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Dynamic Modeling of Mechanisms with Spur Gears

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***Abstract*—In this work we have taken into consideration a gearing mechanism which does rely upon a mechanical model which does own two torsional freedom degrees and which does represent a new modality of expression for the movement equations. When adopting certain hypotheses of work, we have studied the mathematical model of the movement. We have applied to the obtained mathematical models the unilateral Laplace transformation in respect to time. This method does present the advantage transform into algebra calculation the concerned problem and this fact simplify the resolving of some differential equations or of some systems of differential equations.**

***Index Terms*—gearbox, dynamic analysis, Laplace transform, differential equation.**

I. Introduction

IN the components made use of for the transmission of power the gearing does appear as the main source of excitation. The instantaneous movements of each of the wheels are represented by six degrees of freedom (three translations and three rotations). The fluctuations which do occur in the rigidity of the gearing mechanism and in the transmission, error are the main causes of the excitations that are usually associated to them. For the transmission error it is particularly indispensable to make the accurate distinction between the effects which are due to elastic deformations and the kinematic effects which are associated to the gearing of some non-conjugated profiles [2]. There are several authors (see [3]) which do make use of these sizes in order to define the interface which is created through the gearing procedure. In the specialized literature are usually discussed about three distinct levels when it comes to the modeling of the gearing mechanisms.

1. Models made of multiple rigid bodies which do represent the gearbox as an aggregate of masses and of inertial behaviors which are connected among themselves through springs and cushions. This type of models is applied in the case of the epicyclical gearbox which do present a straight [4] or helicoidally [13] tooth structure.
2. Models made of finite elements [8] who are allowing a highly realistic simulation of the respective physical behaviors.
3. Models made of multiple flexible bodies [5]. They consist in the superposition of a model with finite elements above a classical model made of rigid bodies. In order to reduce the cost of the simulation which is rather important for the models made of finite elements the method of Craig & Bampton is proposed for the reduction of the model [4]. This method is described in [12] and it does consist in the total condensation of the models made of finite elements upon an assembly of degrees of freedom held by the interface to which come to be joined the modes of vibrations presented by the immobile interface. In [9] is presented – in a multidisciplinary approach – the simplified mechanical model made use of in order to study the vibratory and acoustic behavior of a gearing mechanism where its elements are considered as behaving like some solid and rigid bodies. The objective of this approach is constituted by the process of optimizing a mechatronics product.

II. Gearing as a Source of Excitation. Contribution Brought by the Transmission Error

For reasons of simplicity the gearing mechanism below does rely upon a mechanical model with two torsional degrees of freedom - as in Fig. 1- a fact which does represent a new modality of illustrating the movement equations.

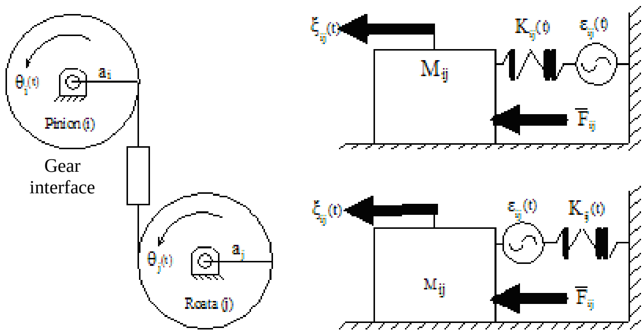

Fig. 1. Model with two torsional degrees of freedom (a) and equivalent linear model (b).

The rotation movement of every wheel is expressed through the angle:

$$\theta_i(t) = \theta_i^*(t) + \theta_i^d(t), \tag{1}$$

where:

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$\theta_i^*(t)=\Omega_i \bullet t$ - the nominal component;

$\theta_i^d(t)$ - the dynamic component;

Ω_i - the angular velocity of the cog wheels.

The nominal movements $\theta_i^*(t)=\Omega_i \bullet t$ and $\theta_j^*(t)=\Omega_j \bullet t$ are related through a kinematic relationship which is defined by the movement of the cog wheels taken into consideration as rigid un-deformable bodies that is to say

$$\Omega_i = r_{ij} \bullet \Omega_j, \tag{2}$$

where:

$$r_{ij} = -\frac{a_j}{a_i} \tag{3}$$

is the reduction ratio.

