Comparison of the Effectiveness of Degradation Test and Life Test Based on Wiener Degradation Failure Product

Xinyu Ma, Guang Jin

***Abstract*—For the Wiener degradation failure products, the effects of life test and degradation test on product reliability evaluation are studied and compared respectively. Firstly, in the cases of no measurement error, measurement error and different number of measurement in the degradation test, the asymptotic variance of the estimated p-percentile of the product's lifetime distribution is given, and we research the influence of measurement error on evaluation accuracy. Furthermore, under the constraint that the total experimental cost does not exceed a predetermined budget, we set the asymptotic variance of the estimated p-percentile of the product's lifetime distribution as reliability evaluation accuracy index. In the same evaluation accuracy, we compare the optimal design problem in time censored life test with that in degradation test in different situations. Researches have shown that compared with the life test, the degradation test has obvious advantages for improving the accuracy of product reliability evaluation, which can significantly reduce the sample size and fully use the advantages of test time. The research in this paper can provide reference value for the optimization of long-life product reliability evaluation.**

***Index Terms*—Wiener process; life test; degradation test; Fisher information; optimal design.**

I. Introduction

In order to clarify the life distribution of the product, estimate the reliability indicators of the product, study the failure mechanism of the product, we often conduct reliability tests. Traditional reliability test evaluations use life tests to estimate parameter distributions from life data. However, with the continuous extension of product life, the general life test cannot obtain enough life information under the limited time and constraints of cost. Then people propose a degradation-based reliability technology. Degradation-based reliability technology provides a new technical approach to solve the problem of long-life product reliability assessment. Through the degradation test, from studying the product failure mechanism, we analyze the product failure correlation and degradation failure law to obtain performance information. There are some studies on life test and degradation test. A detailed discussion of the maximum likelihood estimation of the failure time data is given by Lawless[1]. Lu and Meeker[2] discuss the method of using degraded data to estimate the failure time distribution. Lu and Meeker[3] define the relative efficiencies to compare the asymptotic efficiency of degradation analysis with that of traditional failure time in life analysis.

We generally consider that, compared with the life test, the test data can be fully utilized by degradation test and the degradation process modeling, and high reliability evaluation accuracy can be obtained. However, the current conclusions are mainly qualitative judgments and lack of quantitative comparative studies. In this paper, the asymptotic variance of product's reliable life (the percentile of the product's lifetime) is used as the accuracy index. The quantitative comparison study is carried out, based on the role of life test and degradation test in the modeling and evaluation of long-life product reliability. Firstly, in the cases of no measurement error, measurement error and different number of measurement in the degradation test, we give limit form of asymptotic variance of reliable life of products based on degradation test data. Then we study the optimal design problems for degradation tests and life tests. The optimal variables are sample size and censored time. With constraint of total experimental cost, the optimal settings of these variables are obtained by minimizing reliable life assessment accuracy. Finally, we compare the result in time censored life test with that in degradation test under different number of measurement.

II. Wiener Degradation Product Reliability Model

We know that the lifetime of classical Wiener degradation failure product obeys inverse Gaussian (IG) [4], and its distribution function and density function are respectively

$$F(t)=\Phi\left(\frac{\mu t-l}{\sigma\sqrt{t}}\right)+\exp\left(\frac{2\mu l}{\sigma^2}\right)\Phi\left(-\frac{\mu t+l}{\sigma\sqrt{t}}\right) \tag{2.1}$$

$$f(t)=\frac{l}{\sqrt{2\pi\sigma^2 t^3}}\exp\left[-\frac{(l-\mu t)^2}{2\sigma^2 t}\right] \tag{2.2}$$

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Xinyu Ma, 1995.3, female, master, College of Systems Engineering, National University of Defense Technology, Changsha, 410073, major research direction reliability engineering, phone: 18890067587; E-mail: 1157607735@qq.com.

Guang Jin, 1973.1, male, researcher, Ph.D., College of Systems Engineering, National University of Defense Technology, Changsha, 410073, main research direction: reliability engineering, prediction and health management, system testing and evaluation, E-mail: jinguang@nudt.edu.cn

ξ_p is $100p^{th}$ percentile of the product's lifetime distribution, and $\hat{\xi}_p = F^{-1}(p)$. By using the δ-method, the asymptotic variance of $100p^{th}$ percentile of the product's lifetime distribution $\mathrm{Aver}(\hat{\xi}_p)$ is

$$\mathrm{Aver}(\hat{\xi}_p) = \frac{1}{(f(F^{-1}(p)))^2}\mathbf{H}\cdot\mathbf{I}\cdot\mathbf{H}' \tag{2.3}$$

Where

$$\mathbf{H} = \begin{pmatrix} \frac{\partial F}{\partial \theta_1} & \frac{\partial F}{\partial \theta_2} & \cdots & \frac{\partial F}{\partial \theta_q} \end{pmatrix}$$

which is the value of the first-order partial derivative of inverse Gaussian distribution function;

$$\mathbf{I} = \mathrm{E}\begin{bmatrix} -\frac{\partial^2 L}{\partial \theta_1^2} & -\frac{\partial^2 L}{\partial \theta_1 \partial \theta_2} & \cdots & -\frac{\partial^2 L}{\partial \theta_1 \partial \theta_q} \\ & -\frac{\partial^2 L}{\partial \theta_2^2} & \cdots & -\frac{\partial^2 L}{\partial \theta_2 \partial \theta_q} \\ & & \ddots & \vdots \\ & & & -\frac{\partial^2 L}{\partial \theta_q^2} \end{bmatrix}^{-1} = \begin{bmatrix} E(-\frac{\partial^2 L}{\partial \theta_1^2}) & E(-\frac{\partial^2 L}{\partial \theta_1 \partial \theta_2}) & \cdots & E(-\frac{\partial^2 L}{\partial \theta_1 \partial \theta_q}) \\ & E(-\frac{\partial^2 L}{\partial \theta_2^2}) & \cdots & E(-\frac{\partial^2 L}{\partial \theta_2 \partial \theta_q}) \\ & & \ddots & \vdots \\ & & & E(-\frac{\partial^2 L}{\partial \theta_q^2}) \end{bmatrix}^{-1}$$

which is Fisher information matrix.

