Approximate-analytical Solutions of the Generalized Newell-Whitehead-Segel Model by He's Polynomials Method

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***Abstract*— This paper considers approximate-analytical solutions of the generalized Newell-Whitehead-Segel model by means of He's polynomials solution method. The method is technically presented and applied to both linear and nonlinear forms of the Newell-Whitehead-Segel model. The results guarantee the efficiency and reliability of the proposed method.**

***Index Terms*— Analytical solutions; He's polynomials; Newell-Whitehead-Segel model.**

I. INTRODUCTION

In real life settings, modelling involves partial differential equations (PDEs), which may appear in linear or nonlinear forms. However, providing solutions to these models has become a great task before researchers. Hence, the development of numerical schemes, semi-analytical methods, and even modified semi-analytical methods [1-8]. In this work, emphasis will be on one of the vital models known as Newell-Whitehead-Segel Model (NWSM) whose general form is:

$$\begin{cases} w_t(x,t) = kw_{xx}(x,t) + aw(x,t) - bw^j(x,t), \\ w(x,0) = g(x), \end{cases} \quad (1.1)$$

where $a, b \in \mathbb{R}$, and $k, j \in \mathbb{Z}^+$.

The NWSM is a vital in fluid mechanics, engineering, and other aspects of pure and applied sciences. Recently, many researchers have considered, and adopted good number of solution techniques in a bid to solving (1.1) [9-11]. The purpose of this work is to consider in a general form, the solution of the NWSM by means of He's polynomial method whose basic merit is hinged on easy handling on nonlinear terms [12-16].

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II. ANALYSIS OF THE METHOD [12, 13]

Let Ξ be an integral or a differential operator on the equation of the form:

$$\Xi(\Im) = 0. \quad (2.1)$$

Let $H(\Im, p)$ be a convex homotopy defined by:

$$H(\Im, p) = p\Xi(\Im) + (1-p)G(\Im), \quad (2.2)$$

where $G(\Im)$ is a functional operator with $\Im_0$ as a known solution. Thus, we have:

$$H(\Im,0) = G(\Im) \text{ and } H(\Im,1) = \Xi(\Im), \quad (2.3)$$

whenever $H(\Im, p) = 0$ is satisfied, and $p \in (0,1]$ is an embedded parameter. In Homotopy Perturbation Method (HPM), p is used as an expanding parameter to obtain:

$$\Im = \sum_{j=0}^{\infty} p^j \Im_j = \Im_0 + p\Im_1 + p^2\Im_2 + \cdots. \quad (2.4)$$

From (2.4) the solution is obtained as $p \to 1$. The method considers $N(\Im)$ (the nonlinear term) as:

$$N(\Im) = \sum_{j=0}^{\infty} p^j H_j, \quad (2.5)$$

where H_k's are the so-called He's polynomials, which can be computed using:

$$H(\Im) = \frac{1}{i!}\frac{\partial^i}{\partial p^i}\left(N\left(\sum_{j=0}^{i} p^j \Im_j\right)\right)_{p=0}, \ n \ge 0, \quad (2.6)$$

where $H(\Im) = H_i(\Im_0, \Im_1, \Im_2, \Im_3, \cdots, \Im_i)$.

III. THE HE'S POLYNOMIALS ON THE GENERALIZED NWSM

Here, the He's Polynomials method is applied to the generalized NWSM as follows.

In integral form, with $I_0^t(\cdot)$ denoting an integral operator, we write (1.1) as:

$$\begin{cases} w(x,t) = w(x,0) + I_0^t\left(kw_{xx} + aw - bw^j\right), \\ w(x,0) = g(x), \ w(x,t) = w. \end{cases} \quad (3.1)$$

Note: In HPM, the series solution is expressed as:

$$w(x,t)=\sum_{n=0}^{\infty}p^n w_n\ , \qquad (3.2)$$

which is evaluated as $p\to 1$. Thus, by applying convex homotopy method to (3.1), we have:

$$\begin{cases}\sum_{n=0}^{\infty}p^n w_n = g(x)\\ \quad +I_0^t\left(k\sum_{n=0}^{\infty}p^{n+1}w_{xx,n}+a\sum_{n=0}^{\infty}p^{n+1}w_n - bH_n\right),\end{cases} \qquad (3.3)$$

where H_n , $n\in\mathbb{N}\cup\{0\}$ represent He's polynomials associated with the nonlinear term, $w^j(x,t)$.