Should we take into consideration these kinematic relationships we could define the nominal movement of the system by starting from a unique parameter which we are going to denote:

$$\theta^*(t)=\Omega^* \bullet t \tag{4}$$

and, furthermore, the dynamic component $\theta_i^d(t)$ could be written under the form:

$$\theta_i^d(t)=\theta_i^{d\varepsilon}(t)+\theta_i^{de}(t). \tag{5}$$

The component $\theta_i^{d\varepsilon}(t)$ does express the effect of the shaping and position errors of the teeth structures upon the transmission of the movement while the $\theta_i^{de}(t)$ component is generated by the elastic deformations. The mechanical model from Fig. 2 could be reduced into a linear system which is equivalent to one only degree of freedom.

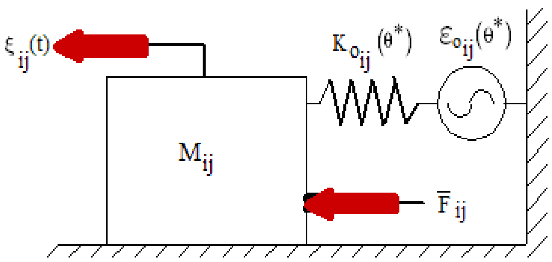

Fig. 2. Gearbox mechanical model.

The mathematical model which does define the dynamic movement of the un-cushioned system could be written under the form of a second order differential equation such as the one below:

$$M_{ij} \bullet \ddot{\xi}_{ij}(t)+K_{ij}(t)\left[\xi_{ij}(t)-\varepsilon_{ij}(t)\right]=F_{ij}, \tag{6}$$

where:

$$\xi_{ij}(t)=a_i \bullet \theta_i^d + a_j \bullet \theta_j^d \tag{7}$$

represent the error of the dynamic transmission and

$$\varepsilon_{ij}(t)=a_i \bullet \theta_i^{d\varepsilon} + a_j \bullet \theta_j^{d\varepsilon} \tag{8}$$

is the transmission error generated by the gearing of the non-conjugated profiles (the errors of shaping and of position). The last term is:

$$M_{ij}=\frac{I_i \bullet I_j}{a_i^2 \bullet I_j + a_j^2 \bullet I_i} \tag{9}$$

is the equivalent mass where I_i and I_j are the polar inertial moment of the cog wheels *i* and *j* as well as simultaneously:

$$F_{ij}=\frac{C_i}{a_i} \tag{10}$$

The above term defines the nominal effort which is associated to the static couple C_i.

We have represented here only the sources of excitation which are associated with the gearing mechanism. The other exterior actions such as the loading fluctuations have not been taken into consideration for reasons of simplicity. The terms $\varepsilon_{ij}(t)$ and $K_{ij}(t)$ are dependent upon the conditions of contact and their produced effects are not separable one from the other. They could be written under the form:

$$\varepsilon_{ij}(t)=\varepsilon_{ij}\left[F_{ij}(t),K_{ij}(t),\theta^*(t),\xi_{ij}(t)\right],$$

$$K_{ij}(t)=K_{ij}\left[F_{ij}(t),\varepsilon_{ij}(t),\theta^*(t),\xi_{ij}(t)\right].$$

Should we be interested in the elastic deformations only of the teeth structure the above-mentioned mathematical model could be written as:

$$M_{ij} \bullet \ddot{\xi}_{ij}(t)+K_{ij}(t)\xi_{ij}(t)=F_{ij}-m_{ij}\ddot{\varepsilon}_{ij}(t), \tag{11}$$

through the denotation:

$$\xi_{ij}(t)=a_i \bullet \theta_i^{de} + a_j \bullet \theta_j^{de}. \tag{12}$$

The contribution brought by the transmission error $\varepsilon_{ij}(t)=a_i \bullet \theta_i^{d\varepsilon} + a_j \bullet \theta_j^{d\varepsilon}$ which is generated by the gearing of some non-conjugated profiles would be then associated with the introduction of an inertial term within the second member. Some authors have preferred this written form (see [11]). Yet even under these conditions it does require for the calculation of the second order derivative in respect to time of the function $\varepsilon_{ij}(t)$. Or this size is *a priori* unknown because it is discretely dependent upon the instantaneous circumstances of the made contact. Therefore, it is usually substituted by the transmission error which is obtained under a quasi-static regime which we have qualified as being the quasi-static uncharged transmission error. The calculation of the second order derivative would become easy to perform should we dispose of the respective analytic expression. Such is the case, for example, of the quasi-static uncharged transmission error associated with an eccentricity default. The models developed in the specialized literature (see [7]) do make mostly use of the expressions obtained under a quasi-static regime in order to define the functions $\varepsilon_{ij}(t)$ and $K_{ij}(t)$. Blankenship & Singh (in the work [1]) have described in a highly precise manner the intrinsic