III. Reliability Modeling Based on Life Test Data

Considering the case of the timing censored life test, supposing the sample size is n, the censored time is T, and the failure time data is $X_1 < X_2 < \ldots < X_D < T$, D is the number of failed sample. Denoted $\mathbf{X}$ as the sample data, the likelihood function is [5]

$$L(\mu,\sigma^2|\mathbf{x}) = \left[\Phi\left(\sqrt{\frac{1}{\sigma^2 T}}(l-\mu T)\right) - e^{\frac{2\mu l}{\sigma^2}}\Phi\left(-\sqrt{\frac{1}{\sigma^2 T}}(\mu T + l)\right)\right]^{n-D} \times \prod_{i=1}^{D}\left(\frac{l}{2\pi\sigma^2 x_i^3}\right)^{1/2}\exp\left(-\frac{(\mu x_i - l)^2}{2\sigma^2 x_i}\right)$$

Calculate the second order partial derivative of the log-likelihood function for the parameter μ and σ^2, we get

$$\begin{aligned}\frac{\partial^2 L}{\partial(\sigma^2)^2} = {} & \frac{Dl}{2(\sigma^2)^2} - \sum_{i=1}^{D}\frac{(\mu x_i - l)^2}{x_i(\sigma^2)^3} + (n-D)\times \\ & \left[\frac{\varphi(\psi_{1T})L_{11T} - \psi_{1T}\varphi(\psi_{1T})L_{1T}^2 - (\frac{2\mu l}{(\sigma^2)^2})^2 e^{\frac{2\mu l}{\sigma^2}}\Phi(\psi_{2T}) - e^{\frac{2\mu l}{\sigma^2}}\varphi(\psi_{2T})M_{11T}4}{1-F(T,\sigma^2,\mu)} + \right. \\ & \frac{\frac{4\mu l}{(\sigma^2)^2}e^{\frac{2\mu l}{\sigma^2}}\varphi(\psi_{2T})M_{1T} - \frac{4\mu l}{(\sigma^2)^3}e^{\frac{2\mu l}{\sigma^2}}\Phi(\psi_{2T}) + e^{\frac{2\mu l}{\sigma^2}}\psi_{2T}\varphi(\psi_{2T})M_{1T}^2}{1-F(T,\sigma^2,\mu)} - \\ & \left.\frac{(\varphi(\psi_{1T})L_{1T} + \frac{2\mu l}{(\sigma^2)^2}e^{\frac{2\mu l}{\sigma^2}}\Phi(\psi_{2T}) - e^{\frac{2\mu l}{\sigma^2}}\varphi(\psi_{2T})M_{1T})^2}{(1-F(T,\sigma^2,\mu))^2}\right]\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 L}{\partial \mu^2} = {} & -\sum_{i=1}^{D}\frac{x_i}{\sigma^2} + (n-D)\left[\frac{\varphi(\psi_{1T})L_{22T} - \psi_{1T}\varphi(\psi_{1T})L_{2T}^2 - (\frac{2l}{\sigma^2})^2 e^{\frac{2\mu l}{\sigma^2}}\Phi(\psi_{2T})}{1-F(T,\sigma^2,\mu)} + \right. \\ & \frac{e^{\frac{2\mu l}{\sigma^2}}\psi_{2T}\varphi(\psi_{2T})M_{2T}^2 - \frac{4l}{\sigma^2}e^{\frac{2\mu l}{\sigma^2}}\varphi(\psi_{2T})M_{2T} - e^{\frac{2\mu l}{\sigma^2}}\varphi(\psi_{2T})M_{22T}}{1-F(T,\sigma^2,\mu)} - \\ & \left.\frac{(\varphi(\psi_{1T})L_{2T} - \frac{2l}{\sigma^2}e^{\frac{2\mu l}{\sigma^2}}\Phi(\psi_{2T}) - e^{\frac{2\mu l}{\sigma^2}}\varphi(\psi_{2T})M_{2T})^2}{(1-F(T,\sigma^2,\mu))^2}\right]\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 L}{\partial \sigma^2 \partial \mu} = {} & \sum_{i=1}^{D}(\frac{\mu x_i}{(\sigma^2)^2} - \frac{l}{(\sigma^2)^2}) + (n-D)\left[\frac{e^{\frac{2\mu l}{\sigma^2}}\varphi(\psi_{2T})\psi_{2T}M_{1T}M_{2T}}{1-F(T,\sigma^2,\mu)} + \right. \\ & \frac{\varphi(\psi_{1T})L_{12T} - e^{\frac{2\mu l}{\sigma^2}}\varphi(\psi_{2T})M_{12T} - \psi_{1T}\varphi(\psi_{1T})L_{2T}L_{1T} + \frac{2l}{(\sigma^2)^2}e^{\frac{2\mu l}{\sigma^2}}\Phi(\psi_{2T})}{1-F(T,\sigma^2,\mu)} + \\ & \frac{\frac{4\mu l^2}{(\sigma^2)^3}e^{\frac{2\mu l}{\sigma^2}}\Phi(\psi_{2T}) + \frac{2\mu l}{(\sigma^2)^2}e^{\frac{2\mu l}{\sigma^2}}\varphi(\psi_{2T})M_{2T} - \frac{2l}{\sigma^2}e^{\frac{2\mu l}{\sigma^2}}\varphi(\psi_{2T})M_{1T}}{1-F(T,\sigma^2,\mu)} - \\ & \frac{(\varphi(\psi_{1T})L_{2T} - \frac{2l}{\sigma^2}e^{\frac{2\mu l}{\sigma^2}}\Phi(\psi_{2T}) - e^{\frac{2\mu l}{\sigma^2}}\varphi(\psi_{2T})M_{2T})}{(1-F(T,\sigma^2,\mu))^2}\times \\ & \left.\frac{(\varphi(\psi_{1T})L_{1T} + \frac{2\mu l}{(\sigma^2)^2}e^{\frac{2\mu l}{\sigma^2}}\Phi(\psi_{2T}) - e^{\frac{2\mu l}{\sigma^2}}\varphi(\psi_{2T})M_{1T})}{(1-F(T,\sigma^2,\mu))^2}\right]\end{aligned}$$