So, by comparing the powers of the $p's$ in (3.3), we have:

$$p^{(0)}: w_0 = g(x)$$
$$p^{(1)}: w_1 = I_0^t\left(kw_{xx,0}+aw_0-bH_0\right)$$
$$p^{(2)}: w_2 = I_0^t\left(kw_{xx,1}+aw_1-bH_1\right)$$
$$p^{(3)}: w_3 = I_0^t\left(kw_{xx,2}+aw_2-bH_2\right)$$
$$\vdots$$
$$p^{(i)}: w_i = I_0^t\left(kw_{xx,i-1}+aw_{i-1}-bH_{i-1}\right), i\geq 1.$$

Hence, the solution: $w(x,t)=\sum_{n=0}^{\infty}p^n w_n \to \sum_{n=0}^{\infty}w_n$ as $p\to 1$.

IV. Illustrative Examples

Problem 1: Consider the following linear NWSM [10, 11]:

$$\begin{cases}w_t(x,t)=w_{xx}(x,t)-3w(x,t),\\ w(x,0)=e^{2x},\end{cases} \qquad (4.1)$$

whose exact solution is:

$$w(x,t)=e^{2x+t}. \qquad (4.2)$$

Procedure w.r.t Problem 1:

Comparing (4.1) with (1.1) gives: $k=1,\ a=-3,\ b=0$, and $g(x)=e^{2x}$. Therefore, using the detail in section 3 gives the recursive relation:

$$\begin{cases}w_0=e^{2x},\\ w_i=I_0^t\left(w_{xx,i-1}-3w_{i-1}\right),\ i\geq 1,\end{cases} \qquad (4.3)$$

such that:

$$w_0=e^{2x},\ w_1=e^{2x}t,\ w_2=\frac{e^{2x}t^2}{2},$$
$$w_3=\frac{e^{2x}t^3}{6},\ w_4=\frac{e^{2x}t^4}{24},\ w_5=\frac{e^{2x}t^5}{120},\cdots$$

$\therefore$

$$w(x,t)=e^{2x}+e^{2x}t+\frac{e^{2x}t^2}{2}+\frac{e^{2x}t^3}{6}+\frac{e^{2x}t^4}{24}+\frac{e^{2x}t^5}{120}+\cdots$$
$$=\left(1+t+\frac{t^2}{2}+\frac{t^3}{6}+\frac{t^4}{24}+\frac{t^5}{120}+\cdots\right)e^{2x}$$
$$=e^{2x+t} \qquad (4.4)$$

Problem 2: Consider the following nonlinear NWSM [9-11]:

$$\begin{cases}w_t(x,t)=5w_{xx}(x,t)+2w(x,t)+w^2(x,t),\\ w(x,0)=\eta,\end{cases} \qquad (4.5)$$

whose exact solution is:

$$w(x,t)=\frac{2\eta e^{2t}}{2+\eta\left(1-e^{2t}\right)} \qquad (4.6)$$

Procedure w.r.t Problem 2:

Comparing (4.5) with (1.1) gives: $k=5,\ a=2,\ b=-1$, $j=2$ and $g(x)=\eta$. Therefore, using the detail in section 3 gives the recursive relation:

$$\begin{cases}w_0=\eta,\\ w_i=I_0^t\left(5w_{xx,i-1}+2w_{i-1}+H_n\right),\ i\geq 1,\end{cases} \qquad (4.7)$$

where $H_0=w_0^2$, $H_1=2w_0w_1$, $H_2=2w_0w_2+w_1^2$, $H_3=2\left(w_0w_3+w_1w_2\right)$, $\cdots$, such that:

$$w_0=\eta,\ w_1=\left(\eta^2+2\eta\right)t,\ \ w_2=\left(1+\eta\right)\left(\eta^2+2\eta\right)t^2,$$
$$w_3=\left(\frac{2}{3}+\frac{2\eta}{3}+2\eta\left(\frac{1}{3}+\frac{\eta}{3}\right)+\frac{1}{3}\right)\left(\eta^2+2\eta\right)t^2,$$
$$w_4=\frac{\eta t^4}{3}\left(\eta+2\right)\left(\eta+1\right)\left(3\eta^2+6\eta+1\right),\ \cdots$$

$\therefore$

$$w(x,t)=\frac{1}{3}\begin{pmatrix}3+6t+6t^2+3\eta t+4t^3+9\eta t^2+2t^4\\ +14\eta t^3+3\eta^2t^2+15\eta t^4+12\eta^2t^3\\ +25\eta^2t^4+3\eta^3t^3+15\eta^3t^4+3\eta^4t^4+\cdots\end{pmatrix} \qquad (4.8)$$

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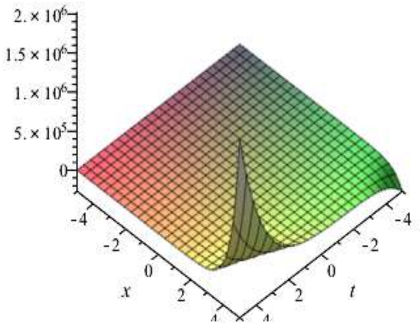

Fig. 1a: Approximate solution of problem 1

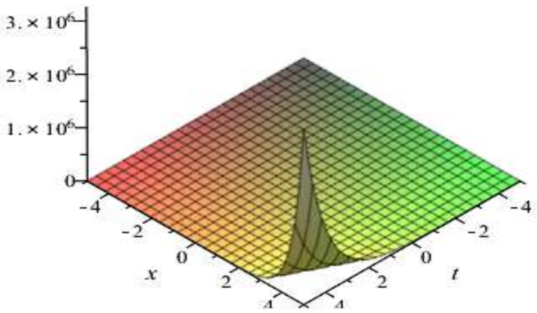

Fig. 1b: Exact solution of problem 1

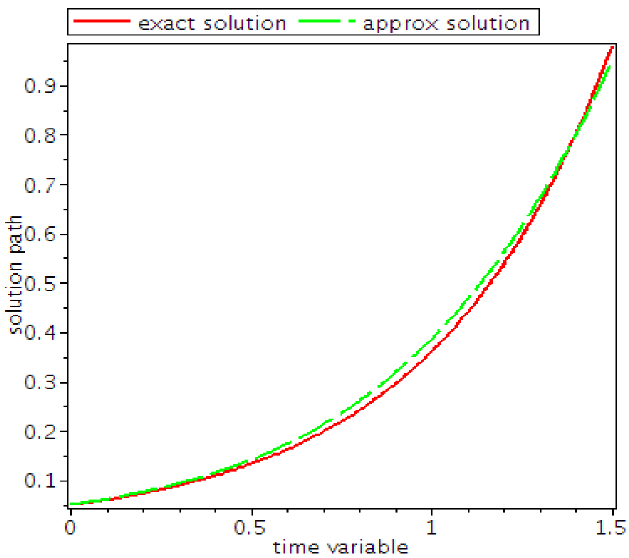

Fig. 2: Exact and He's polynomial solutions of problem 2

V. Concluding Remarks

In this paper, approximate-analytical solutions of the generalized Newell-Whitehead-Segel model by means of He's polynomials solution method were considered. Based on the solved illustrative problems: linear and nonlinear forms of the NWSM with efficiency and reliability of the proposed method being guaranteed by the results. We therefore, recommend the method for applications regarding problems arising from other areas of pure and applied sciences.

