Exact Solutions of Some Complex Non-Linear Equations

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***Abstract*—Exact solutions of the coupled Higgs and Maccari system are obtained. Travelling wave solutions of coupled Higgs equation and Maccari system in the form of Jacobi's elliptical functions are presented.**

***Index Terms*—exact solutions, coupled Higgs equation, Maccari system**

I. Introduction

COMPLEX physical phenomena in various fields of sciences, especially in fluid mechanics, solid state physics, plasma physics, plasma wave and chemical physics are represented by nonlinear evolution equations (NEEs). Analytical solutions of such equations are of fundamental importance. In the literature, quite a few methods have been proposed for constructing explicit travelling and solitary wave solutions of nonlinear evolution equations, such as the inverse scattering method [1], tanh-sech method by author(s) [8], [10], sine cosine method [2], [9], [11], ansatz method [4], etc.

The coupled Higgs equation [6]

$$\begin{array}{l} u_{tt} - u_{xx} + |u|^2 u - 2uv = 0 \\ v_{tt} + v_{xx} - (|u|^2)_{xx} = 0, \end{array} \tag{1}$$

describes a system of conserved scalar nucleons interacting with neutral scalar mesons.

Attilio Maccari derived a new integrable (2 + 1)-dimensional nonlinear system [5]

$$\begin{array}{l} iu_t + u_{xx} + uv = 0 \\ v_t + v_y + (|u|^2)_x = 0. \end{array} \tag{2}$$

The integrability property was explicitly demonstrated and the Lax pair was also obtained.

Bekir in [2] looked for exact solutions of the coupled Higgs [6] and Maccari system [5] using the tanh-coth [10] and the sine-cosine [7], [11] methods.

II. Travelling wave solutions

A. Coupled Higgs equation

Assume that coupled Higgs equation (1) has a travelling wave solution in the form

$$u = e^{i\theta}U(\xi), \quad v = V(\xi), \quad \theta = px + rt, \quad \xi = x + ct, \tag{3}$$

where p, r, c are arbitrary constants. Substitution of (3) into Eq. (1) reduces the PDEs to system of ordinary differential equations (ODEs)

$$\begin{array}{l} (c^2-1)U'' + r^2(c^2-1)U - 2UV + U^3 = 0 \\ (c^2+1)V'' - 2(U')^2 - 2UU'' = 0, \end{array} \tag{4}$$

with condition $p = rc$.

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Integrating the second equation in the system and neglecting the constant of integration we get

$$(c^2+1)V = U^2 \tag{5}$$

Substituting (5) into first equation of system (4), we find

$$(c^2+1)U'' + r^2(c^2+1)U + U^3 = 0. \tag{6}$$

Integrating equation (6) we get

$$U'^2 = -\frac{U^4}{2(c^2+1)} - r^2U^2 + \frac{C_1}{2(c^2+1)}, \tag{7}$$

where C_1 is arbitrary constant. Again integrating we get following solution of equation (6)

$$U(\xi) = \frac{a_3\sqrt{2}sn\left(1/2\sqrt{2\,a_2 + 2\sqrt{a_2{}^2 + 4\,a_1a_3}}\xi + C_2, m\right)}{\sqrt{a_3\left(a_2 + \sqrt{a_2{}^2 + 4\,a_1a_3}\right)}}, \tag{8}$$

where sn is Jacobi elliptic sine function and

$$\begin{array}{l} m = \frac{\sqrt{-2\left(2\,a_1a_3 + a_2{}^2 + a_2\sqrt{a_2{}^2 + 4\,a_1a_3}\right)a_1a_3}}{2\,a_1a_3 + a_2{}^2 + a_2\sqrt{a_2{}^2 + 4\,a_1a_3}}, \\ a_1 = \frac{1}{2(c^2+1)}, \; a_2 = r^2, \; a_3 = \frac{C_1}{2(c^2+1)}. \end{array} \tag{9}$$

Corresponding solution of Higgs field equation is

$$\begin{array}{l} u(x,t) = \frac{a_3\sqrt{2}sn\left(1/2\sqrt{2\,a_2+2\sqrt{a_2{}^2+4\,a_1a_3}}(x+ct)+C_2,m\right)}{\sqrt{a_3\left(a_2+\sqrt{a_2{}^2+4\,a_1a_3}\right)}} e^{(ir(cx+t))} \\ v(x,t) = \frac{2a_3sn\left(1/2\sqrt{2\,a_2+2\sqrt{a_2{}^2+4\,a_1a_3}}(x+ct)+C_2,m\right)^2}{(c^2+1)\left(a_2+\sqrt{a_2{}^2+4\,a_1a_3}\right)}, \end{array} \tag{10}$$

where m, a_1, a_2 and a_3 are given by (9).

