Performance Analysis of the Fast-NLMS type Algorithm

M. Arezki, A. Namane and A. Benallal

***Abstract*—The performance analysis of Fast normalized least-mean-square (FNLMS) algorithm is presented. We propose to use convergence bounds on the adaptation step-size obtained for the NLMS algorithm using the approximate mean-square analysis and we will try to see how the FNLMS algorithm affects the adaptation gain compared to the NLMS algorithm. We provide a theoretical justification for this algorithm by formulating a new stability condition. It will be followed by an analytical analyze of the FNLMS algorithm convergence and we show, both theoretically and experimentally, its robustness.**

***Index Terms*—Fast RLS, Estimation, Adaptive Filtering, Propagation of Errors, Numerical Stability.**

I. Introduction

RECENTLY, a new adaptive algorithm with fast convergence and low complexity is proposed [1]. This Fast Normalized Least Mean Square (FNLMS) algorithm derived from the Fast Recursive Least Squares (FRLS) algorithm where the adaptation gain is obtained by discarding completely the forward and backward predictors. In the following section, we propose to use convergence bounds on the adaptation step-size obtained for the NLMS algorithm using the approximate mean-square analysis [2] and we will try to see how the FNLMS algorithm affects the adaptation gain compared to the NLMS algorithm. We provide a theoretical justification for this algorithm by formulating a new stability condition. It will be followed by an analytical analyze of the FNLMS algorithm convergence and we show, both theoretically and experimentally, its robustness.

II. Adaptive Algorithms

The main identification block diagram of a linear system with finite impulse response (FIR), by adaptive filtering using an adaptation algorithm, is represented in Fig 1. The output a priori error $\bar{\varepsilon}_n$ of this system at time n is:

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$$\bar{\varepsilon}_{L,n} = d_n - \hat{y}_n \tag{1}$$

where $\hat{y}_n = \mathbf{w}^{\mathrm{T}}_{L,n-1}\mathbf{x}_{L,n}$ is the model filter output, $\mathbf{x}_{L,n}$ is a vector containing the last L samples of the input signal x_n, $\mathbf{w}_{L,n}$ is the coefficient vector of the adaptive filter and L is the filter length. We assume that the desired signal from the model is:

$$d_n = v_n + \mathbf{w}^{\mathrm{T}}_{\mathrm{opt},L}\mathbf{x}_{L,n} \tag{2}$$

where $\mathbf{w}_{\mathrm{opt},L}$ is the unknown system impulse response vector and v_n is a stationary, zero-mean, and independent noise sequence that is uncorrelated with any other signal. The superscript $^{\mathrm{T}}$ describes transposition.
The error signal $\bar{\varepsilon}_n$ can be used to adapt the adaptive filter $\mathbf{w}_{L,n-1}$ using some algorithm for filter adaptation. Several different algorithms for filter adaptation have been proposed. The filter is updated at each instant by feedback of the estimation error proportional to the adaptation gain, denoted as $\mathbf{g}_{L,n}$, and according to:

$$\mathbf{w}_{L,n} = \mathbf{w}_{L,n-1} + \mathbf{g}_{L,n}\,\bar{\varepsilon}_n \tag{3}$$

The different algorithms are distinguished by the adaptation gain calculation.

A. *NLMS and FRLS Algorithms*

For the NLMS algorithm, the adaptation gain is given by:

$$\mathbf{g}_{L,n} = \frac{\mu}{L\pi_{x,n} + c_0}\mathbf{x}_{L,n} \tag{4}$$

where μ is referred to as the adaptation step, c_0 is a small positive constant used to avoid division by zero in absence of the input signal and $\pi_{x,n}$ is the power of input signal [3]. The computational complexity of this algorithm is $2L$ multiplications per sample.

For the FRLS algorithm, the adaptation gain is given by:

$$\mathbf{g}_{L,n} = \gamma_n \tilde{\mathbf{k}}_{L,n} \tag{5}$$

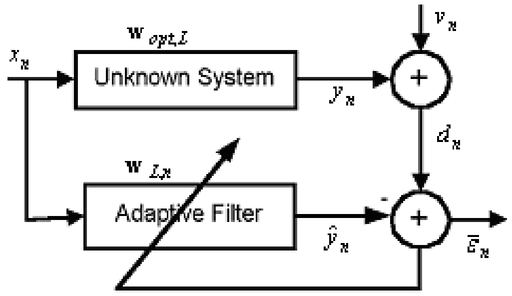

Fig. 1. System identification block diagram.

where the variables γ_n and $\tilde{\mathbf{k}}_{L,n}$ respectively indicate the likelihood variable and normalized Kalman gain vector. This latter is calculated, independently of the filtering part $\mathbf{w}_{L,n}$, by a FRLS algorithm using forward/backward linear prediction analysis over the signal x_n [4]. The calculation complexity of a FRLS algorithm is of order L.