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hypotheses for each of these methods. To do so they have studied under a quasi-static regime the above presented equation (4.1): $\left(\Omega^* \to 0, \ddot{\xi}_{ij} = 0\right)$. This regime is defined by the indices 0. Let us assume the fact that $\varepsilon_{0,ij}(t)$ and $K_{0,ij}(t)$ are separable. Then this equation should become:

$$\begin{aligned} &K_{0,ij}\left(F_{ij},\theta_0^*\right)\xi_{0,ij}\left(F_{ij},\theta_0^*\right) = \\ &= F_{ij} + K_{0,ij}\left(F_{ij},\theta_0^*\right)\varepsilon_{0,ij}\left(F_{ij},\theta_0^*\right), \end{aligned} \tag{13}$$

a fact which would allow us to express the size $\xi_{0,ij}\left(F_{ij},\theta_0^*\right)$ which does represent the quasi-static transmission error under charge under the form:

$$\xi_{0,ij}\left(F_{ij},\theta_0^*\right) = \frac{F_{ij}}{K_{0,ij}\left(F_{ij},\theta_0^*\right)} + \varepsilon_{0,ij}\left(F_{ij},\theta_0^*\right). \tag{14}$$

In the particular case where $F_{ij} \to 0$ the resulted quasi-static transmission error should be:

$$\hat{\varepsilon}_{0,ij}\left(\theta_0^*\right) = \varepsilon_{0,ij}\left(F_{ij} \to 0, \theta_0^*\right). \tag{15}$$

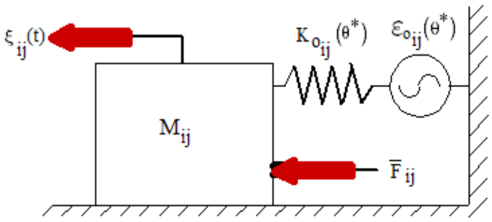
For the quality of the gearing mechanism this expression is an adequate indicator. For the whole of this context the quasi-static transmission error under charge is expressed through the relationship:

$$\xi_{0,ij}\left(F_{ij},\theta_0^*\right) = \frac{F_{ij}}{K_{0,ij}\left(F_{ij},\theta_0^*\right)} + \hat{\varepsilon}_{0,ij}\left(\theta_0^*\right).$$

Should we make use of (14) and (15). Almost all of the dynamic modeling presented in the specialized literature do make use of $\xi_{0,ij}\left(F_{ij},\theta_0^*\right)$ or $\hat{\varepsilon}_{0,ij}\left(\theta_0^*\right)$ and $K_{0,ij}\left(F_{ij},\theta_0^*\right)$ as primary sources of excitation. These sizes do present the advantage of being accessible through the simulation of a quasi-static behavior or through measurements. The specialized literature does impart to Kubo & col. (see [6]) as well as to Umezawa & col. (see [10]) the first ever modeling attempts which do make use of $K_{ij}(t) \cong K_{0,ij}\left[F_{ij},\theta^*\right]$ and $\varepsilon_{ij}(t) \cong \hat{\varepsilon}_{0,ij}\left(\theta_0^*\right)$.

III. Mathematical Models

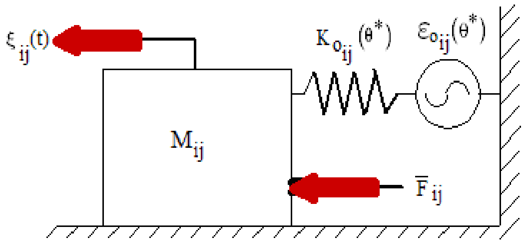
For the model obtained in the works [6] and [10] the equivalent model and the dynamic equation are presented in figure 3.



$$M_{ij}\bullet\ddot{\xi}_{ij}(t) + K_{0,ij}\left(F_{ij},\theta_0^*\right)\xi_{ij}(t) = F_{ij} + K_{0,ij}\left(F_{ij},\theta_0^*\right)\bullet\hat{\varepsilon}_{0,ij}\left(\theta_0^*\right)$$

Fig. 3. Dynamic model.

