Modelling of Renewable Resource Systems with an Analysis of Changes of the Individual Parameter

O. Karelin, A. Tarasenko, O. Barabash, M. Gonzalez-Hernandez, and J. Medina-Marin

Abstract— **Here we present a study on the dynamics of the individual parameters of resources. We carry out a detailed investigation of the graphics of the function of change of the individual parameter (weight) with time (cycles). We introduce an age parameter for elements of the system. Applying the principles of modelling of renewable resource systems developed in previous works and the theory of functional operators with shift, we have obtained mathematical models that take into account the age of elements of the systems with fixed individual parameters. Based on these models, possibilities to formulate economic ecological problems that use renewable resource systems and account of age are opened.**

Index Terms—**functional equations, shift, individual parameter, age of resource**

I. Introduction

In works [1], [2], principles of modelling were proposed and applied to the study of systems with one renewable resource. Cyclic models, where the initial state of the system coincides with the final state, were considered. The balance relation has the form

$$\nu(\mathrm{x})=\mathrm{d}(\mathrm{x})\nu[\beta(\mathrm{x})]+P(\mathrm{x},\nu(\mathrm{x}))+\mathrm{g}(\mathrm{x}), \qquad (1)$$

where $\nu(\mathrm{x})$ is the initial density of the elements of the system.

Here, we take natural mortality into account with the coefficient $\mathrm{d}(\mathrm{x})$, the process of reproduction is represented by the term $\mathrm{P}(\mathrm{x},\nu(\mathrm{x}))$, the process of artificial entry of elements into the system and extraction from the system are accounted for by the term $\mathrm{g}(\mathrm{x})$.

Manuscript received March 04, 2017; revised March 21, 2017. This work was supported by Project CB-2014-01/236816/10017 CONACYT.

O. Karelin is with Institute of Basic Sciences and Engineering, Hidalgo State Autonomous University, C.P. 42184, Pachuca, Hgo, Mexico (e-mail: karelin@uaeh.edu.mx).

A. Tarasenko is with Institute of Basic Sciences and Engineering, Hidalgo State Autonomous University, C.P. 42184, Pachuca, Hgo, Mexico (corresponding author, phone: 771-717-2000, fax: 771-717-2133, e-mail: anataras@uaeh.edu.mx).

M. Gonzalez-Hernandez is with Institute of Basic Sciences and Engineering, Hidalgo State Autonomous University, C.P. 42184, Pachuca, Hgo, Mexico (e-mail: mghdez@uaeh.edu.mx).

J. Medina-Marin is with Institute of Basic Sciences and Engineering, Hidalgo State Autonomous University, C.P. 42184, Pachuca, Hgo, Mexico (e-mail: jmedina@uaeh.edu.mx).

O. Barabash is with the Department of Enterprise Economics, Institute of Business, Economics and Information Technology, Odessa National Polytechnic University, Shevchenko Avenue 1, Odessa, Ukraine (e-mail: odessaaa48@gmail.com).

Conditions for the existence and uniqueness of the solution are formulated.

In works [3] - [5], we have continued the study of systems whose state depends on time and whose resources are renewable, using functional operators with shift. We made generalizations to systems with two resources. For functional operators with shift, inverse operators in weighted Hölder spaces were constructed. In modelling, interactions and reciprocal influence between these two resources were taken into account. We applied our results on invertibility of the operators to the study of balance equations. The balance relation has the form

$$\nu(\mathrm{x})=\mathrm{d}(\mathrm{x})\nu(\alpha(\mathrm{x}))+P_\nu(\mathrm{x})+W_\omega(\mathrm{x})+g(\mathrm{x}),$$

$$\omega(\mathrm{y})=\mathrm{c}(\mathrm{y})\omega(\gamma(\mathrm{y}))+P_\omega(\mathrm{y})+\mathrm{W}_\nu(\mathrm{y})+q(\mathrm{y}),$$

where $\nu(\mathrm{x})$ and $\omega(\mathrm{y})$ represent initial densities of the distribution of the group parameters by the individual parameters x and y for the resources λ_1 and λ_2.