Where $\varphi(\cdot)$ is the density function of the standard normal distribution, $\Phi(\cdot)$ is the standard normal distribution function, in addition,

$$\varphi'(y) = -y\varphi(y),\ \psi_{1T} = \sqrt{\frac{1}{\sigma^2 T}}(l-\mu T),\ \psi_{2T} = -\sqrt{\frac{1}{\sigma^2 T}}(l+\mu T),$$

$$L_{1T} = \frac{\partial \psi_{1T}}{\partial \sigma^2} = -\frac{1}{2\sqrt{T(\sigma^2)^3}}(l-\mu T),\ L_{2T} = \frac{\partial \psi_{1T}}{\partial \mu} = -\sqrt{\frac{T}{\sigma^2}},$$

$$M_{1T} = \frac{\partial \psi_{2T}}{\partial \sigma^2} = \frac{1}{2\sqrt{T(\sigma^2)^3}}(l+\mu T),\ M_{2T} = \frac{\partial \psi_{2T}}{\partial \mu} = -\sqrt{\frac{T}{\sigma^2}} = L_{2T},$$

$$L_{11T} = \frac{\partial^2 \psi_{1T}}{\partial(\sigma^2)^2} = \frac{3}{4\sqrt{T(\sigma^2)^5}}(l-\mu T),\ L_{12T} = \frac{\partial^2 \psi_{1T}}{\partial \sigma^2 \partial \mu} = \sqrt{\frac{T}{4(\sigma^2)^3}},$$

$$L_{22T} = \frac{\partial^2 \psi_{1T}}{\partial \mu^2} = 0,\ M_{11T} = \frac{\partial^2 \psi_{2T}}{\partial(\sigma^2)^2} = -\frac{3}{4\sqrt{T(\sigma^2)^5}}(l+\mu T),$$

$$M_{12T} = \frac{\partial^2 \psi_{2T}}{\partial \sigma^2 \partial \mu} = \sqrt{\frac{T}{4(\sigma^2)^3}},\ M_{22T} = \frac{\partial^2 \psi_{2T}}{\partial \mu^2} = L_{22T}.$$

Then, in the time censored life test, the variance-covariance matrix of the model parameters' maximum likelihood estimation is

$$\mathbf{I}(\mu,\sigma^2) = E\begin{bmatrix} -\frac{\partial^2 L}{\partial \mu^2} & -\frac{\partial^2 L}{\partial \mu \partial \sigma^2} \\ -\frac{\partial^2 L}{\partial \mu \partial \sigma^2} & -\frac{\partial^2 L}{\partial(\sigma^2)^2} \end{bmatrix}^{-1} \tag{3.1}$$

$$\mathbf{H} = (\frac{\partial F}{\partial \mu}, \frac{\partial F}{\partial \sigma^2})\Big|_{\mu=\hat{\mu},\sigma^2=\hat{\sigma}^2} \tag{3.2}$$

Where

$$\begin{aligned}\frac{\partial F}{\partial \sigma^2} = {} & \frac{1}{2\sqrt{\xi_p(\sigma^2)^3}}(1-\mu\xi_p)\varphi(\sqrt{\frac{1}{\sigma^2\xi_p}}(\mu\xi_p - 1)) + \frac{1}{2\sqrt{\xi_p(\sigma^2)^3}}(1+\mu\xi_p)\times \\ & e^{\frac{2\mu}{\sigma^2}}\varphi(-\sqrt{\frac{1}{\sigma^2\xi_p}}(1+\mu\xi_p)) - \frac{2\mu}{(\sigma^2)^2}e^{\frac{2\mu}{\sigma^2}}\Phi(-\sqrt{\frac{1}{\sigma^2\xi_p}}(1+\mu\xi_p))\end{aligned}$$

$$\begin{aligned}\frac{\partial F}{\partial \mu} = {} & \frac{\sqrt{\xi_p}}{\sigma^2}\varphi(\sqrt{\frac{1}{\sigma^2\xi_p}}(\mu\xi_p - 1)) + \frac{2}{\sigma^2}e^{\frac{2\mu}{\sigma^2}}\Phi(-\sqrt{\frac{1}{\sigma^2\xi_p}}(1+\mu\xi_p)) - \\ & \frac{\sqrt{\xi_p}}{\sigma^2}e^{\frac{2\mu}{\sigma^2}}\varphi(-\sqrt{\frac{1}{\sigma^2\xi_p}}(1+\mu\xi_p))\end{aligned}$$

Substituting the **H** and **I** matrices into the formula (2.3), the asymptotic variance of $\hat{\xi}_p$ can be calculated.

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IV. Reliability Modeling Based on Degradation Test Data

Suppose the sample size is n, the test duration is T, the number of measurement for sample i is m_i, the observation interval is Δt_{ij} and the observed data is Δx_{ij}, which is the performance change of sample i between two adjacent measuring times.

A. Case 1: without measurement error

When there is no measurement error, the likelihood function of the degradation test data is as the following, according to the independent increment property of Wiener process:

$$L(\mu,\sigma^2)=\prod_{i=1}^{n}\prod_{j=1}^{m_i}\frac{1}{\sqrt{2\sigma^2\pi\Delta t_{ij}}}\exp[-\frac{(\Delta x_{ij}-\mu\Delta t_{ij})^2}{2\sigma^2\Delta t_{ij}}]$$

After calculating log likelihood function, the second order partial derivative of the log-likelihood function for the parameter μ and σ^2 are

$$\frac{\partial^2 l}{\partial\mu^2}=-\frac{nT}{\sigma^2}\qquad \frac{\partial^2 l}{\partial(\sigma^2)^2}=\frac{mn}{2(\sigma^2)^2}-\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{(\Delta x_{ij}-\mu\Delta t_{ij})^2}{(\sigma^2)^3\Delta t_{ij}}$$

$$\frac{\partial^2 l}{\partial\mu\partial\sigma^2}=\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{-\Delta x_{ij}+\mu\Delta t_{ij}}{(\sigma^2)^2}$$

Expected values are

$$E(\frac{\partial^2 l}{\partial\mu^2})=-\frac{nT}{\sigma^2}\qquad E(\frac{\partial^2 l}{\partial(\sigma^2)^2})=-\frac{mn}{2(\sigma^2)^2}\qquad E(\frac{\partial^2 l}{\partial\mu\partial\sigma^2})=0$$

So the Fisher information is

$$\mathbf{I}(\mu,\sigma^2)=\begin{bmatrix}\frac{nT}{\sigma^2} & 0\\ 0 & \frac{mn}{2(\sigma^2)^2}\end{bmatrix}^{-1}$$

The estimation accuracy of $\hat{\xi}_p$ is

$$\mathrm{Aver}(\hat{\xi}_p)=\frac{1}{\left[f\left(\xi_p\right)\right]^2}\mathbf{H}\cdot\mathbf{I}\cdot\mathbf{H}'$$

$$=2\pi\sigma^2\xi_p^3\times\exp[\frac{(1-\mu\xi_p)^2}{\sigma^2\xi_p}]\times[\frac{\sigma^2}{nT}(\frac{\partial F}{\partial\mu})^2+\frac{2\sigma^4}{mn}(\frac{\partial F}{\partial\sigma^2})^2]$$