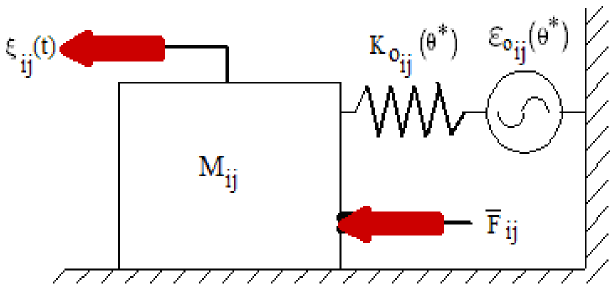
Should we regroup the terms from the second member and should we make use of $\varepsilon_{0,ij}\left(F_{ij},\theta_0^*\right)$ we would be led towards the second type of modeling which is generally attributed to the first ever works written about the dynamics of gearing mechanisms. The corresponding model and the equation associated to it are presented in figure 4.



$$M_{ij}\bullet\ddot{\xi}_{ij}(t) + K_{0,ij}\left(F_{ij},\theta_0^*\right)\bullet\xi_{ij}(t) = K_{0,ij}\left(F_{ij},\theta_0^*\right)\bullet\xi_{0,ij}\left(F_{ij},\theta_0^*\right)$$

Fig. 4. Dynamic model.

In the work [7] the obtained results have been very much alike to the ones obtained for the two previous modeling's when $K_{0,ij}\left(F_{ij},\theta_0^*\right)$ has been substituted by $K_{0,ij}\left[F_{ij}\right]$ (Fig. 4). Then the excitation effects which are associated to the variations occurring into the rigidity of the gearing mechanism are introduced only through the fluctuations of the quasi-static transmission error which does occur under the charge $\xi_{0,ij}\left(F_{ij},\theta_0^*\right)$. The main advantage of this latter method is the one of being described through a mathematical model which corresponds to a differential equation with constant coefficients which is submitted to a deterministic excitation. Under these circumstances let us at first consider the loading $\xi_{0,ij}\left(F_{ij},\theta_0^*\right)$ as constant. Or this last equation could be integrated through the applying of the integral Laplace transformation. Due to this fact the problem does become algebraic.



$$M_{ij}\bullet\ddot{\xi}_{ij}(t) + K_{0,ij}\left(F_{ij}\right)\bullet\xi_{ij}(t) = K_{0,ij}\left(F_{ij}\right)\bullet\xi_{0,ij}\left(F_{ij},\theta_0^*\right)$$

Fig. 5. Dynamic model.

IV. Determining of the Dynamic Answer

We have integrated the mathematical models we are working with by making use of the unilateral Laplace transformation in respect to time. This method does provide to us the advantage of rendering algebraic the problem and this fact does simplify a lot the solving of some differential equations or the one of some systems of differential

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equations.

Let us look again at the last equation (the one which does define the Fig. 5):

$$M_{ij}\bullet\ddot{\xi}_{ij}(t)+K_{0,ij}(F_{ij})\bullet\xi_{ij}(t)= =K_{0,ij}(F_{ij})\bullet\xi_{0,ij}(F_{ij},\theta_0^*), \qquad (16)$$

written under the form:

$$\ddot{\xi}_{ij}(t)+\frac{K_{0,ij}(F_{ij})}{M_{ij}}\bullet\xi_{ij}(t)=\frac{K_{0,ij}(F_{ij})\bullet\xi_{0,ij}(F_{ij},\theta_0^*)}{M_{ij}}, \qquad (17)$$

to which we are applying the unilateral Laplace transformation in respect to time. So does result the algebraic equation bearing the unknown $\tilde{\xi}_{ij}(s)$:

$$s^2\tilde{\xi}_{ij}(s)-s\xi_{ij}(0)-\dot{\xi}_{ij}(0)+\frac{K_{0,ij}(F_{ij})}{M_{ij}}\tilde{\xi}_{ij}(s)= =\frac{K_{0,ij}(F_{ij})\bullet\xi_{0,ij}(F_{ij},\theta_0^*)}{M_{ij}}\frac{1}{s}, \qquad (18)$$

with the solution:

$$\tilde{\xi}_{ij}(s)=\frac{s\xi_{ij}(0)+\dot{\xi}_{ij}(0)}{\left[s^2+\frac{K_{0,ij}(F_{ij})}{M_{ij}}\right]}+\frac{K_{0,ij}(F_{ij})\bullet\xi_{0,ij}(F_{ij},\theta_0^*)}{s\left[s^2+\frac{K_{0,ij}(F_{ij})}{M_{ij}}\right]M_{ij}}. \qquad (19)$$

Should we apply to the relationship (19) the reversed Laplace transformation we would obtain the solution of the equation (18) under the form:

$$\xi_{ij}(t)=\xi_{0,ij}(F_{ij},\theta_0^*)+ +\left[\xi_{ij}(0)-\xi_{0,ij}(F_{ij},\theta_0^*)\right]\cos\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right]+ +\frac{\dot{\xi}_{ij}(0)}{\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}}\sin\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right]. \qquad (20)$$

From Fig. 6 and taking into consideration the geometric features of the gearing mechanism as we have established them above - is provided the representation of the function (20). From it we can see that the studied movement does own the feature of being harmonic.