The terms $P_\nu(\mathrm{x})$, $P_\omega(\mathrm{y})$ are responsable for the reproduction and the terms $W_\omega(\mathrm{x})$, $W_\nu(\mathrm{y})$ are responsable for the mutual influence and contain integrals with degenerate kernels:

$$P_\nu(\mathrm{x})=\sum_{i=1}^{n}P_i p_i(\mathrm{x}),\ P_1=\int_{\nu_0}^{\nu_1}\nu(\mathrm{x})dx...P_n=\int_{\nu_{n-1}}^{\nu_n}\nu(\mathrm{x})dx$$

$$P_\omega(\mathrm{y})=\sum_{i=1}^{n}Q_i q_i(\mathrm{y}),\ Q_1=\int_{\mu_0}^{\mu_1}\omega(\mathrm{y})dy...Q_m=\int_{\mu_{n-1}}^{\mu_n}\omega(\mathrm{y})dy$$

$$0=\nu_0<\nu_1<...<\nu_n=x_{\max},\ 0=\mu_0<\mu_1<...<\mu_n=y_{\max},$$

$$W_\nu(\mathrm{x})=\sum_{i=1}^{k}R_i r_i(\mathrm{x}),\ R_1=\int_{\tau_0}^{\tau_1}\nu(\mathrm{x})dx...\mathrm{R}_n=\int_{\tau_{k-1}}^{\tau_k}\nu(\mathrm{x})dx$$

$$W_\omega(\mathrm{y})=\sum_{i=1}^{l}F_i f_i(\mathrm{y}),\ F_1=\int_{\varepsilon_0}^{\varepsilon_1}\omega(\mathrm{y})dy...F_\varepsilon=\int_{\varepsilon_{n-1}}^{\varepsilon_n}\omega(\mathrm{y})dy$$

ISBN: 978-988-14047-9-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$0=\tau_0<\tau_1<...<\tau_k=x_{\max}$, $0=\varepsilon_0<\varepsilon_1<...<\varepsilon_l=y_{\max}$.

For the solution of the balance integral equation with degenerate kernels and inverse operators, a modified Fredholm method [6], [7] for equations of second type is proposed. The equilibrium state of the system is found.

Systems whose state depends on time and whose resources are renewable form an important class of general systems. A great number of works has been dedicated to systems with renewable resources [8], [9]. The core of the mathematical apparatus used for the study of such systems consists of differential equations in which the sought for function is dependent on time [10] - 12]. Our approach presupposes discretization of the processes with respect to time. We move away from tracking the changes in the system continuously to tracking the changes at fixed time points. This discretization and the identification of the individual parameter and the group parameter lead us to functional equations with shift.

II. Change of the Individual Parameter

In this article, we carry out an investigation of the changes in the individual parameter of elements of the system in time.

The function $\alpha=\alpha(x)$ shows how the individual parameter x changes, during at the time interval T that corresponds to one cycle. If an object has weight x inside period T, its weight is transformed to the weight $\alpha(x)$, as shown in

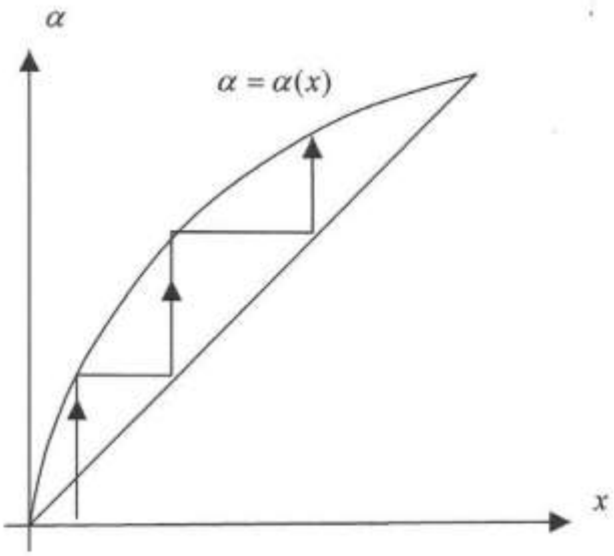

Fig. 1.

A graphical representation of how the object with individual parameters x increases its weight.