B. The Fast-NLMS type Algorithm

Recently, a new adaptive algorithm with fast convergence and low complexity is proposed [1]. This algorithm FNLMS derived from the FRLS algorithm where the adaptation gain is obtained by discarding completely the forward and backward predictors. Thus in this algorithm, we proposed a simplified adaptation gain:

$$\begin{bmatrix}\tilde{\mathbf{k}}_{L,n}\\ q_n\end{bmatrix}=\begin{bmatrix}\dfrac{e_n}{\lambda\alpha_{n-1}+c_0}\\ \tilde{\mathbf{k}}_{L,n-1}\end{bmatrix} \tag{6}$$

where the prediction error e_n is evaluated using a first order prediction model:

$$e_n = x_n - a_n x_{n-1} \tag{7}$$

where a_n is a prediction parameter which minimizes the cost function $\mathrm{E}\{e_n^2\}$. An estimate of this parameter is given by the following equation:

$$a_n=\frac{r_{1x,n}}{r_{0x,n}+c_a} \tag{8}$$

where $r_{1x,n}$ is an estimate of the first lag correlation function of x_n, $r_{0x,n}$ an estimate of the input signal power and c_a is a small positive constant used to avoid the possibility of a dividing term very close to zero. The forward prediction error variance is now evaluated by:

$$\alpha_n = \lambda\alpha_{n-1} + e_n^2 \tag{9}$$

The small positive constant c_0 is added to the dividing term $\lambda\alpha_{n-1}$ to overcome divisions by small values as in the case of the NLMS algorithm. Thus, we obtain an algorithm similar to the NLMS algorithm in term of the computational complexity; i.e. $2L$ multiplications per iteration. It is given in Table I.

III. Performance Analysis of FNLMS Algorithm

In this section, the stability condition and error analysis filtering part of the FNLMS algorithm will be presented.

A. The Stability Condition

We use convergence bounds on the adaptation step-size μ obtained for the NLMS algorithm using the approximate mean-square analysis [2] and we will try to see how the FNLMS algorithm affects the adaptation gain compared to the NLMS algorithm. For this purpose, we suppose that the input signal is white Gaussian stationary signal and consider that all recursive variables of the algorithm have reached their true asymptotic values. In particular [6], we replace the following slowing quantities by their asymptotic values:

TABLE I
FNLMS ALGORITHM (2L)

Initialization:

$\mathbf{w}_{L,0}=\tilde{\mathbf{k}}_{L,0}=0_L$; $r_{0,0}=E_0$; $r_{1,0}=0$;

$\alpha_0=\lambda^L E_0$; $\gamma_0=1$; $E_0\geq L\sigma_x^2/100$

Variables available at the discrete-time index n:

$r_{1x,n-1}$; $r_{0x,n-1}$; α_{n-1} ; γ_{n-1}; $\tilde{\mathbf{k}}_{L,n-1}$; $\mathbf{w}_{L,n-1}$

New information: x_n, d_n.

$r_{1x,n}=(1-\lambda_a)\,r_{1x,n-1}+\lambda_a\,x_n\,x_{n-1}$;

$r_{0x,n}=(1-\lambda_a)\,r_{0x,n-1}+\lambda_a\,x_n^2$;

$a_n=\dfrac{r_{1x,n}}{r_{0x,n}+c_a}$;

$e_n=x_n-a_n\,x_{n-1}$; $\alpha_n=\lambda\alpha_{n-1}+e_n^2$;

- **Adaptation Gain:**

$\begin{bmatrix}\tilde{\mathbf{k}}_{L,n}\\ q_n\end{bmatrix}=\begin{bmatrix}\dfrac{e_n}{\lambda\alpha_{n-1}+c_0}\\ \tilde{\mathbf{k}}_{L,n-1}\end{bmatrix}$;