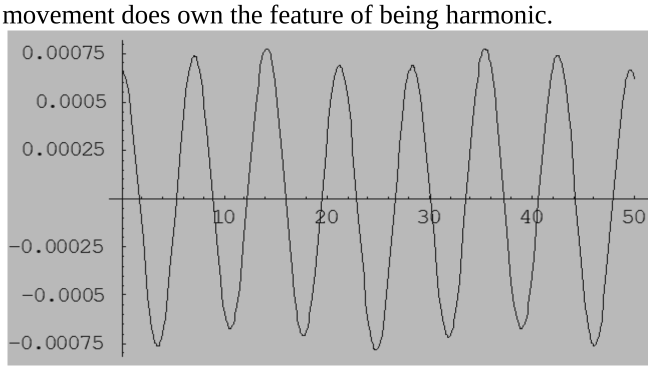

Fig. 6. the representation of the function (20)

Let us look now at the loading in respect to time:

$$\xi_{0,ij}(F_{ij},\theta_0^*)=\xi_{0,ij}(t).$$

When substituted in the equation (17) it should lead us to the differential equation:

$$\ddot{\xi}_{ij}(t)+\frac{K_{0,ij}(F_{ij})}{M_{ij}}\bullet\xi_{ij}(t)=\frac{K_{0,ij}(F_{ij})}{M_{ij}}\xi_{0,ij}(t). \qquad (21)$$

Should we apply to the equation (21) the unilateral Laplace transformation in respect to time we would come to the algebraic equation bearing the unknown $\tilde{\xi}_{ij}(s)$:

$$s^2\tilde{\xi}_{ij}(s)-s\xi_{ij}(0)-\dot{\xi}_{ij}(0)+\frac{K_{0,ij}(F_{ij})}{M_{ij}}\tilde{\xi}_{ij}(s)= =\frac{K_{0,ij}(F_{ij})}{M_{ij}}\tilde{\xi}_{0,ij}(s) \qquad (22)$$

with the solution:

$$\tilde{\xi}_{ij}(s)= =\frac{s\xi_{ij}(0)+\dot{\xi}_{ij}(0)}{s^2+\frac{K_{0,ij}(F_{ij})}{M_{ij}}}+\frac{K_{0,ij}(F_{ij})}{M_{ij}}\bullet\frac{1}{\left[s^2+\frac{K_{0,ij}(F_{ij})}{M_{ij}}\right]}\tilde{\xi}_{0,ij}(s). \qquad (23)$$

Should we apply to the relationship (23) the reversed Laplace transformation and taking as well into consideration the convolution theorem we would obtain the solution of the equation (22) under the form:

$$\xi_{ij}(t)=\frac{\dot{\xi}_{ij}(0)}{\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}}\sin\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right]+ \xi_{ij}(0)\cos\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right]+ +\frac{K_{0,ij}(F_{ij})}{M_{ij}\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}}\int_0^t\xi_{0,ij}(\tau)\sin\left[\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}(t-\tau)\right]d\tau. \qquad (24)$$

Let us consider a sinusoidal load bearing the form:

$$\xi_{0,ij}(t)=\xi_0\sin(\omega_e t),$$

where ξ_0 is the amplitude of the load (of the excitation) while ω_e is its respective pulse the solution (24) does acquire the form:

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$$\xi_{ij}(t)=\frac{\dot{\xi}_{ij}(0)}{\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}}\sin\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right]+$$
$$\xi_{ij}(0)\cos\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right]+ \tag{25}$$
$$+\frac{\xi_0 K_{0,ij}(F_{ij})}{M_{ij}\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}}\int_0^t \sin(\omega_e\tau)\sin\left[\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}(t-\tau)\right]d\tau,$$

It all does become reduced to the solving of the integral:

$$I=\int_0^t \sin(\omega_e\tau)\sin\left[\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}(t-\tau)\right]d\tau \quad , \tag{26}$$

For three cases such as:

1. should its own pulse:

$$\omega_n=\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}$$

be a lot different from ω_e the pulse of the excitation;

2. $\omega_n=\omega_e$;

3. $\omega_n\cong\omega_e$

Case 1.