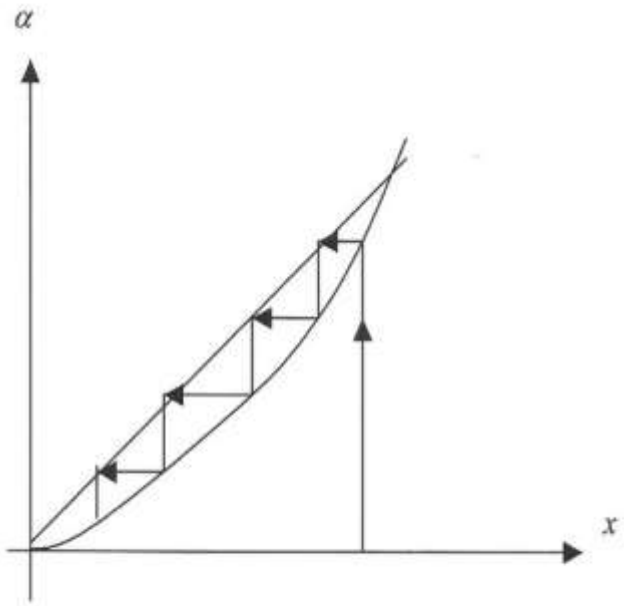

Fig. 2.

A graphical representation of how the object loses its weight from one cycle to the other.

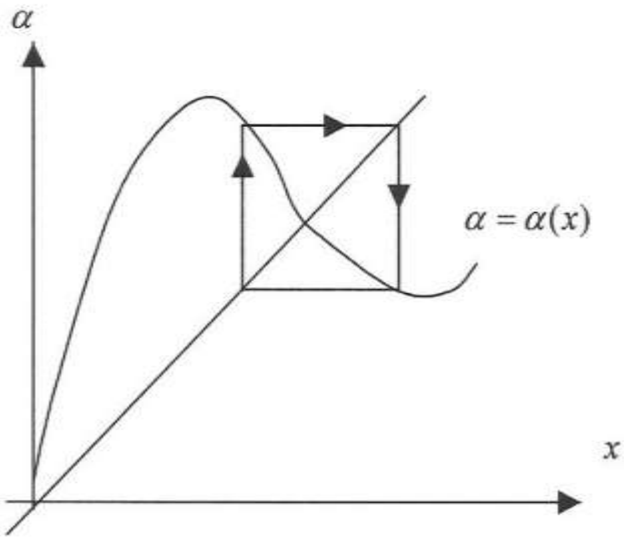

Fig. 3

In Figure 3, the condition for cyclic trajectories is the existence symmetrical points with respect to the bisector $y=x$. The point has the coordinates $(x_0,\alpha(x_0))$ and the other has the coordinates $(\alpha(x_0),x_0)$ and both belong to the graphic of the function $y=\alpha(x)$. In the special case, then both points coincide the degenerate cycle result. In this case $\alpha(x_0)=x_0$.

More complex cyclic trajectories are represented in Figure 4.

ISBN: 978-988-14047-9-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

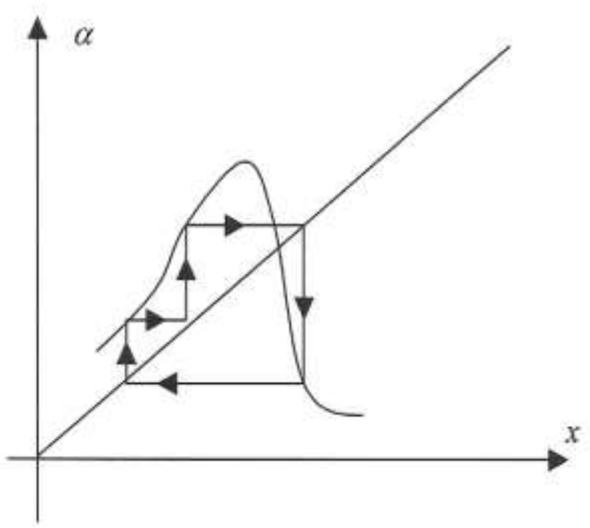

Fig. 4.

Let us consider the following figure.