B. Case 2: with measurement error

For the convenience of description, only consider equal measurement interval. That is to say, for each sample i and measurement time j, observation interval $\Delta t_{ij}=\delta$. The number of measurements is the same for each sample, which is m and the observation interval $\delta=T/m$. Let the measurement error (standard error) is σ_R. For each sample i, the degradation test data is written as $\Delta\mathbf{X}_i=\left(\Delta x_{i,1},\cdots,\Delta x_{i,m}\right)^T$, then the likelihood function of parameters μ, σ^2, σ_R^2 is [6]

$$L(\mu,\sigma^2,\sigma_R^2)=(2\pi\sigma_R^2)^{-\frac{mn}{2}}\times|\Sigma|^{-\frac{n}{2}}\times$$
$$\prod_{i=1}^{n}\left\{\exp\left[-\frac{1}{2\sigma_R^2}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})^T\Sigma^{-1}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})\right]\right\}$$

There, $\mathbf{1}$ is m-dimensional column vector with all elements being 1.

$$\Sigma=\frac{\sigma^2}{\sigma_R^2}D+P$$

$$D=\frac{T}{m}\cdot\begin{bmatrix}1&0&\cdots&0\\0&1&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&1\end{bmatrix}_{m\times m},\quad P=\begin{bmatrix}2&-1&&0\\-1&2&\ddots&\\&\ddots&\ddots&-1\\0&&-1&2\end{bmatrix}_{m\times m}$$

Set the measurement accuracy index $\gamma=\frac{\sigma_R^2}{\sigma^2}$, we get

$$\Sigma=\frac{1}{\gamma}D+P$$

The second order partial derivative of the log-likelihood function for the μ, σ^2 and σ_R^2 are

$$\frac{\partial^2 L}{\partial\mu^2}=-\frac{n}{\sigma_R^2}(\delta\cdot\mathbf{1})^T\Sigma^{-1}(\delta\cdot\mathbf{1})$$

$$\frac{\partial^2 L}{\partial(\sigma^2)^2}=-\frac{n}{2}tr\left(-\frac{1}{\sigma_R^4}\Sigma^{-1}\cdot D\cdot\Sigma^{-1}\cdot D\right)-$$
$$\sum_{i=1}^{n}\frac{1}{\sigma_R^6}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})^T\Sigma^{-1}D\Sigma^{-1}D\Sigma^{-1}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})$$

$$\frac{\partial^2 L}{\partial(\sigma_R^2)^2}=\frac{mn}{2\sigma_R^4}-\frac{n}{2}tr\left(-\frac{\sigma^4}{\sigma_R^8}\Sigma^{-1}\cdot D\cdot\Sigma^{-1}\cdot D+\frac{2\sigma^2}{\sigma_R^6}\Sigma^{-1}\cdot D\right)-$$
$$\sum_{i=1}^{n}\frac{1}{\sigma_R^6}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})^T\Sigma^{-1}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})+$$
$$\sum_{i=1}^{n}\frac{2\sigma^2}{\sigma_R^8}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})^T\Sigma^{-1}D\Sigma^{-1}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})-$$
$$\sum_{i=1}^{n}\frac{\sigma^4}{\sigma_R^{10}}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})^T\Sigma^{-1}D\Sigma^{-1}D\Sigma^{-1}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})$$

$$\frac{\partial^2 L}{\partial\mu\partial\sigma^2}=-\frac{1}{2\sigma_R^4}\sum_{i=1}^{n}(\delta\cdot\mathbf{1})^T\Sigma^{-1}D\Sigma^{-1}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})-$$
$$\frac{1}{2\sigma_R^4}\sum_{i=1}^{n}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})^T\Sigma^{-1}D\Sigma^{-1}(\delta\cdot\mathbf{1})$$

$$\frac{\partial^2 L}{\partial\mu\partial\sigma_R^2}=-\frac{1}{2\sigma_R^4}\sum_{i=1}^{n}(\delta\cdot\mathbf{1})^T\Sigma^{-1}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})-$$
$$\frac{1}{2\sigma_R^4}\sum_{i=1}^{n}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})^T\Sigma^{-1}(\delta\cdot\mathbf{1})+$$
$$\frac{\sigma^2}{2\sigma_R^6}\sum_{i=1}^{n}(\delta\cdot\mathbf{1})^T\Sigma^{-1}D\Sigma^{-1}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})+$$
$$\frac{\sigma^2}{2\sigma_R^6}\sum_{i=1}^{n}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})^T\Sigma^{-1}D\Sigma^{-1}(\delta\cdot\mathbf{1})$$

$$\frac{\partial^2 L}{\partial\sigma^2\partial\sigma_R^2}=-\frac{n}{2}tr\left(\frac{\sigma^2}{\sigma_R^6}\Sigma^{-1}D\Sigma^{-1}D-\frac{1}{\sigma_R^4}\Sigma^{-1}D\right)-$$
$$\frac{1}{\sigma_R^6}\sum_{i=1}^{n}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})^T\Sigma^{-1}D\Sigma^{-1}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})+$$
$$\sum_{i=1}^{n}\frac{\sigma^2}{\sigma_R^8}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})^T\Sigma^{-1}D\Sigma^{-1}D\Sigma^{-1}(\Delta\mathbf{X}_i-\mu\cdot\delta\cdot\mathbf{1})$$

We can get the $\mathbf{H}$ matrix as

$$\boldsymbol{H}=(\frac{\partial F}{\partial\sigma^2},\frac{\partial F}{\partial\mu},\frac{\partial F}{\partial\sigma_R^2})\bigg|_{\sigma^2=\hat{\sigma}^2,\mu=\hat{\mu},\sigma_R^2=\hat{\sigma}_R^2}$$

