(1,1) & p_{uv}(1,0) & p_{uv}(1,1)\end{bmatrix}$$

is the boundary information matrix.

If we rewrite the equation (2), we have

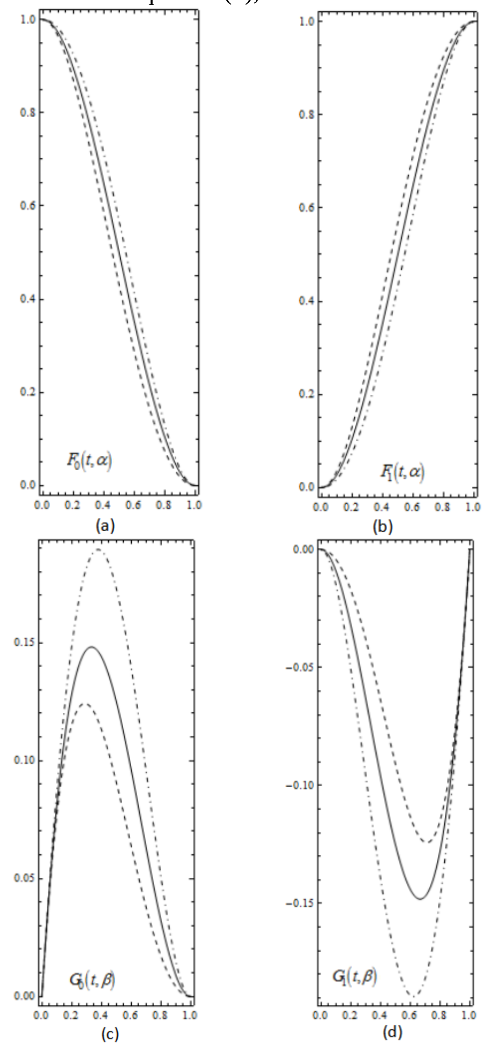


Fig.1 the graph of the four AT Hermite bases

$$p(u,v)=UMCM^TV^T,$$

where

$$U=\left[1 \quad u \quad u^2 \quad \sin\frac{\pi}{2}u \quad \cos\frac{\pi}{2}u\right](0\le u\le 1),$$

$$V=\left[1 \quad v \quad v^2 \quad \sin\frac{\pi}{2}v \quad \cos\frac{\pi}{2}v\right](0\le v\le 1),$$

ISBN: 978-988-14047-4-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$M=\begin{bmatrix} \frac{2-\pi}{4-\pi}-\frac{2\alpha}{\pi} & \frac{2\alpha}{\pi}+\frac{2}{4-\pi} & -\frac{4}{\pi(4-\pi)}-\frac{2\beta}{\pi} & \frac{4-2\pi}{\pi(4-\pi)}+\frac{2\beta}{\pi} \\ \frac{\pi}{4-\pi}-\alpha & \frac{\pi}{4-\pi}-\alpha & \frac{2}{4-\pi}-\beta & \frac{2}{4-\pi}+\beta \\ \alpha & -\alpha & \beta & -\beta \\ \frac{2\alpha}{\pi}-\frac{2}{4-\pi} & \frac{2}{4-\pi}-\frac{2\alpha}{\pi} & \frac{4-2\pi}{\pi(4-\pi)}+\frac{2\beta}{\pi} & -\frac{4}{\pi(4-\pi)}-\frac{2\beta}{\pi} \\ \frac{2\alpha}{\pi}+\frac{2}{4-\pi} & -\frac{2\alpha}{\pi}-\frac{2}{4-\pi} & \frac{4}{\pi(4-\pi)}+\frac{2\beta}{\pi} & \frac{4-2\pi}{\pi(4-\pi)}-\frac{2\beta}{\pi} \end{bmatrix},$$

Eq. (2) yields the parametric equations

$$\begin{cases} x(u,v)=UMC_xM^TV^T; \\ y(u,v)=UMC_yM^TV^T; \\ z(u,v)=UMC_zM^TV^T, \end{cases} \tag{3}$$

where C_x, C_y and C_z are the coordinates components of the boundary information matrix C.