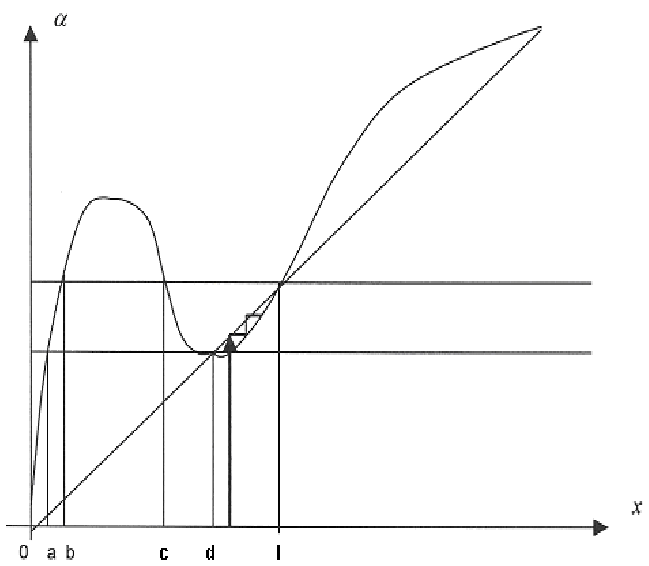

Fig. 5.

Objects with weight from the interval (d,l) transform into objects with weight from the same interval $x \in (d,l)$, $\alpha(x) \in (d,l)$, $d = \alpha(d)$, $l = \alpha(l)$.
Objects this weight from the interval (d,l) transform.

Objects with weight $x \in (d,l)$ will not be able to jump from the interval (d,l) and their weight will not be able to exceed l. In this interval, the weight of objects is found for which the conditions are disadvantageous, these objects are worn out and decrease their weight. According to Figure 5, objects with weight $x \in (a,b) \bigcup (c,d)$ also transform to the interval (d,l) and safer disadvantageous conditions. The objects with weight $x \in (b,c)$ come out of this interval $x \in (b,c)$ and thereafter increase their weight in proximal cycles.

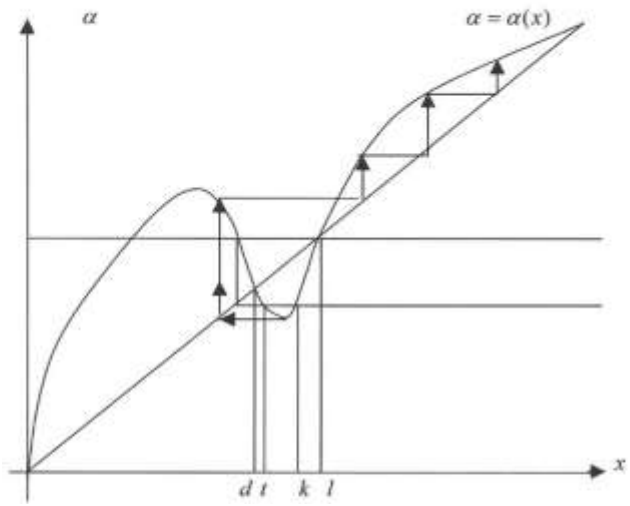

Fig. 6.

The interval (d,l) on Figure 6 has a more complex behavior. The objects that are in this interval can leave it. Those objects that have weight lesser than the value $\alpha(d)$: $x \in (t,k)$ come out of the rectangular area as indicated by the arrows.

Let us consider dependence on age. In Figure 7, two different objects at one time have positions x^* and x^{**} on the axis of weight *OX*. Both reach the weight *M*. The life histories of the objects x^* and x^{**} are, however, different: object x^* is "young" and in two cycles achieves position *M*. Object x^{**} uses four cycles to obtain weight *M*, *so* it is "old" when it achieves position *M*. The future of these two objects will have to be different.