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References

[1] A.M. Wazwaz, M.S. Mehanna, 'The combined Laplace–Adomian method for handling singular integral equation of heat transfer", *Int J Nonlinear Sci.* **10** (2010): 248-52.

[2] S. O. Edeki, *Member*, G. O. Akinlabi., and A. S. Osheku, "On a Modified Iterative Method for the Solutions of Advection Model", World Congress on Engineering 2017, WCE 2017, London, U.K. (*Accepted*).

[3] J.H. He, "A coupling method of homotopy techniques and perturbation technique for nonlinear problems", *International Journal of Non-Linear Mechanics*, **35**(1) (2000): 37-43.

[4] S. O. Edeki, G. O. Akinlabi, and S. A. Adeosun, "On a modified transformation method for exact and approximate solutions of linear Schrödinger equations", AIP Conference Proceedings **1705**, 020048 (2016); doi: 10.1063/1.4940296.

[5] G.O. Akinlabi and S.O. Edeki "On Approximate and Closed-form Solution Method for Initial-value Wave-like Models", International Journal of Pure and Applied Mathematics, 107(2), (2016): 449–456.

[6] H. K. Mishra and A. K. Nagar, "He-Laplace Method for Linear and Nonlinear Partial Differential Equations," Journal of Applied Mathematics, Vol. 2012, (2012): 1-16.

[7] G.O. Akinlabi and S. O. Edeki, "The solution of initial-value wave-like models via Perturbation Iteration Transform Method," Lecture Notes in Engineering and Computer Science: Proceedings of the International MultiConference of Engineers and Computer Scientists 2017, IMECS 2017, 15-17 March, 2017, Hong Kong, pp 1015-1018.

[8] S.O. Edeki, G.O. Akinlabi and S.A. Adeosun, "Analytic and Numerical Solutions of Time-Fractional Linear Schrödinger Equation", Comm Math Appl, 7(1), (2016): 1–10.

[9] G.O. Akinlabi and S.O. Edeki "Perturbation Iteration Transform Method for the Solution of Newell-Whitehead-Segel Model Equations", Journal of Mathematics and Statistics, arXiv preprint arXiv:1703.06745 .

[10] P. Pue-on, Laplace Adomian Decomposition Method for Solving Newell-Whitehead-Segel Equation, *Applied Mathematical Sciences*, **7**, 2013, no. 132, 6593 – 6600.

[11] J. Patade, and S. Bhalekar, "Approximate analytical solutions of Newell-Whitehead-Segel equation using a new iterative method", *World Journal of Modelling and Simulation*, **11** (2), (2015): 94-103.

[12] S.O. Edeki, O.O. Ugbebor, and E.A. Owoloko, "He's polynomials for analytical solutions of the Black-Scholes pricing model for stock option valuation," Lecture Notes in Engineering and Computer Science: Proceedings of the World Congress on Engineering 2016, WCE 2016, 29 June – 1 July, 2016, London, U.K., pp 632-634.

[13] A. Ghorbani and J. S. Nadjfi, "He's homotopy perturbation method for calculating Adomian's polynomials", *Int. J. Nonlin. Sci. Num. Simul*. **8** (2) (2007): 229-332.

[14] J.H. He, "Homotopy perturbation method: A new nonlinear analytical technique", *Appl. Math. Comput.* **135** (2003):73-79.

[15] J. Saberi-Nadjafi, A. Ghorbani, "He's homotopy perturbation method: an effective tool for solving nonlinear integral and integro-differential equations", *Computers & Mathematics with Applications*, **58**, (2009):1345–1351.

[16] J. Singh, D. Kumar and S. Rathore, "Application of Homotopy Perturbation Transform Method for Solving Linear and Nonlinear Klein-Gordon Equations", *Journal of Information and Computing Science*, **7** (2), (2012): 131-139.