Where

$$\frac{\partial F}{\partial\mu}=\frac{\sqrt{\xi_p}}{\sigma^2}\varphi(\sqrt{\frac{1}{\sigma^2\xi_p}}(\mu\xi_p-1))+\frac{2}{\sigma^2}e^{\frac{2\mu}{\sigma^2}}\Phi(-\sqrt{\frac{1}{\sigma^2\xi_p}}(1+\mu\xi_p))-$$
$$\frac{\sqrt{\xi_p}}{\sigma^2}e^{\frac{2\mu}{\sigma^2}}\varphi(-\sqrt{\frac{1}{\sigma^2\xi_p}}(1+\mu\xi_p))$$

$$\frac{\partial F}{\partial \sigma^2}=\frac{1}{2\sqrt{\xi_p(\sigma^2)^3}}(1-\mu\xi_p)\varphi(\sqrt{\frac{1}{\sigma^2\xi_p}}(\mu\xi_p-1))+$$
$$\frac{1}{2\sqrt{\xi_p(\sigma^2)^3}}(1+\mu\xi_p)e^{\frac{2\mu}{\sigma^2}}\varphi(-\sqrt{\frac{1}{\sigma^2\xi_p}}(1+\mu\xi_p))-$$
$$\frac{2\mu}{(\sigma^2)^2}e^{\frac{2\mu}{\sigma^2}}\Phi(-\sqrt{\frac{1}{\sigma^2\xi_p}}(1+\mu\xi_p))$$
$$\frac{\partial F}{\partial \sigma_R^2}=0$$

By substituting the **H** and **I** matrices into the formula (2.3), the asymptotic variance of the $\hat{\xi}_p$ under the degradation test can be calculated. When the numbers of measurement are 1, 2 and 3 respectively, limit form of measurement accuracy of degradation test and the asymptotic variance of the $\hat{\xi}_p$ is shown in the following.

When the number of measurement is 1

For each sample is only measured once, that is m=1, there is

$$\mathbf{I}(\mu,\sigma^2,\sigma_R^2)=\begin{bmatrix}\frac{nT}{\sigma^2}\cdot\frac{1}{1+2\gamma/T} & 0 & 0\\ 0 & \frac{n}{2(\sigma^2)^2}\cdot\frac{1}{(1+2\gamma/T)^2} & \frac{n}{(\sigma^2)^2}\cdot\frac{1/T}{(1+2\gamma/T)^2}\\ 0 & \frac{n}{(\sigma^2)^2}\cdot\frac{1/T}{(1+2\gamma/T)^2} & \frac{n}{(\sigma^2)^2}\cdot\frac{2/T^2}{(1+2\gamma/T)^2}\end{bmatrix}^{-1}$$

When measurement accuracy γ→0, the Fisher information matrix can be written as

$$\mathbf{I}(\mu,\sigma^2,\sigma_R^2)=\begin{bmatrix}\frac{nT}{\sigma^2} & 0 & 0\\ 0 & \frac{n}{2(\sigma^2)^2} & \frac{n/T}{(\sigma^2)^2}\\ 0 & \frac{n/T}{(\sigma^2)^2} & \frac{2n/T^2}{(\sigma^2)^2}\end{bmatrix}^{-1}$$

We can find that the equation is a singular matrix. This means when the number of measurements is 1 and every sample is measured at the same time, we cannot use the test data to distinguish between the diffusion parameters and the measurement error. Therefore, in the case of measurement errors, different measurement times are required for different samples.

When the number of measurement is 2

Each sample is measured twice, i.e. $m=2$, and we can get

$$E(-\frac{\partial^2 L}{\partial\mu^2})=\frac{nT}{\sigma^2}\cdot\frac{1}{1+\gamma/\delta}$$
$$E(-\frac{\partial^2 L}{\partial\mu\partial\sigma^2})=0$$
$$E(-\frac{\partial^2 L}{\partial\mu\partial\sigma_R^2})=0$$
$$E(-\frac{\partial^2 L}{\partial(\sigma^2)^2})=\frac{n}{(\sigma^2)^2}\frac{1+16\gamma/T+96(\gamma/T)^2+256(\gamma/T)^3+240(\gamma/T)^4}{(1+\gamma/\delta)^3(1+3\gamma/\delta)^3}$$
$$E(-\frac{\partial^2 L}{\partial\sigma^2\partial\sigma_R^2})=\frac{n/T}{(\sigma^2)^2}\cdot\frac{4+56\gamma/T+288(\gamma/T)^2+672(\gamma/T)^3+576(\gamma/T)^4}{(1+\gamma/\delta)^3(1+3\gamma/\delta)^3}$$
$$E(-\frac{\partial^2 L}{\partial(\sigma_R^2)^2})=\frac{n/T^2}{(\sigma^2)^2}\cdot\frac{20+256\gamma/T+1152(\gamma/T)^2+2304(\gamma/T)^3+1728(\gamma/T)^4}{(1+\gamma/\delta)^3(1+3\gamma/\delta)^3}$$

We can get

$$\lim_{\gamma\to0}E(-\frac{\partial^2 L}{\partial(\sigma^2)^2})=\frac{n}{(\sigma^2)^2}\qquad \lim_{\gamma\to0}E(-\frac{\partial^2 L}{\partial\sigma^2\partial\sigma_R^2})=\frac{4n/T}{(\sigma^2)^2},$$
$$\lim_{\gamma\to0}E(-\frac{\partial^2 L}{\partial(\sigma_R^2)^2})=\frac{20n/T^2}{(\sigma^2)^2}$$

We can obtain that when the measurement accuracy γ→0 (that is, the measurement error is particularly small relative to the diffusion coefficient), the Fisher information matrix is

$$\mathbf{I}(\mu,\sigma^2,\sigma_R^2)=\begin{bmatrix}\frac{nT}{\sigma^2} & 0 & 0\\ 0 & \frac{n}{(\sigma^2)^2} & \frac{4n/T}{(\sigma^2)^2}\\ 0 & \frac{4n/T}{(\sigma^2)^2} & \frac{20n/T^2}{(\sigma^2)^2}\end{bmatrix}^{-1}=\begin{bmatrix}\frac{\sigma^2}{nT} & 0 & 0\\ 0 & \frac{5(\sigma^2)^2}{n} & -\frac{T(\sigma^2)^2}{n}\\ 0 & -\frac{T(\sigma^2)^2}{n} & \frac{T^2(\sigma^2)^2}{4n}\end{bmatrix}$$

The limit of estimation accuracy is

$$\text{Aver}(\hat{\xi}_p)=\left[f\left(\xi_p\right)\right]^{-2}\mathbf{H}\cdot\mathbf{I}\cdot\mathbf{H}'$$
$$=2\pi\sigma^2\xi_p^3\times\exp[\frac{(1-\mu\xi_p)^2}{\sigma^2\xi_p}]\times[\frac{\sigma^2}{nT}(\frac{\partial F}{\partial\mu})^2+\frac{5\sigma^4}{n}(\frac{\partial F}{\partial\sigma^2})^2]$$

From the Fisher information matrix under the measurement accuracy γ→0, when there is no measurement error, if the reliability model parameters are estimated in a way with measurement errors, the accuracy of the estimation will be lower than the actual result without measurement errors.