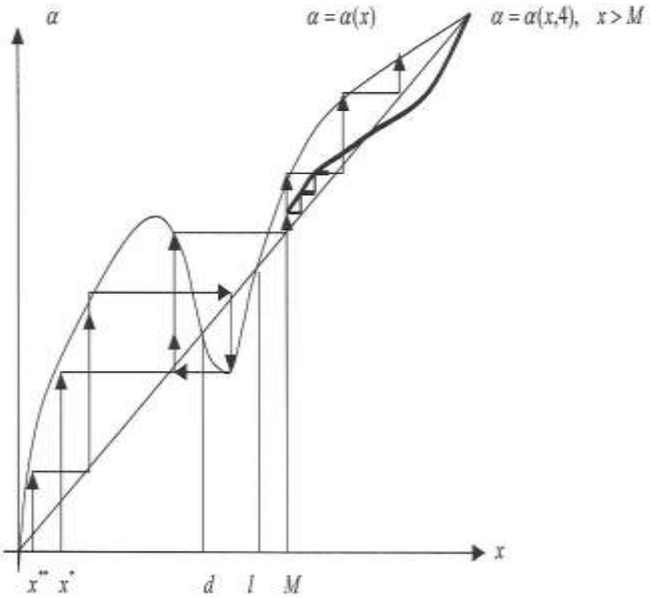

Fig. 7. Dependence on age.

ISBN: 978-988-14047-9-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

We propose to take into account cycles (age). Consider the function $\alpha = \alpha(x,e)$ with a continuous variable - individual parameter x and another discrete variable - number of cycles. The graph of development of object x^* is $\alpha = \alpha(x,2), \quad x > M$ and the graph of development of object x^{**} is $\alpha = \alpha(x,4), \quad x > M$.

We assume that the individual and the group parameters for both resources are the same, and the ranges of variation of the individual parameters are different. Resource λ_1 has a range of variation equal to $I_1 = (0, \mathrm{x}^1{}_{\max})$ and resource λ_2 has a range of variation equal to $I_2 = (0, \mathrm{x}^2{}_{\max})$. Time interval $[t_0, t_0 + T]$ is divided into subintervals T_k, $[t_0, t_0+T] = \bigcup T_k$. The subintervals are chosen taking into account the changes taking place in system S and the human activity.

To avoid introducing new notation, the symbols are independent and are not related to the symbols in the previous sections.

Let $b_{1k}(\mathrm{x}, e)$ be a shift which describes the change of the individual parameter x of the resource λ_1 during $T_k = [\mathrm{t}_{k-1}, \mathrm{t}_k]$. For the resource λ_2 we denote the corresponding shift by β_{2k}.

Let $b_{1k}(\mathrm{x}, e)$, $b_{2k}(\mathrm{x}, e)$ be differentiable functions and their inverse functions be $a_{1k}(\mathrm{x}, e)$, $a_{2k}(\mathrm{x}, e)$.

Applying the approach proposed in [5] we obtain a system of balance relations with non-Carleman shifts $a_{1k}(\mathrm{x}, e)$, $a_{2k}(\mathrm{x}, e)$:

$$\nu_{1k+1}(\mathrm{x}) = (\mathrm{A}_{1k}\, \nu_{1k})(\mathrm{x}),\ \ \nu_{2k+1}(\mathrm{x}) = (\mathrm{A}_{2k}\, \nu_{2k})(\mathrm{x}),$$

where

$$(\mathrm{A}_{1k}\, \nu_{1k})(\mathrm{x}) =$$

$$(\mathrm{B}_{1k}\, \nu_{1k})(\mathrm{x}) + (\mathrm{B}_{1k}\, q_{1k})(\mathrm{x}) + P_{1k}(x) + R_{1k}(x) - g_{1k}(x),$$
$$(\mathrm{B}_{1k}\, n_{1k})(\mathrm{x}) = d_{1k}(x,e) n_{1k}\left[a_{1k}(x,e)\right],$$

and

$$(\mathrm{A}_{2k}\, \nu_{2k})(\mathrm{x}) =$$

$$(\mathrm{B}_{2k}\, \nu_{2k})(\mathrm{x}) + (\mathrm{B}_{2k}\, q_{2k})(\mathrm{x}) + P_{2k}(x) + R_{2k}(x)\, \mathrm{p}_{2k}(x) - g_{2k}(x),$$

where $\nu_{1k}(x)$ and $\nu_{2k}(x)$ are densities at the moment t_k of the objects with individual parameters x for the first and the second resources respectively.