$\delta_n=\dfrac{x_n\,e_n}{\lambda\alpha_{n-1}+c_0}-q_n\,x_{n-L}$; $\gamma_n=\dfrac{\gamma_{n-1}}{1+\delta_n\,\gamma_{n-1}}$

- **Filtering Part:**

$\bar{\varepsilon}_n=d_n-\mathbf{w}_{L,n-1}^{\mathrm{T}}\mathbf{x}_{L,n}$; $\bar{\varepsilon}_n=d_n-\mathbf{w}_{L,n-1}^{\mathrm{T}}\mathbf{x}_{L,n}$

$\mathbf{w}_{L,n}=\mathbf{w}_{L,n-1}+\mu_0\,\bar{\varepsilon}_n\,\gamma_n\,\tilde{\mathbf{k}}_{L,n}$

$$\alpha_n \approx \frac{\sigma_x^2}{1-\lambda} \tag{10a}$$

$$\tilde{\mathbf{k}}_{L,n}\approx\frac{\mathbf{x}_{L,n}}{\dfrac{\lambda}{1-\lambda}\sigma_x^2+c_0} \tag{10b}$$

$$\gamma_n=\frac{1}{1+\tilde{\mathbf{k}}_{L,n}^{\mathrm{T}}\mathbf{x}_{L,n}}\approx\frac{1}{1+\dfrac{L\sigma_x^2}{\dfrac{\lambda\sigma_x^2}{1-\lambda}+c_0}} \tag{10c}$$

where $\sigma_x^2=\mathrm{E}\{x_n^2\}$. Using approximations (10), the adaptation gain for the FNLMS algorithm is:

$$\mathbf{g}_{L,n}=\mu_0\,\gamma_n\tilde{\mathbf{k}}_{L,n}\approx G_{\mathrm{F}}\,\mathbf{x}_{L,n} \tag{11}$$

where

$$G_{\mathrm{F}}=\frac{\mu_0}{L\sigma_x^2\left(1+\dfrac{\lambda}{(1-\lambda)L}+\dfrac{c_0}{L\sigma_x^2}\right)} \tag{12}$$

The error $\bar{\varepsilon}_n$ calculated by (1) is called a priori, because it employs the coefficients before updating. The a posteriori error is defined as:

$$\varepsilon_n=d_n-\mathbf{w}_{L,n}^{\mathrm{T}}\mathbf{x}_{L,n} \tag{13}$$

and it can be computed after (1) and (3) have been completed. Now, from (1) and (3), (13) can be written as:

$$\varepsilon_n=\left(1-\mathbf{x}_{L,n}^{\mathrm{T}}\,\mathbf{g}_{L,n}\right)\bar{\varepsilon}_n \tag{14}$$

The system can be considered stable if the expectation of the a posteriori error magnitude is smaller than that of the a priori error, which is logical since more information is incorporated in ε_n. If the error $\bar{\varepsilon}_n$ is assumed to be independent of the L most recent input data, which is

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approximately true after convergence, the stability condition is:

$$\left|1-\mathrm{E}\left\{G_{\mathrm{F}}\mathbf{x}_{L,n}^{\mathrm{T}}\mathbf{x}_{L,n}\right\}\right|<1 \qquad (15)$$

which yields

$$0<G_{\mathrm{F}}\,L\sigma_x^2<2 \qquad (16)$$

from which we obtain the stability condition :

$$0<\mu_{\mathrm{F}}<2 \qquad (17)$$

with

$$\mu_{\mathrm{F}}=\frac{\mu_0}{\left(1+\dfrac{\lambda}{(1-\lambda)L}+\dfrac{c_0}{L\sigma_x^2}\right)} \qquad (18)$$

The influence of the power signal in the adaptation gain can be reduced by choosing $\frac{c_0}{L\sigma_x^2}<<1$.

B. Error Analysis Filtering Part

The analysis uses the common independence assumption that the current input signal vector is statistically independent of the current coefficient vector of the adaptive filter [5]. We define the weight-error vector at time *n* as:

$$\Delta\mathbf{w}_{L,n}=\mathbf{w}_{\mathrm{opt},L}-\mathbf{w}_{L,n} \qquad (19)$$

The output a priori error $\bar{\varepsilon}_n$ can be written as:

$$\bar{\varepsilon}_n=v_n+\mathbf{x}_{L,n}^{\mathrm{T}}\Delta\mathbf{w}_{L,n-1} \qquad (20)$$