B. Maccari system

Let us consider the travelling wave solution of Maccari system (2) in the form

$$u = e^{i\theta}U(\xi), \quad v = V(\xi), \theta = px + qy + rt, \xi = x + y + ct, \tag{11}$$

and corresponding system of ODEs is

$$\begin{array}{l} U'' - (r + p^2)U + UV = 0 \\ (1-2p)V' + 2UU' = 0. \end{array} \tag{12}$$

with condition $c = -2p$.

Integrating the second equation and neglecting the constant of integration we find

$$(2p-1)V = U^2. \tag{13}$$

Substituting (13) into first equation of the system (12), we find

$$(1-2p)U'' - (1-2p)(r+p^2)U - U^3 = 0. \tag{14}$$

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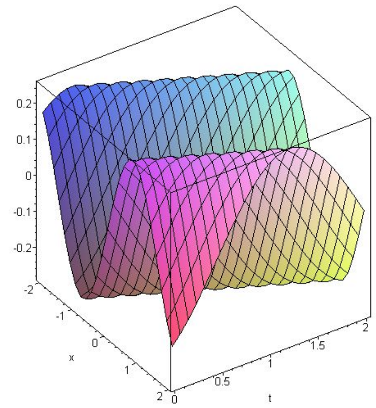

Fig. 1. Periodic solution $u(x,t)$ given by (10), for Higgs equation (1) with parameters $r=-1, c=-1, C_1=1, C_2=0$

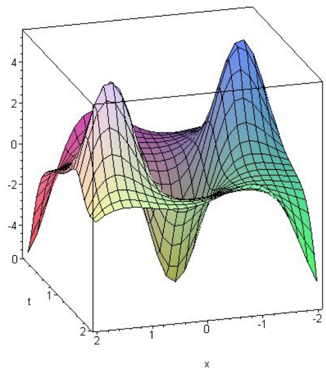

Fig. 2. Dromion solution u(x,t) given by (18) for Maccari system (2) with parameters $r=1, p=2, q=-1, y=1, C_1=-1, C_2=0$

Now we will give Jacobi elliptic sine function solution of Maccari system (2). Integrating equation (14) we get

$$U'^2 = \frac{U^4}{2(1-2p)} + (r+p^2)U^2 + \frac{C_1}{2(1-2p)}, \qquad (15)$$

where C_1 is arbitrary constant. Again integrating we get

$$U(\xi) = \frac{a_3\sqrt{2}sn\left(1/2\sqrt{-2\,a_2+2\sqrt{{a_2}^2-4\,a_1a_3}}\xi+C_2,m\right)}{\sqrt{a_3\left(-a_2+\sqrt{{a_2}^2-4\,a_1a_3}\right)}}, \qquad (16)$$

where sn is Jacobi elliptic sine function and m, a_1, a_2, a_3 are given by

$$m = \frac{\sqrt{-2\left(2\,a_1a_3-{a_2}^2+a_2\sqrt{{a_2}^2-4\,a_1a_3}\right)a_1a_3}}{2\,a_1a_3-{a_2}^2+a_2\sqrt{{a_2}^2-4\,a_1a_3}}, \qquad (17)$$
$$a_1 = \frac{1}{2(1-2p)}, \quad a_2 = r+p^2, \quad a_3 = \frac{C_1}{2(1-2p)}.$$

Corresponding solution of Maccari system (2) is given as

$$u(x,t) = \frac{a_3\sqrt{2}sn\left(1/2\sqrt{-2\,a_2+2\sqrt{{a_2}^2-4\,a_1a_3}}(x+y-2pt)+C_2,m\right)}{\sqrt{a_3\left(-a_2+\sqrt{{a_2}^2-4\,a_1a_3}\right)}}e^{(i(px+qy+rt))},$$
$$v(x,t) = \frac{2a_3sn\left(1/2\sqrt{-2\,a_2+2\sqrt{{a_2}^2-4\,a_1a_3}}(x+y-2pt)+C_2,m\right)^2}{(2p-1)\left(-a_2+\sqrt{{a_2}^2-4\,a_1a_3}\right)}, \qquad (18)$$

where m, a_1, a_2 and a_3 are given by (17).

III. Conclusion

In this paper, we have given the exact travelling wave solutions of both the complex systems,coupled Higgs field equation and Maccari system, in form of Jacobi elliptical functions and authenticity of all the solutions have been checked using software MAPLE.

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