Its own pulse ω_n being a lot different from ω_e the pulse of the excitation.

Should we perform the integral (26) and substitute the result of the integration within the equation (25) we would be led to the dynamic answer of the system expressed under the form of the time function:

$$\xi_{ij}(t)=\frac{\dot{\xi}_{ij}(0)}{\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}}\sin\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right]+\xi_{ij}(0)\cos\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right]+$$
$$+\frac{\xi_0 K_{0,ij}(F_{ij})\omega_e}{M_{ij}\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\left(\omega_n^2-\omega_e^2\right)}\sin\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right]+\frac{\xi_0 K_{0,ij}(F_{ij})}{M_{ij}\left(\omega_n^2-\omega_e^2\right)}\sin(\omega_e t) \tag{27}$$

or should we group the terms in respect to:

$\sin(\omega_n t)$, $\cos(\omega_n t)$ and $\sin(\omega_e t)$,

$$\xi_{ij}(t)=\left[\frac{\dot{\xi}_{ij}(0)\left(\omega_n^2-\omega_e^2\right)M_{ij}+\xi_0 K_{0,ij}(F_{ij})\omega_e}{M_{ij}\left(\omega_n^2-\omega_e^2\right)\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}}\right]$$
$$\sin\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right]+\xi_{ij}(0)\cos\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right]+$$
$$+\frac{\xi_0 K_{0,ij}(F_{ij})}{M_{ij}\left(\omega_n^2-\omega_e^2\right)}\sin(\omega_e t). \tag{28}$$

The former two terms of the function (28) do represent a vibratory movement of the elastic system with its own pulse ω_n while the last term does define some sustained vibrations bearing the pulse ω_e. This fact does show us the fact that the resultant movement is composed of a superposition of vibrations that is to say a vibration of its own of pulse ω_n as well as a forced vibration of pulse ω_e. Since the values of these two pulses are much different one from the other it results that the resultant movement is a non-harmonic vibration which is represented in Fig. 7.

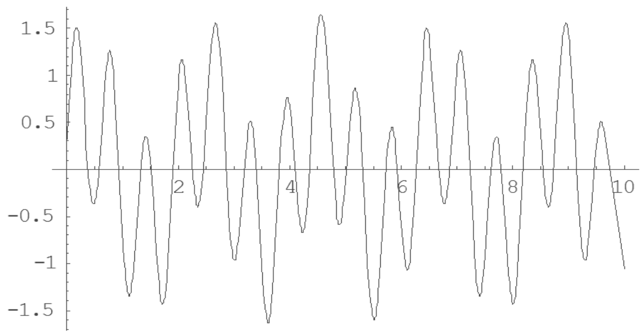

Fig. 7. Resultant movement, in [mm] versus time

Should we suppose the initial conditions to be homogeneous, that is to say with $\dot{\xi}_{ij}(0)=0$ and $\xi_{ij}(0)=0$ the function (28) would acquire the particular form:

$$\xi_{ij}(t)=\left[\frac{\xi_0 K_{0,ij}(F_{ij})\omega_e}{M_{ij}\left(\omega_n^2-\omega_e^2\right)\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}}\right]$$
$$\sin\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right]+\frac{\xi_0 K_{0,ij}(F_{ij})}{M_{ij}\left(\omega_n^2-\omega_e^2\right)}\sin(\omega_e t) \tag{29}$$

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The representation of which is provided in Fig. 8.

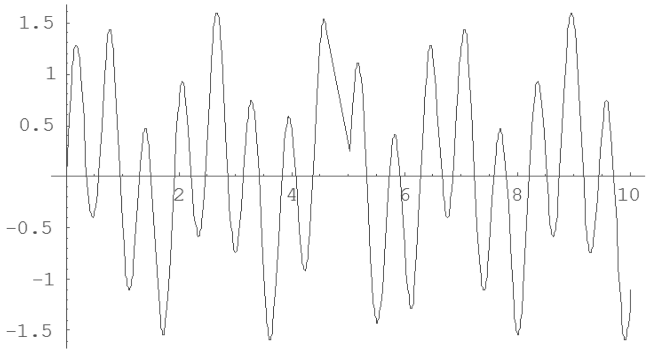

Fig. 8. Resultant movement, in [mm] versus time

Case 2. The intrinsic pulse $\omega_n = \omega_e$.