B. *The properties of AT Coons patch*

(i) Interpolation. The AT Coons patch interpolates the four corner points $p(i,j)$ $(i,j=0,1)$, the four tangent vectors $p_u(i,j)$, $p_v(i,j)(i,j=0,1)$ and the four twist vectors $p_{uv}(i,j)$ $(i,j=0,1)$.

(ii) Geometric invariance. The shape and the properties of the AT Coons surface patch have nothing to do with the choice of the coordinate system due to the parameterized method of the surface construction.

(iii) Adjustability. the AT Coons patch has four parameters , $\alpha_1,\beta_1,\alpha_2$ and β_2. Given the boundary conditions, the shape of the AT Coons patch can be adjusted by changing the values of parameters.

For example, given the values of the boundary information matrix as follows,

$$C=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{\pi}{2} \\ \frac{\pi}{2} & 0 & 0 & \frac{\pi^2}{4} \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

various surfaces for different parameters may be produced (see Fig. 2).

IV. Connecting AT Coons surface patches smoothly

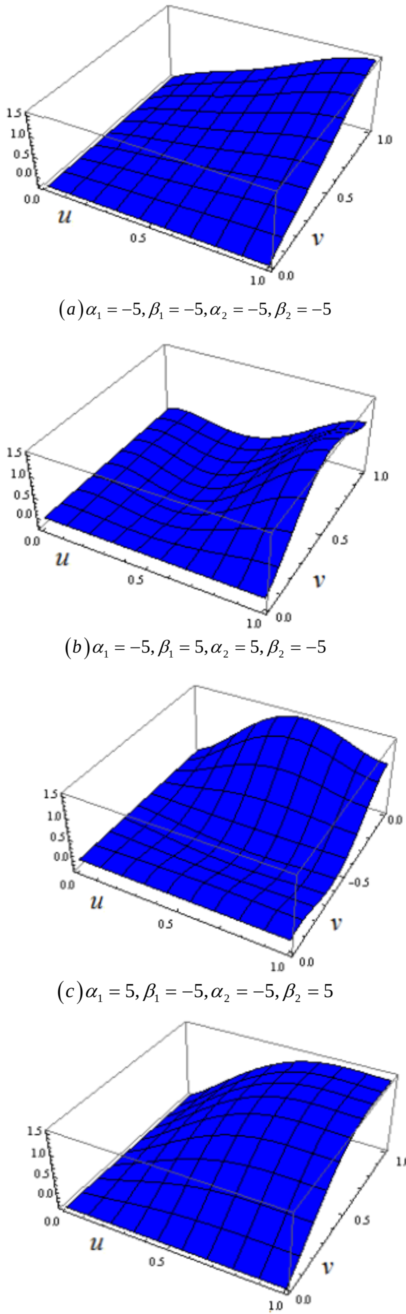
A. *Sufficient condition of connecting two adjacent Coons patches smoothly*

In CAGD, a complex surface is frequently generated by several small surfaces to meet the needs of described complex shapes. Generally, a complex surface can be divided into several simple patches, and the simple pieces are then connected together with smooth conditions. In this section, we provide the sufficient conditions and methods of connecting two adjacent AT Coons patches with G^1-continuity.

Suppose that $p(u,v)$ and $q(u,v)$ are two AT Coons patches with four parameters. Let α_1 and β_1 be parameters of u-direction of $p(u,v)$, and α_2 and β_2 be parameters of v-direction of $p(u,v)$. Let α_3 and β_3 be parameters of u-direction of $q(u,v)$, and α_4 and β_4 be parameters of v-direction of $q(u,v)$. Let C_1 and C_2 be the boundary information matrix as follows:

$$C_1=\begin{bmatrix} p(0,0) & p(0,1) & p_v(0,0) & p_v(0,1) \\ p(1,0) & p(1,1) & p_v(1,0) & p_v(1,1) \\ p_u(0,0) & p_u(0,1) & p_{uv}(0,0) & p_{uv}(0,1) \\ p_u(1,0) & p_u(1,1) & p_{uv}(1,0) & p_{uv}(1,1) \end{bmatrix},$$

$$C_2=\begin{bmatrix} q(0,0) & q(0,1) & q_v(0,0) & q_v(0,1) \\ q(1,0) & q(1,1) & q_v(1,0) & q_v(1,1) \\ q_u(0,0) & q_u(0,1) & q_{uv}(0,0) & q_{uv}(0,1) \\ q_u(1,0) & q_u(1,1) & q_{uv}(1,0) & q_{uv}(1,1) \end{bmatrix}$$