Here, the processes of reducing the group parameters (natural mortality) are described by d_{ik}. The terms P_{1k}, P_{2k} are responsible for the processes of natural increase of the group parameters (reproduction), the terms R_{1k}, R_{2k} are responsible for the reciprocal influence between resources. They have the same form as their corresponding terms in the balance equations and contain integrals with degenerate kernels.

The extractions from the system as a result of human economic activity are represented by $g_{ik}(x)$.

The artificial input into the system is accounted for by q_{ik} the beginning of period T_k and the actions related to g_{ik} take place at the end of period T_k.

We will assume that the actions related to q_{ik} take place at

We will assume that the actions related to q_{ik} take place at the beginning of period T_k and the actions related to g_{ik} take place at the end of period T_k.

State of the system $\nu_{1k}(x)$, $\nu_{2k}(x)$ at the moments t_k are described by the two presiding equations.

Note that we do not require for the final state and the initial state of system S to coincide. We obtain an open system $\nu_{1k+1}(x) \neq \nu_{1k}(x), \quad \nu_{2k+1}(x) \neq \nu_{2k}(x)$.

III. Exploitation Problems for Open Resource Systems

Let us formulate problems for the exploitation of open resource systems.

We introduce into consideration a functional that estimates the system during the period $[t_0, t_0 + T]$

$$E = E^g - E^q.$$

Let C^g_{ikj} be some known constants

$$C^g_{ikj} = \int_{x_{ij}}^{x_{ij+1}} g_{ik}(\tau) d\tau,$$

where $g_{ik}(\tau)$ is the substraction of resource λ_i from the system during the period $T_k = [\mathrm{t}_{k-1}, \mathrm{t}_k]$.

Here, interval $\left[0,\, x^i_{\max}\right]$ is divided into

$0 = x_{i0} < x_{i1} < x_{i2} < \ldots < x^i_{\max}$.

We write the functional

$$E^g_{ik} = \sum C^g_{ikj} G^g_{ikj},$$

where C^g_{ikj} are some known constants (product prices). It provides an economical estimate of the cost of $g_{ik}(\tau)$.

The structure of the term E^q is similar.

ISBN: 978-988-14047-9-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Based on the presented model, it is possible to formulate some problems for the rational use of system S. What should the strategy for the selection of q_{ik} and g_{ik} on each interval T_k be, in order to reach the maximum value of E at the final moment t_0+T.

Thus, of course, you must comply with various restrictions. We list some. (To simplify the presentation and make it brief, we omit the dependence on age.)

Restrictions on functions $q_{ik}(x)$:

$$Q_{ikj} \le Q_{ikj}^{\max}, \quad Q_{ikj} = \int_{Iij} q_{ik}(\tau)d\tau,$$

and

$$E_{ikj}^{q\min} \le E_{ikj}^{q} \le E_{ikj}^{q\max}, \quad E_{ikj}^{q} = C_{ikj}^{q}Q,$$

$$E_{ikj}^{q\min} \le \sum_{j} E_{ikj}^{q} \le E_{ikj}^{q\max},$$

$$F_{i0}^{q\min} \le \sum_{k}\sum_{j} E_{ikj}^{q} \le F_{i0}^{q\max},$$

$$F_{0k}^{q\min} \le \sum_{i}\sum_{j} E_{ikj}^{q} \le F_{0k}^{q\max},$$

$$E^{q\min} \le E^{q} \le F^{q\max}.$$

Restrictions on functions $g_{ik}(x)$:

$$G_{ikj}^{g\min} \le G_{ikj}^{g} \le G_{ikj}^{g\max}, \quad G_{ikj}^{g} = \int_{Iij} g_{ik}(\tau)d\tau,$$

$$G_{ik}^{q\min} \le \sum_{j} G_{ikj}^{q} \le G_{ikj}^{g\max},$$

$$K_{i0}^{q\min} \le \sum_{k}\sum_{j} E_{ikj}^{q} \le K_{i0}^{q\max},$$

$$K_{0k}^{q\min} \le \sum_{i}\sum_{j} G_{ikj}^{g} \le K_{0k}^{g\max},$$

$$G^{g\min} \le G^{g} \le F^{g\max},$$

$$G_{00}^{g\min} \le G_{00}^{g} \le G_{00}^{g\max}, \quad G_{00}^{g} = \sum_{ik} G_{ik}.$$

The state of the system cannot be worse than the permissible state and ecologically acceptable norms must be met.