The performing of the integral (26) where the pulse ω_e is substituted by the natural (intrinsic) pulse ω_n and the result of the integration is substituted in the equation (24) should lead us to the dynamic answer of the system expressed under the form of the time function:

$$\xi_{ij}(t) = \frac{\dot{\xi}_{ij}(0)}{\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}} \sin\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right] +$$

$$+\xi_{ij}(0)\cos\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right] +$$

$$+\frac{\xi_0 K_{0,ij}(F_{ij})}{2M_{ij}\frac{K_{0,ij}(F_{ij})}{M_{ij}}} \sin\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right] -$$

$$-\frac{\xi_0 K_{0,ij}(F_{ij})}{2M_{ij}\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}}\, t\cos\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right]. \tag{30}$$

Should we suppose the initial conditions to be homogeneous that is to say with:

$$\dot{\xi}_{ij}(0) = 0 \text{ and } \xi_{ij}(0) = 0,$$

the function (30) would acquire the particular form:

$$\xi_{ij}(t) = \frac{\xi_0 K_{0,ij}(F_{ij})}{2M_{ij}\frac{K_{0,ij}(F_{ij})}{M_{ij}}} \sin\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right] -$$

$$-\frac{\xi_0 K_{0,ij}(F_{ij})}{2M_{ij}\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}}\, t\cos\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right]. \tag{31}$$

The time function which does define the forced vibration is:

$$\xi_{ij,e}(t) = -\frac{\xi_0 K_{0,ij}(F_{ij})}{2M_{ij}\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}}\, t\cos\left[\left(\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}\right)t\right] \tag{32}$$

and it does define a quasi-harmonic vibration with a modulation in amplitude for which the function:

$$A_e(t) = -\frac{\xi_0 K_{0,ij}(F_{ij})}{2M_{ij}\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}}\, t, \tag{33}$$

is the function which does modulate the amplitude. This is how the equation (30) does represent three harmonic movements of constant amplitudes and a fourth quasi-harmonic movement the amplitude of which is linearly increasing in respect to time as we can see from the relationship (33). Its graphical representation within the system of coordinates (t, ξ_{ij}) is provided in Fig. 9.

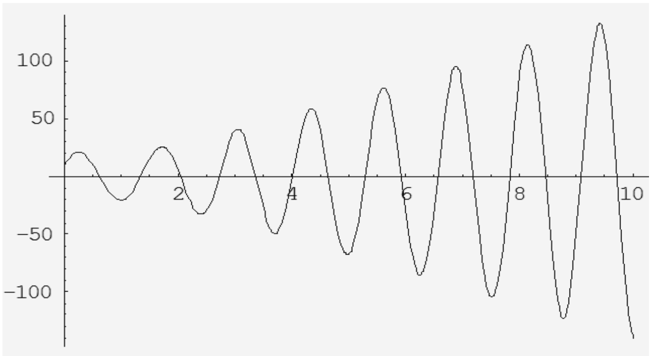

Fig. 9. Amplitude of the vibration.

The movement expressed through the equation (32) is an unstable one and its amplitude is increasing infinitely (the phenomenon of resonance) as it is demonstrated by its representation within the system of coordinates (t, $\xi_{ij,e}$) from Fig. 10.

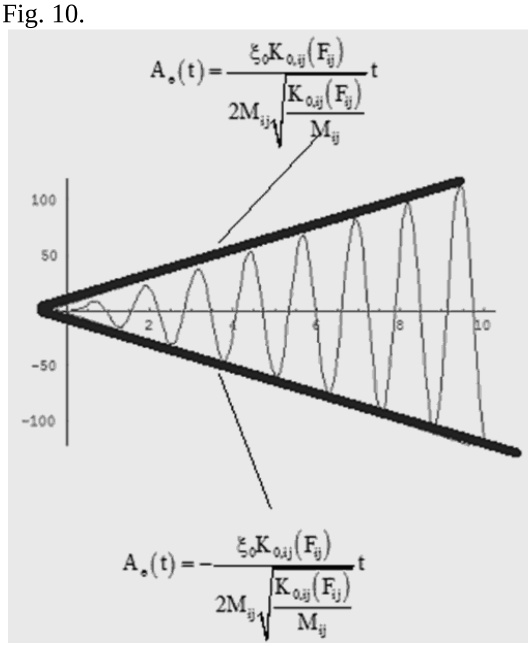

Fig. 10. Increasing amplitude

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The representative curve $\xi_{ij,e} = \xi_{ij,e}(t)$ is tangent to the straight lines:

$$A_e(t) = \pm \frac{\xi_0 K_{0,ij}(F_{ij})}{2M_{ij}\sqrt{\frac{K_{0,ij}(F_{ij})}{M_{ij}}}} t, \tag{34}$$

which do represent the wrapper of the concerned curve. In order to avoid the destruction of the gearing mechanism the phenomenon of resonance ought to be prevented. The period of the movement is:

$$T_e = \frac{2\pi}{\omega_e}. \tag{35}$$

Case 3. The intrinsic pulse $\omega_n = \omega_e$.

Should the vibration be situated nearby the vicinity of the resonance frequency and should we substitute:

$$\frac{\omega_n}{\omega_e} \cong 1,\ \omega_n - \omega_e \cong \varepsilon,\ \omega_n + \omega_e \cong 2\varepsilon,$$

ε being a positive and small enough number the expression (29) would lead us to the time function modulated through amplitude:

$$\xi_{ij}(t) = -\left[\frac{\xi_0 K_{0,ij}(F_{ij})}{M_{ij}\omega_e \varepsilon}\right]\sin\left(\frac{\varepsilon t}{2}\right)\cos(\omega_e t). \tag{36}$$

In the equation (36) amplitude is the time function:

$$A(t) = -\left[\frac{\xi_0 K_{0,ij}(F_{ij})}{M_{ij}\omega_e \varepsilon}\right]\sin\left(\frac{\varepsilon t}{2}\right) \tag{37}$$

and it does represent the wrapper of the curve expressed through the equation (36). Such a movement is usually designated as a beat that is to say a vibration of pulse ω_e the amplitude of which does own the pulse $\frac{\varepsilon}{2}$. The period of the beats is:

$$T_b = \frac{2\pi}{\omega_n - \omega_e}. \tag{38}$$

One last type of approximation could be the one where the effects of the fluctuation which usually does occur in the rigidity of the gearing mechanism should not be taken into consideration. It does consist in making use of a Fourier series development in order to represent the sizes:

$$K_{0,ij}(F_{ij},\theta_0^*),\ \xi_{0,ij}(F_{ij},\theta_0^*) \text{ and } \hat{\varepsilon}_{0,ij}(\theta_0^*).$$

The modeling's we have described above do rely upon the main hypothesis of the choice to make use for the nominal load destined to be the primary source of excitation of some sizes which would be obtained under the quasi-static regime. Yet in the respective cases of some alignment defaults or of some important shaping errors this latter point of view might be considered as highly doubtful. In such a context the rigidity of the gearing mechanism is largely dependent upon the instantaneous load which might come to be applied upon the teeth structure. This latter phenomenon could be taken into consideration through the movement equation:

$$M_{ij}\bullet\ddot{\xi}_{ij}(t) + K_{ij}(F_{ij},\theta^*)\bullet\xi_{ij}(t) = \\ = F_{ij} + K_{ij}(F_{ij},\theta_0^*)\bullet\hat{\varepsilon}_{0,ij}(\theta^*), \tag{39}$$

where F_{ij} is the dynamic instantaneous load which might come to be applied upon the teeth structure.

V. Conclusion

The fluctuations undergone by both the transmission error and the intrinsic rigidity of the gearing mechanism are the main causes of the excitations which are associated with them. For the transmission error it is particularly indispensable to distinguish the effects due to elastic deformations from the cinematic effects associated to the gearing of some non-conjugated profiles. There are several scientific schools (see [3]) which do make use of these size units in order to describe the interface created through the gearing mechanism. Nowadays the research activities do focus upon the development of some multidisciplinary experimental, theoretical and numerical competencies which would have to be relied upon when the structures and elements of the machines – or more extensively the mechanical systems – should be conceived. The purposes of present researches are the ones of improving our knowledge of the behavior shown by materials and structures, of developing some models and instruments that could be useful in the designing process which does concern structures and machines as well as of searching to take advantage from an already existing technical culture insofar the methodologies of analysis, conception and fabrication could be concerned. These researches are relying upon domains such as the science of materials, the branches of non-linear mechanics which do respectively involve solid bodies, fluids and coupled systems, acoustics, some techniques made use of in the forming and processing technical procedures, some experimental measurement methods and numerical modeling.

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