$(a)\alpha_1=-5,\beta_1=-5,\alpha_2=-5,\beta_2=-5$

$(b)\alpha_1=-5,\beta_1=5,\alpha_2=5,\beta_2=-5$

$(c)\alpha_1=5,\beta_1=-5,\alpha_2=-5,\beta_2=5$

$(d)\alpha_1=5,\beta_1=5,\alpha_2=5,\beta_2=5$

Fig. 2 The AT Coons patch with different parameters

ISBN: 978-988-14047-4-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

When two AT Coons patch are joined together, to achieve G^1-continuity, two adjacent surfaces should have a common boundary curve and a common tangent plane. Considering the directivities of the Coons patch, we discuss the following 3 different cases:

(i) Joining two pieces both in the u-direction

First, two adjacent surfaces share a common boundary, let $p(u,0)=q(u,1)$, we have

$$\begin{aligned}&F_0(u,\alpha_1)\,p(0,0)+F_1(u,\alpha_1)\,p(0,1)\\&+G_0(u,\beta_1)\,p_v(0,0)+G_1(u,\beta_1)\,p_v(0,1)\\&=F_0(u,\alpha_3)\,q(0,1)+F_1(u,\alpha_3)\,q(1,1)\\&+G_0(u,\beta_3)\,q_v(0,1)+G_1(u,\beta_3)\,q_v(1,1)\end{aligned}\tag{4}$$

Second, two adjacent surfaces share a common tangent plane which means that the normal vector of every point on the boundary is continuous, namely

$$p_v(u,0)\times q_u(u,0)=kq_v(u,1)\times q_u(u,1),k\in R^+$$

since $p(u,0)=q(u,1)$, and $p_u(u,0)=q_u(u,1)$,we have $p_v(u,0)=kq_v(u,1)$, namely

$$\begin{aligned}&F_0(u,\alpha_1)\,p_v(0,0)+F_1(u,\alpha_1)\,p_v(1,0)\\&+G_0(u,\beta_1)\,p_{uv}(0,0)+G_1(u,\beta_1)\,p_{uv}(1,0)\\&=k\begin{bmatrix}F_0(u,\alpha_3)\,q_v(0,1)+F_1(u,\alpha_3)\,q_v(1,1)\\+G_0(u,\beta_3)\,q_{uv}(0,1)+G_1(u,\beta_3)\,q_{uv}(1,1)\end{bmatrix},\end{aligned}\tag{5}$$

When $\alpha_1=\alpha_3$ and $\beta_1=\beta_3$, we get

$$\begin{aligned}&\begin{bmatrix}p(0,0) & p(1,0) & p_u(0,0) & p_u(1,0)\end{bmatrix}\\&=\begin{bmatrix}q(0,1) & q(1,1) & q_u(0,1) & q_u(1,1)\end{bmatrix}\end{aligned}\tag{6}$$

$$\begin{aligned}&\begin{bmatrix}p_v(0,0) & p_v(1,0) & p_{uv}(0,0) & p_{uv}(1,0)\end{bmatrix}\\&=k\begin{bmatrix}q_v(0,1) & q_v(1,1) & q_{uv}(0,1) & q_{uv}(1,1)\end{bmatrix}\end{aligned}\tag{7}$$

From Eq. (6) and Eq. (7), we see that the first column elements in the information matrix C_1 equal to the corresponding second column elements in the information matrix C_2, the third column elements of C_1 equal to k times the corresponding column elements of the fourth column of C_2. To summarize, we arrive at the following:

$$C_1=\begin{bmatrix}q(0,1) & p(0,1) & kq_v(0,1) & p_v(0,1)\\ q(1,1) & p(1,1) & kq_v(1,1) & p_v(1,1)\\ q_u(0,1) & p_u(0,1) & kq_{uv}(0,1) & p_{uv}(0,1)\\ q_u(1,1) & p_u(1,1) & kq_{uv}(1,1) & p_{uv}(1,1)\end{bmatrix}$$

$$C_2=\begin{bmatrix}q(0,0) & q(0,1) & q_v(0,0) & q_v(0,1)\\ q(1,0) & q(1,1) & q_v(1,0) & q_v(1,1)\\ q_u(0,0) & q_u(0,1) & q_{uv}(0,0) & q_{uv}(0,1)\\ q_u(1,0) & q_u(1,1) & q_{uv}(1,0) & q_{uv}(1,1)\end{bmatrix}$$