When the number of measurement is 3

Each sample is measured three times, that is $m=3$, and the expectation value is

$$E(-\frac{\partial^2 L}{\partial\mu^2})=\frac{nT}{\sigma^2}\frac{1+16\gamma/T+60(\gamma/T)^2}{(1+2\gamma/\delta)(1+4\gamma/\delta+2(\gamma/\delta)^2)}$$
$$E(-\frac{\partial^2 L}{\partial\mu\partial\sigma^2})=0$$
$$E(-\frac{\partial^2 L}{\partial\mu\partial\sigma_R^2})=0$$
$$E(-\frac{\partial^2 L}{\partial(\sigma^2)^2})=\frac{n}{2(\sigma^2)^2}\left(\frac{3+126\gamma/T+2214(\gamma/T)^2+21060(\gamma/T)^3}{(1+2\gamma/\delta)^3(1+4\gamma/\delta+2(\gamma/\delta)^2)^3}+\frac{116964(\gamma/T)^4+379080(\gamma/T)^5+659016(\gamma/T)^6}{(1+2\gamma/\delta)^3(1+4\gamma/\delta+2(\gamma/\delta)^2)^3}+\frac{454896(\gamma/T)^7}{(1+2\gamma/\delta)^3(1+4\gamma/\delta+2(\gamma/\delta)^2)^3}\right)$$
$$E(-\frac{\partial^2 L}{\partial\sigma^2\partial\sigma_R^2})=\frac{n/T}{(\sigma^2)^2}\left(\frac{9+342\gamma/T+5346(\gamma/T)^2+44388(\gamma/T)^3}{(1+2\gamma/\delta)^3(1+4\gamma/\delta+2(\gamma/\delta)^2)^3}+\frac{210924(\gamma/T)^4+577368(\gamma/T)^5+857304(\gamma/T)^6}{(1+2\gamma/\delta)^3(1+4\gamma/\delta+2(\gamma/\delta)^2)^3}+\frac{524880(\gamma/T)^7}{(1+2\gamma/\delta)^3(1+4\gamma/\delta+2(\gamma/\delta)^2)^3}\right)$$
$$E(-\frac{\partial^2 L}{\partial(\sigma_R^2)^2})=\frac{n/T^2}{(\sigma^2)^2}\left(\frac{72+2592\gamma/T+37908(\gamma/T)^2+289656(\gamma/T)^3}{(1+2\gamma/\delta)^3(1+4\gamma/\delta+2(\gamma/\delta)^2)^3}+\frac{1236384(\gamma/T)^4+2939328(\gamma/T)^5+3674160(\gamma/T)^6}{(1+2\gamma/\delta)^3(1+4\gamma/\delta+2(\gamma/\delta)^2)^3}+\frac{1889568(\gamma/T)^7}{(1+2\gamma/\delta)^3(1+4\gamma/\delta+2(\gamma/\delta)^2)^3}\right)$$

There

$$\lim_{\gamma\to0}E(-\frac{\partial^2 L}{\partial(\sigma^2)^2})=\frac{3n}{2(\sigma^2)^2}\qquad \lim_{\gamma\to0}E(-\frac{\partial^2 L}{\partial\sigma^2\partial\sigma_R^2})=\frac{9n/T}{(\sigma^2)^2}$$
$$\lim_{\gamma\to0}E(-\frac{\partial^2 L}{\partial(\sigma_R^2)^2})=\frac{72n/T^2}{(\sigma^2)^2}$$

When the measurement accuracy γ→0, the Fisher information matrix and limit of estimation accuracy are shown as follows

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$$I_0(\mu,\sigma^2,\sigma_R^2)=\begin{bmatrix}\frac{nT}{\sigma^2} & 0 & 0\\ 0 & \frac{3n}{2(\sigma^2)^2} & \frac{9n/T}{(\sigma^2)^2}\\ 0 & \frac{9n/T}{(\sigma^2)^2} & \frac{72n/T^2}{(\sigma^2)^2}\end{bmatrix}^{-1}=\begin{bmatrix}\frac{\sigma^2}{nT} & 0 & 0\\ 0 & \frac{8(\sigma^2)^2}{3n} & -\frac{T(\sigma^2)^2}{3n}\\ 0 & -\frac{T(\sigma^2)^2}{3n} & \frac{T^2(\sigma^2)^2}{18n}\end{bmatrix}$$

$$\begin{aligned}\mathrm{Aver}(\hat{\xi}_p)&=\left[f\left(\xi_p\right)\right]^{-2}\mathbf{H}\cdot\mathbf{I}\cdot\mathbf{H}'\\&=2\pi\sigma^2\xi_p^3\times\exp[\frac{(1-\mu\xi_p)^2}{\sigma^2\xi_p}]\times[\frac{\sigma^2}{nT}(\frac{\partial F}{\partial\mu})^2+\frac{8\sigma^4}{3n}(\frac{\partial F}{\partial\sigma^2})^2]\end{aligned}$$

V. CASE STUDY

In order to compare the effectiveness of degradation and life test, the life test data and the parameter estimation based on the degradation test data are represented by subscripts T and D, respectively. As can be seen from the fourth section, when the number of measurements is 1, the Fisher information amount is a singular matrix, so the following is only studied when the number of measurements is 2 or 3. We study the effect of measurement error on reliability assessment and compare the effects of time censored life tests with degradation tests on reliability assessment. Without loss of generality, in the following study, we set the drift parameter μ=1, diffusion parameter σ=1, and product failure threshold $l=1$ for the Wiener degradation model.

A. *Effect of measurement error σ_R^2 on degradation process modeling*

When test duration T=1, the number of sample size n_D=1, figure 1 and 2 show the relationship between measurement error and the drift parameter as well as diffusion parameter estimation accuracy of in degradation test. rom the figure, we find that when the measurement error increases, the estimation accuracy of the drift parameter increases linearly, which is proportional to the measurement error, while the estimation accuracy of the diffusion parameter increases nonlinearly and the acceleration speed increases, which in proportion to the second of the measurement error. Fixed measurement error, the higher the number of measurements, the higher the accuracy of parameter estimation. Ideally, cut back the measurement error and raise the number of measurements can obtain higher accuracy.