Restrictions on functions $\nu_{ik}(x)$:

$$N_{ikj}^{\min} \le N_{ikj}, \quad N_{ikj} = \int_{Iij} \nu_{ik}(\tau)d\tau,$$

$$N_{ik0}^{\min} \le \sum_{j} N_{ikj}, \quad N_{0k0}^{\min} \le \sum_{i} N_{ik}.$$

The functions $\nu_{ik}(x),\ q_{ik}(x),\ g_{ik}(x)$ must be non-negative.

Authors plan to make simplifications of the form of functions, functionals and algorithms for applications to solving specific problems, and submit it to the following WCE congresses.

IV. CONCLUCION

Equations with shift appear on modelling systems with renewable resources. The theory of linear functional operators with shift is the adequate mathematical instrument for the investigation of such systems. Based on the models, it is possible to formulate and to study problems for the rational use of open resource systems.

REFERENCES

[1] A. Tarasenko, O. Karelin, G. P. Lechuga, and M. G. Hernandez, “Modelling systems with renewable resources based on functional operators with shift,” *Applied Mathematics and Computations*, vol. 216, Issue 7, pp. 1938-1944, 2010.

[2] O. Karelin, A. Tarasenko, and M. G. Hernandez, "Application of functional operators with shift to the study of renewable systems when the reproductive processed is described by integrals with degenerate kernels," *Applied Mathematics (AM)*, vol. 4, pp. 1376--1380, 2013.

[3] O. Karelin, M. Gonzalez-Hernandez, N. Hernandez-Romero, and J Seck-Tuoh-Mora, “Study of renewable systems based on functional operators with shift”, *Applied Mathematical Sciences*, vol. 9, no. 144, Hikari Ltd, pp. 7167-7173, 2015.

[4] O. Karelin, A. Tarasenko, V. Zolotov, and M. Gonzalez Hernandez, “Mathematical Model for the Study of the Equilibrium State Renewable Systems Based on functional operators with shift,” *Lecture Notes in Engineering and Computer Science: Proceedings of the World Congress on Engineering, 5-7 July, 2017, London, U.K.*, vol. 1, pp. 40—44, 2017.

[5] O. Karelin, A. Tarasenko, V. Zolotov, and M. Gonzalez Hernandez, “Study of the equilibrium state of systems with two renewable resources based on functional operators with shift,” IAENG International Journal of Applied Mathematics, vol. 48, no. 1, pp. 98—104, 2018.

[6] K. E. Atkinson, The Numerical Solution of Integral Equations of the Second Kind, Cambridge University Press, 1997.

[7] A. J. Jerry, Introduction to Integral Equations with Application, 2nd ed. Yohn Wiley@Sons, 1999.

[8] C. W. Clark, Mathematical Bioeconomics: The Optimal Management of Renewable Resources, 2nd ed. Yohn Wiley@Sons, New York, 1990.

[9] Y. Xiao, D. Cheng, and H. Qin, "Optimal Impulsive control in periodic ecosystem," *Systems and Control Letters*, vol. 55, no.2, pp. 558-565, 2006.

[10] M. Bohner, and A. Peterson, Dynamic Equations on Time Scales: An Introduction with Applications, Birkhauser, Boston, 2001.

[11] L. Wang, and M. Hu, "Nonzero Periodic Solutions in Shifts Delta (+/-) for a Higher-Dimensional Nabla Dynamic Equation on Time Scales," IAENG International Journal of Applied Mathematics, vol. 47, no.1, pp. 37-42, 2017.

[12] L. Yang, Y. Yang, Y. Li, and T. Zhang, "Almost Periodic Solution for a Lotka-Volterra Recurrent Networks with Harvesting Terms on time Scales," *Engineering Letters*, vol. 24, no.4, pp. 455-460, 2016.