(ii) Joining pieces in the u-direction and the v-direction

If the u-direction of the $p(u,v)$ and the v-direction of the $p(u,v)$ needed to be connected together, then common boundary satisfies $p(u,1)=q(0,v)$, namely

$$\begin{aligned}&F_0(u,\alpha_2)\,p(0,1)+F_1(u,\alpha_2)\,p(1,1)\\&+G_0(u,\beta_2)\,p_u(0,1)+G_1(u,\beta_2)\,p_u(1,1)\\&=F_0(u,\alpha_3)\,q(0,0)+F_1(u,\alpha_3)\,q(0,1)\\&+G_0(u,\beta_3)\,q_v(0,0)+G_1(u,\beta_3)\,q_v(0,1).\end{aligned}\tag{8}$$

The common tangent plane must satisfy $p_v(u,1)=kq_u(0,1)$, namely

$$\begin{aligned}&F_0(u,\alpha_1)\,p_v(0,1)+F_1(u,\alpha_1)\,p_v(1,1)\\&+G_0(u,\beta_1)\,p_{uv}(0,1)+G_1(u,\beta_1)\,p_{uv}(1,1)\\&=k\begin{bmatrix}F_0(v,\alpha_4)\,q_u(0,0)+F_1(v,\alpha_4)\,q_u(0,1)\\+G_0(v,\beta_4)\,q_{uv}(0,0)+G_1(u,\beta_4)\,q_{uv}(0,1)\end{bmatrix}.\end{aligned}\tag{9}$$

When $\alpha_1=\alpha_4$ and $\beta_1=\beta_4$, Eq.(8) and Eq.(9) can be simplified, and the corresponding information matrices should have the following format:

$$C_1=\begin{bmatrix}p(0,0) & q(0,1) & p_v(0,0) & kq_v(0,1)\\ p(1,0) & q(1,1) & p_v(1,0) & kq_v(1,1)\\ p_u(0,0) & q_u(0,1) & p_{uv}(0,0) & kq_{uv}(0,1)\\ p_u(1,0) & q_u(1,1) & p_{uv}(1,0) & kq_{uv}(1,1)\end{bmatrix},$$

$$C_2=\begin{bmatrix}q(0,0) & q(0,1) & q_v(0,0) & q_v(0,1)\\ q(1,0) & q(1,1) & q_v(1,0) & q_v(1,1)\\ q_u(0,0) & q_u(0,1) & q_{uv}(0,0) & q_{uv}(0,1)\\ q_u(1,0) & q_u(1,1) & q_{uv}(1,0) & q_{uv}(1,1)\end{bmatrix}.$$

(iii) joining two pieces both in the v-direction

Similarly, we can obtain the conditions of joining two pieces both in the v-direction.

Figure 3 demonstrates smooth union of different AT Coons surfaces, where figure (a) shows connecting pieces in the u-direction and the u-direction, figure (b) illustrates connecting pieces in the u-direction and the v-direction, and figure (c) displays connecting four different AT- Coons surface patches.

V. The applications of the AT Coons patch

In this section, we provide examples of conic surfaces, such as ellipsoids and toruses, represented by AT Coons patches with given boundary information matrix.

A. *Ellipsoid*

Given the coordinates components of the boundary information matrix as follows:

$$C=\begin{bmatrix}(0,0,c) & \left(0,b,\frac{\pi}{2}c\right) & \left(0,\frac{\pi}{2}b,c\right) & (0,b,0)\\ (0,0,c) & \left(a,\frac{\pi}{2}b,\frac{\pi}{2}c\right) & \left(\frac{\pi}{2}a,\frac{\pi^2}{4}b,c\right) & \left(a,\frac{\pi}{2}b,0\right)\\ (0,0,c) & \left(\frac{\pi}{2}a,b,\frac{\pi}{2}c\right) & \left(\frac{\pi^2}{4}a,\frac{\pi}{2}b,c\right) & \left(a,\frac{\pi}{2}b,0\right)\\ (0,0,c) & \left(a,0,\frac{\pi}{2}c\right) & \left(\frac{\pi}{2}a,0,c\right) & (a,0,0,)\end{bmatrix},$$