However, under practical engineering practice, due to cost and instrument limitations, measurement errors cannot be reduced indefinitely or the number of measurements cannot be increased indefinitely. Further we discuss the effect of measurement error on the asymptotic variance. The figure 3 shows the measurement error and the asymptotic variance (reliable life) estimation accuracy curve, we take that the number of measurements is 2 and 3, and p is 0.5 and 0.9 respectively. We can see from the curve in the figure that when the number of measurements is constant, the smaller the p value, the higher the precision; when the parameter p is constant, the more the number of measurements, the higher the accuracy is.

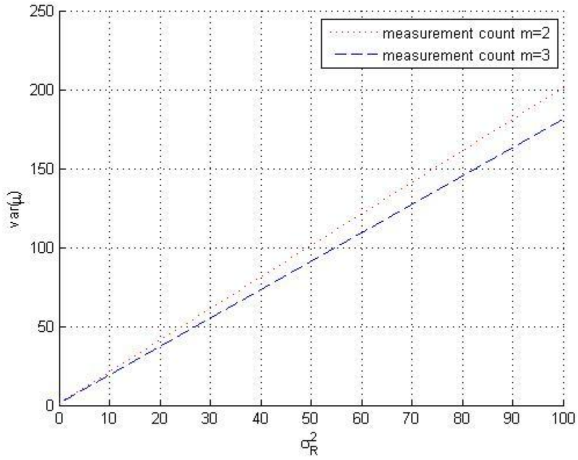

Fig. 1 Effect of measurement error on $\mathrm{var}(\hat{\mu})$

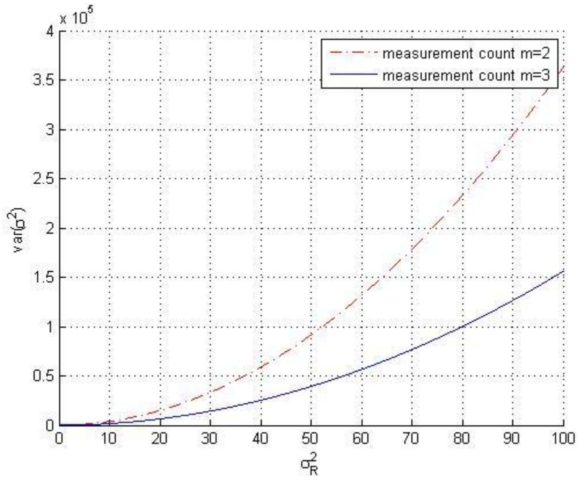

Fig. 2 Effect of measurement error on $\mathrm{var}(\hat{\sigma}^2)$

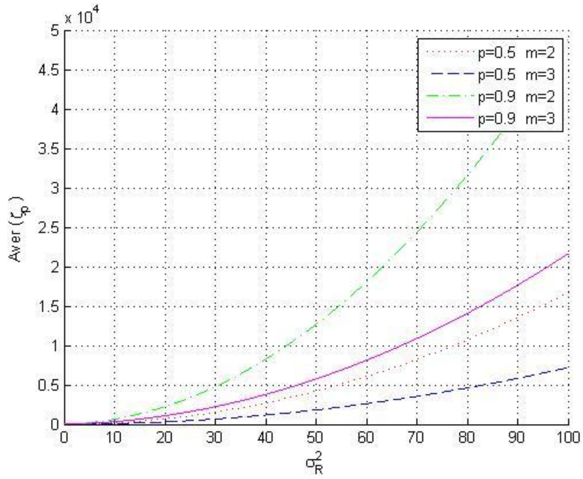

Fig. 3 Effect of measurement error on $\mathrm{Aver}(\hat{\xi}_p)$

B. *Degradation test plan optimization*

In the actual test process, the test design is often subject to realistic conditions, such as test costs. The main factors affecting the cost of the test are the sample size, test time and number of measurements. Assuming that the costs incurred for the individual measurement and the batch measurement are the same, the test cost mainly includes three parts: the cost of the sample size, the labor and public resources cost caused by the test time, and the data measurement cost caused by multiple measurements.

Suppose the price of a single sample is c_0, the labor and public resource cost per unit time is c_p, the cost of each measurement is c_m. If there are n samples to be tested, each sample is measured m times, and the test duration is T, then the total cost of degradation test is

$$TC(n,\Delta t,m)=c_0 n+c_p m\Delta t+c_m mn$$

Generally, in degradation test, the cost of each measurement c_m is not fixed, but is related to the measurement error σ_R of the test. Usually, the smaller the measurement error is, the higher the measurement accuracy is, and the higher the cost per measurement. Here we assume that c_m and σ_R are inversely proportional, that is $c_m = w/\sigma_R$. In reference [7], assuming the price of a single sample $c_0 = 80$, the human and public resource costs per unit time $c_p = 0.07$, $w = 1$, so the total cost in the degradation test is

$$TC(n_D,m_D,T)=80n_D+0.07T+\frac{1}{\sigma_R}m_D n_D$$

From this, when there is one sample in the degradation test at a fixed censoring time $T=1$, the relationship between the measurement error and the measurement cost is as follows:

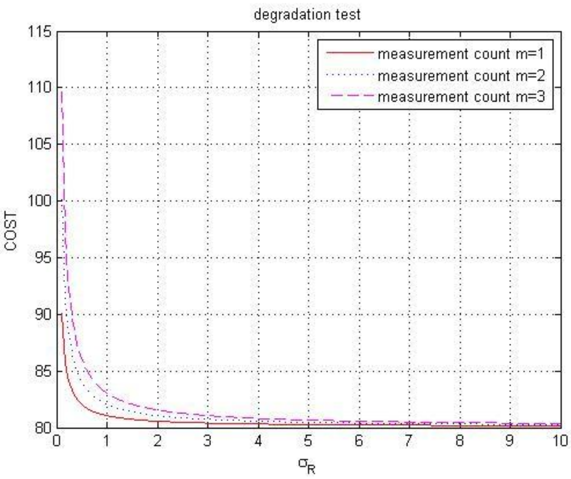

Fig. 4 Relationship between σ_R and total cost

In the reliability test, under the given cost constraint, the higher the accuracy of the reliability index is, the better the model is. The asymptotic variance of the product can be used as the optimization target, and the test cost is used as the constraint. Consider taking degradation test on the same batch of products and taking the parameters $p = 0.9$, censored time $T = 1$ and the number of measurements $m = 3$. The total test cost is required to not exceed 4000, and we use genetic algorithm [9][10] to get the degradation test optimal scheme:

measurement error $\sigma_R^2 = 0.0388$,

sample size $n_D = 42$,

At this time, asymptotic variance reaches the minimum and

$$\mathrm{Aver}(\xi_p)=0.1710$$

We can see that in short censored time, if we need to achieve better evaluation accuracy, you need higher measurement accuracy.