The recursion in (3) on the coefficient error vector is:

$$\Delta\mathbf{w}_{L,n}=\left[\mathbf{I}_L-\mathbf{g}_{L,n}\mathbf{x}_{L,n}^{\mathrm{T}}\right]\Delta\mathbf{w}_{L,n-1}-\mathbf{g}_{L,n}v_n \qquad (21)$$

The mean behavior of the FNLMS coefficient error vector can now be determined by taking the expected value of both sides of (21) and using the independence assumption to yield:

$$\mathrm{E}\{\Delta\mathbf{w}_{L,n}\}=\left[\mathbf{I}_L-G_{\mathrm{F}}\mathbf{R}_{xx}\right]\mathrm{E}\{\Delta\mathbf{w}_{L,n-1}\}-G_{\mathrm{F}}\,\mathrm{E}\{\mathbf{x}_{L,n}v_n\} \qquad (22)$$

where $\mathbf{R}_{xx}=\mathrm{E}\{\mathbf{x}_{L,n}\mathbf{x}_{L,n}^{\mathrm{T}}\}$. Moreover, the input signal is a sequence of uncorrelated Gaussian variables $\mathbf{R}_{xx}=\sigma_x^2\mathbf{I}_L$, we obtain:

$$\mathrm{E}\{\Delta\mathbf{w}_{L,n}\}=G_{\Delta\mathrm{w}}\,\mathrm{E}\{\Delta\mathbf{w}_{L,n-1}\} \qquad (23)$$

where

$$G_{\Delta\mathrm{w}}=\left(1-G_{\mathrm{F}}\,\sigma_x^2\right)=1-\frac{\mu_{\mathrm{F}}}{L} \qquad (24)$$

The steady-state solution of (23) is:
if $G_{\Delta\mathrm{w}}<1 \;\Rightarrow\; \mathrm{E}\{\Delta\mathbf{w}(\infty)\}=0_L$, from which we obtain the steady-state mean coefficient vector of the FNLMS adaptive filter as:

$$\mathrm{E}\{\mathbf{w}_L(\infty)\}=\mathbf{w}_{\mathrm{opt},L} \qquad (25)$$

The mean square error $MSE(n)=\mathrm{E}\{\bar{\varepsilon}_n^2\}$ can be written as:

$$MSE(n)=\left(\sigma_v^2+\mathrm{tr}\left[\mathbf{R}_{xx}\,\mathrm{E}\{\Delta\mathbf{w}_{L,n-1}\Delta\mathbf{w}_{L,n-1}^{\mathrm{T}}\}\right]\right) \qquad (26)$$

from which we obtain:

$$MSE(n)=\sigma_v^2+\sigma_x^2\,\mathrm{E}\left\{\left\|\Delta\mathbf{w}_{L,n-1}\right\|^2\right\} \qquad (27)$$

where $\mathrm{E}\{v_n^2\}=\sigma_v^2$, $\mathrm{tr}[.]$ represents the trace operator and $\|.\|$ denotes the 2-norm vector.

Let the filter misalignment be defined as $\left\|\Delta\mathbf{w}_{L,n}\right\|^2$, we can derive the expected misalignment for the next sample as $Mis(n)=\mathrm{E}\left\{\left\|\Delta\mathbf{w}_{L,n}\right\|^2\right\}$. For that, we need to determine the next expressions:

$$\begin{aligned}\mathrm{E}\{\Delta\mathbf{w}_{L,n}\Delta\mathbf{w}_{L,n}^{\mathrm{T}}\}=&\\ \mathrm{E}\Big\{\left[\mathbf{I}_L-\mathbf{g}_{L,n}\mathbf{x}_{L,n}^{\mathrm{T}}\right]&\Delta\mathbf{w}_{L,n-1}\Delta\mathbf{w}_{L,n-1}^{\mathrm{T}}\left[\mathbf{I}_L-\mathbf{g}_{L,n}\mathbf{x}_{L,n}^{\mathrm{T}}\right]^{\mathrm{T}}\Big\}\\ -\mathrm{E}\Big\{\left[\mathbf{I}_L-\mathbf{g}_{L,n}\mathbf{x}_{L,n}^{\mathrm{T}}\right]&\Delta\mathbf{w}_{L,n-1}\mathbf{g}_{L,n}^{\mathrm{T}}v_n\Big\}\\ -\mathrm{E}\Big\{v_n\,\mathbf{g}_{L,n}\Delta\mathbf{w}_{L,n-1}^{\mathrm{T}}&\left[\mathbf{I}_L-\mathbf{g}_{L,n}\mathbf{x}_{L,n}^{\mathrm{T}}\right]^{\mathrm{T}}\Big\}+\mathrm{E}\left\{\mathbf{g}_{L,n}\mathbf{g}_{L,n}^{\mathrm{T}}v_n^2\right\}\end{aligned} \qquad (28)$$