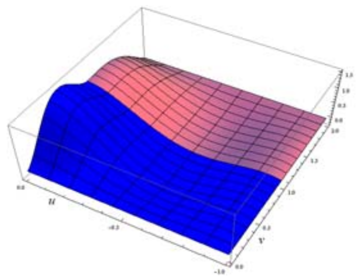
where $a,b,c>0$. If we let $\alpha_1=\beta_1=\alpha_2=\beta_2=0$, in the Eq.(3), the coordinates of AT Coons surface patch are

$$\begin{cases} x(u,v)=a\sin\frac{\pi}{2}u\sin\frac{\pi}{2}v; \\ y(u,v)=b\cos\frac{\pi}{2}u\sin\frac{\pi}{2}v;0\le u,v\le 1. \\ x(u,v)=c\cos\frac{\pi}{2}v, \end{cases}$$

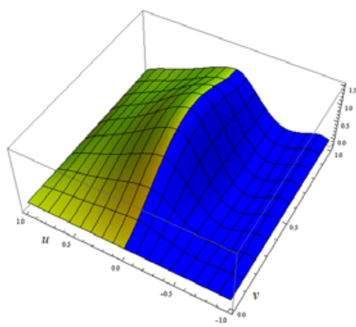
This is the parametric equations of a part of an ellipsoid.

B. *Torus*

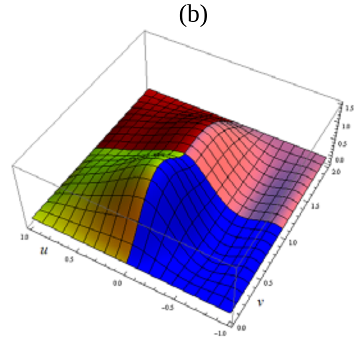
If the coordinates components of the boundary information matrix are given as follows:



(a)



(b)



(c)

Fig.3 Different AT Coons patches connecting smoothly

$$C=\begin{bmatrix} (0,a+b,0) & (0,b,a) & \left(0,0,\frac{\pi}{2}a\right) & \left(0,-\frac{\pi}{2}a,0\right) \\ (a+b,0,0) & (b,0,a) & \left(0,0,\frac{\pi}{2}a\right) & \left(-\frac{\pi}{2}a,0,0\right) \\ \left(\frac{\pi}{2}a+\frac{\pi}{2}b,0,0\right) & \left(\frac{\pi}{2}b,0,0\right) & (0,0,0) & \left(-\frac{\pi^2}{4}a,0,0\right) \\ \left(0,-\frac{\pi}{2}a-\frac{\pi}{2}b,c\right) & \left(0,-\frac{\pi}{2}b,0\right) & (0,0,0) & \left(0,\frac{\pi^2}{4}a,0,\right) \end{bmatrix},$$

where $a,b,c>0$. If we let $\alpha_1=\beta_1=\alpha_2=\beta_2=0$, in the Eq.(3), we get the parametric equations of AT Coons patch as following,

$$\begin{cases} x(u,v)=\left(a+b\cos\frac{\pi}{2}u\right)\sin\frac{\pi}{2}v; \\ y(u,v)=\left(a+b\cos\frac{\pi}{2}u\right)\cos\frac{\pi}{2}v;0\le u,v\le 1 \\ x(u,v)=b\sin\frac{\pi}{2}v, \end{cases}$$

which represents a part of a torus.

VI. Conclusions

In this paper, a class of AT Coons patch using algebraic and trigonometric polynomials with different parameters is constructed, with properties similar to those of Coons surfaces. We discuss the sufficient conditions of G^1 continuity for connecting two adjacent AT Coons patches. Since the AT Coons patch has two parameters for each direction, its shape can be modified easily by adjusting the parameters. Moreover, the special geometric properties of shape parameters make the AT Coons patch a more desirable surface-design tool for users.

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ISBN: 978-988-14047-4-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)