If there is no restriction on the test time, when censoring time is $T = 1089$, the sample size is $n_D = 48$ and measurement error is $\sigma_R^2 = 2.9563$, the asymptotic variance of $\hat{\xi}_p$ reaches minimum and $\mathrm{Aver}(\xi_p)=0.075$. We can see that even if the measurement error is large, the influence can be compensated by a long test time, and a high evaluation accuracy is obtained.

C. Comparison on degradation test and time censored life test

A large number of theoretical studies and practices have shown that degradation testing can make full use of experimental information and improve reliability modeling and evaluation accuracy compared with life test. In order to quantitatively compare the pros and cons of degradation test and life test in reliability modeling and evaluation, based on reference [7], we set the cost of each measurement for the life test $c_m = 10$, so the total cost in the life test is

$$TC(n_L,T)=90n_L+0.07T$$

When the censored time is 1, we find that to achieve the same asymptotic variance as degradation test, figure 4 shows the relationship between the asymptotic variance and the sample size. The life test sample amount required is $n_L = 92$, when its accuracy reach the same asymptotic variance $\mathrm{Aver}(\xi_p)=0.1710$. And its experimental cost is 8280.07, which exceeds the predetermined budge.

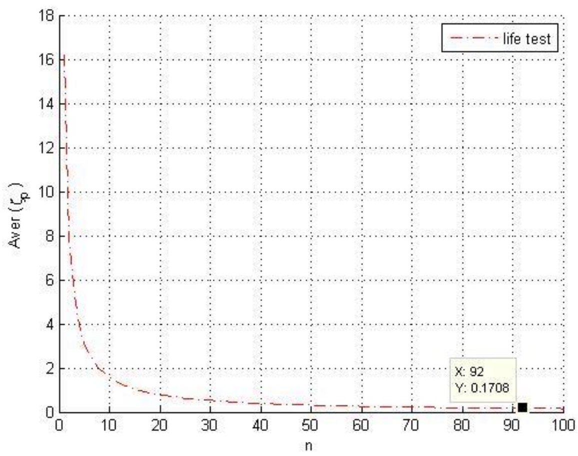

Figure 5 Relationship between n_L and $\mathrm{Aver}(\hat{\xi}_p)$

For life test, without the limitation of test time, under the total experimental cost constraint, enumeration method [8] is used to get the relationship between optimal test plan and reliability evaluation accuracy (asymptotic variance of $\hat{\xi}_p$), as shown in Table I.

We can see from the table that increasing the censoring time and reducing the sample size cannot improve the asymptotic variance of the estimated p-percentile of the product's lifetime distribution. This is because the accuracy of the parameter estimation under the life test is determined by the effective test time, which depends on the test end time of each sample. In the timing censored life test, it depends on the minimum value of the test censoring time and the sample failure time. When the test censoring time is large (as shown in Table I), the effective test time of the life test depends on the failure time of each sample, and is no longer affected by cut-off time. In fact, the life test does not require such a long test time as shown in Table I, because all test samples have

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failed before the test deadline is reached and the test does not need to continue. At this time, increasing the sample size and reducing the test time appropriately can obtain higher estimation accuracy.

Table I the asymptotic variance of $\hat{\xi}_p$ in life test

n_L	44	43	42	41	40
T_L	571	1857	3142	4428	5714
Avar($\hat{\xi}_p$)	***0.1491***	0.1522	0.1562	0.1590	0.1633
n_L	39	38	37	36	35
T_L	7000	8285	9571	10857	12142
Avar($\hat{\xi}_p$)	0.1686	0.1719	0.1775	0.1827	0.1873
n_L	34	33	32	31	30
T_L	13428	14714	16000	17285	18571
Avar($\hat{\xi}_p$)	0.1920	0.1987	0.2043	0.2108	0.2177
n_L	29	28	27	26	25
T_L	19857	21142	22428	23714	25000
Avar($\hat{\xi}_p$)	0.2250	0.2330	0.2425	0.2517	0.2624
n_L	24	23	22	21	20
T_L	26285	27571	28857	30142	31428
Avar($\hat{\xi}_p$)	0.2719	0.2848	0.2975	0.3120	0.3268
n_L	19	18	17	16	15
T_L	32714	34000	35285	36571	37857
Avar($\hat{\xi}_p$)	0.3438	0.3632	0.3848	0.4100	0.4340
n_L	14	13	12	11	10
T_L	39142	40428	41714	43000	44285
Avar($\hat{\xi}_p$)	0.4667	0.5010	0.5484	0.5984	0.6532
n_L	9	8	7	6	5
T_L	45571	46857	48142	49428	50714
Avar($\hat{\xi}_p$)	0.7274	0.8186	0.9326	1.0987	1.3154
n_L	4	3	2	1	
T_L	52000	53285	54571	55857	
Avar($\hat{\xi}_p$)	1.6395	2.1732	3.2953	6.5302	

VI. Conclusion

In this paper, for the Wiener degenerate failure products and two cases which are without and with measurement error in the degradation test, the reliability modeling methods with different number of measurement have been studied respectively. And we compare the effectiveness of life test and degradation test on reliability assessment. The research shows that:

(1) For the reliability evaluation based on the degradation test, it is necessary to make a reasonable analysis and determination of the factors affecting the test. For example, if there is no measurement error in the test, using the evaluation method for measurement error case will reduce accuracy. If the destructive measurement is used or each sample only measure once, different measurement times are required for each sample so as to identify the measurement error and the diffusion parameter.

(2) For the optimization of degradation test, the effects of measurement errors must be considered. Specifically, the measurement error has different influence on the accuracy of different model parameter. For example, the standard error of MLE of drift parameter increases linearly with the measurement error, while the estimation accuracy of the diffusion parameter decreases nonlinearly when the measurement error increases.

(3) Degradation tests have significant advantages for reliability assessment compared to life tests. As far as we know, this paper is the first to quantitatively compare the degradation test and the life test. The results can be of practical significance for the study of reliability evaluation test optimization.

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