Using the independence assumption and approximations (10), we obtain:

$$\mathrm{E}\{\Delta\mathbf{w}_{L,n}\Delta\mathbf{w}_{L,n}^{\mathrm{T}}\}=\left(1-G_{\mathrm{F}}\sigma_x^2\right)^2\mathrm{E}\{\Delta\mathbf{w}_{L,n-1}\Delta\mathbf{w}_{L,n-1}^{\mathrm{T}}\}+G_{\mathrm{F}}^2\,\sigma_x^2\,\sigma_v^2\,\mathbf{I}_L \qquad (29)$$

By using (24), the recursive expression (29) becomes:

$$\mathrm{E}\{\Delta\mathbf{w}_{L,n}\Delta\mathbf{w}_{L,n}^{\mathrm{T}}\}=G_{\Delta\mathrm{w}}^2\,\mathrm{E}\{\Delta\mathbf{w}_{L,n-1}\Delta\mathbf{w}_{L,n-1}^{\mathrm{T}}\}+\left(1-G_{\Delta\mathrm{w}}\right)^2\frac{\sigma_v^2}{\sigma_x^2}\mathbf{I}_L \qquad (30)$$

By taking the trace of both sides of (29), we can write the misalignment at time *n* as:

$$Mis(n)=G_{\Delta\mathrm{w}}^2Mis(n-1)+\left(1-G_{\Delta\mathrm{w}}\right)^2\frac{\sigma_v^2}{\sigma_x^2}L \qquad (31)$$

The stability of the recursion (30) is guaranteed if $G_{\Delta\mathrm{w}}<1$. We define the normalized mean square error $NMSE(n)$ and the normalized misalignment $Nmis(n)$ as follow:

$$NMSE(n)=\left(\frac{\mathrm{E}\{\bar{\varepsilon}_n^2\}}{\mathrm{E}\{d_n^2\}}\right) \qquad (32)$$

$$Nmis(n)=\frac{\mathrm{E}\left\{\left\|\mathbf{w}_{L,n}\right\|^2\right\}}{\left\|\mathbf{w}_{\mathrm{opt},L}\right\|^2} \qquad (33)$$

By using the independence assumption, we obtain:

$$NMSE(n)=\frac{\sigma_v^2+\sigma_x^2\,\mathrm{E}\left\{\left\|\mathbf{w}_{L,n-1}\right\|^2\right\}}{\sigma_v^2+\sigma_x^2\,\left\|\mathbf{w}_{\mathrm{opt},L}\right\|^2} \qquad (34)$$

$$Nmis(n)=G_{\Delta\mathrm{w}}^2\,Nmis\,(n-1)+\frac{\left(1-G_{\Delta\mathrm{w}}\right)^2\sigma_v^2}{\sigma_x^2\left\|\mathbf{w}_{\mathrm{opt},L}\right\|^2}L \qquad (35)$$

Let us consider the signal to noise ratio $SNR_{\mathrm{out}}=\sigma_y^2/\sigma_v^2$, where $\sigma_y^2=\mathrm{E}\{y_n^2\}=\sigma_x^2\left\|\mathbf{w}_{\mathrm{opt},L}\right\|^2$. After convergence, the expressions (34) and (35) become:

$$NMSE(\infty)=\frac{1+\left(\dfrac{1-G_{\Delta\mathrm{w}}}{1+G_{\Delta\mathrm{w}}}\right)L}{1+SNR_{\mathrm{out}}} \qquad (36)$$

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$$Nmis(\infty)=\left(\frac{1-G_{\Delta w}}{1+G_{\Delta w}}\right)\frac{L}{SNR_{out}} \quad (37)$$

IV. SIMULATIONS

In this section, we present some simulation results to verify the proposed mean theoretical results derived in this paper.

For the FNLMS algorithm, we use two different forgetting factors. The first one λ is chosen according to condition (18). We have noticed in practice that the FLNMS algorithm remains stable even for very small values of λ. The second forgetting factor λ_a can be adjusted according to the degree of non-stationarity of the input signal; i.e., very close to one for stationary signal and calculated at least over a rectangular window of 15ms for the speech signal. The initial values for variances and regularisation constants are set to a value comparable with the input signal power σ_x^2; i.e. $c_0=c_a=E_0=1$.

We try to estimate an impulse response $\mathbf{w}_{\text{opt}L}$ of length L=256 the same length is used for the adaptive filter $\mathbf{w}_{L,n}$.

The input signal x_n used in our simulation is a white Gaussian noise, with mean zero and variance equal to 1. The reference signal d_n is obtained by convolving $\mathbf{w}_{\text{opt}L}$ with x_n and adding a white Gaussian noise signal with an SNR_{out} of 50 dB. Performance of the estimation is measured by the normalized mean square error $NMSE(n)$ and the normalized misalignment $Nmis(n)$.

Figure 2, presents the $NMSE(n)$ determined from simulation and from the theoretical expression in (34). From this plot, we observe that simulation and theoretical curves agree very well.

Figure 3, shows the convergence of the normalized misalignment $Nmis(n)$ as obtained from the theoretical analysis (35) and from simulation results. It can be seen that, for long adaptive filter, there is a good agreement between the actual behavior of the algorithm and that predicted by the theoretical expression.

Figure 4, shows theoretical and experimental values for different values of the SNR_{out}. These results and the previous ones have confirmed and validated the good properties of our proposed analysis of the FNLMS algorithm.

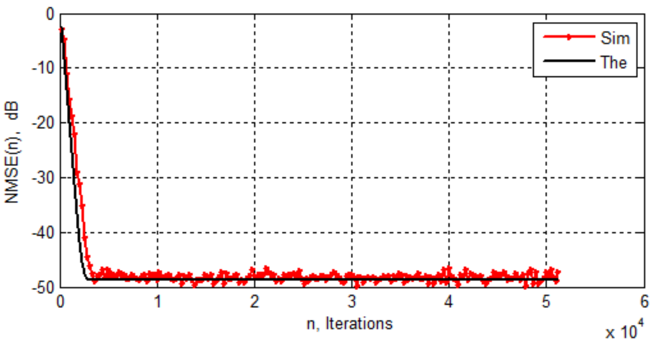

Fig. 2. Comparison of theoretical and simulation curves of the $NMSE(n)$.

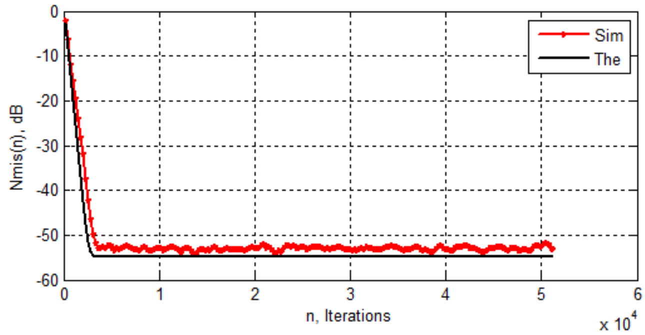

Fig. 3. Comparison of theoretical and simulation curves of the Normalized Misalignment $Nmis(n)$.

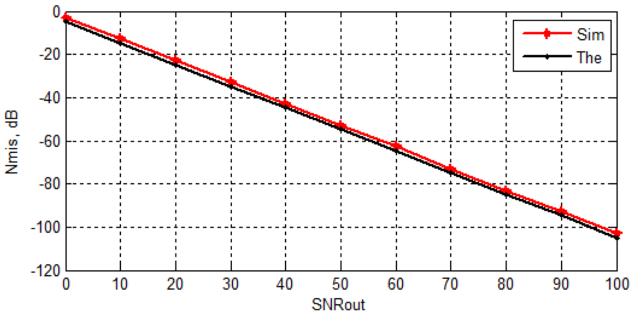

Fig. 4. Comparison of theoretical and simulation curves of the Normalized Misalignment for different values of the SNR_{out}.

V. CONCLUSION

We have analyzed the numerical properties of the FNLMS algorithm by using the common independence assumption that the current input signal vector is statistically independent of the current coefficient vector of the adaptive filter. We also consider that all variables of the algorithm have reached their true asymptotic values. In particular, we replace the following slowing quantities by their asymptotic values. The condition of stabilization was shown to be capable of maintaining a good convergence performance by way of computer simulations.

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