Parallel Manipulator of a Class RoboMech with Two End-Effectors

Zhumadil Baigunchekov, Zhadyra Zhumasheva, Batyr Naurushev, Azamat Mustafa, Rustem Kairov, Bekzat Amanov

***Abstract* –In this paper, the methods of structural and parametric synthesis of a RoboMech class parallel manipulator with two end-effectors are presented. This parallel manipulator is formed by connecting the two moving output objects with the fixed base by two passive, one active and two negative closing kinematic chains. Geometrical parameters of the active and negative closing kinematic chains are determined by the Chebyshev and least-square approximations.**

***Index Terms* - Parallel manipulator, end-effector, structural-parametric synthesis.**

I. INTRODUCTION

Depending on the type of technological operation, the robot manipulator can operate in two modes: a simultaneous manipulation of two objects and a sequential manipulation of one object.

In the simultaneous manipulation of two objects, two serial manipulators *ABC* and *DEF* handle two objects in the initial positions P_1 and P_2 (Fig. 1a), then two objects are moved to the specified position P_3 (Fig. 1b). Next, the manipulators return to their initial positions.

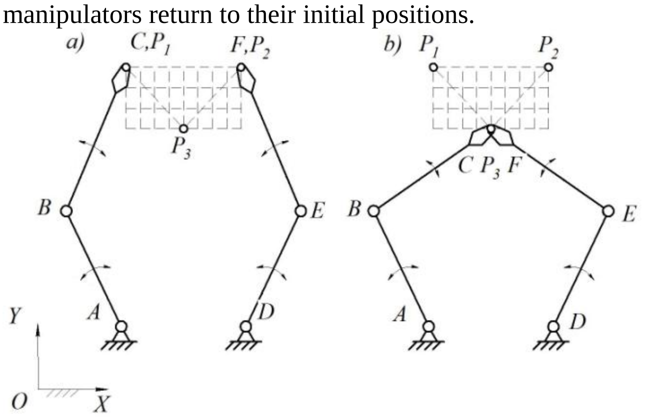

Fig. 1. Two serial manipulators *ABC* and *DEF*.

In the sequential manipulation of one object, the first serial manipulator *ABC* handles the object in the position P_1 (Fig. 1a), then the object is moved to the intermediate position P_3, where the object is transferred to the gripper of the second serial manipulator *DEF* (Fig. 1b). Next, the object is moved by the second serial manipulator *DEF* to the specified position P_2 (Fig. 1b).

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For example, a printing machine operates in the mode of sequential manipulation of one object. In this machine, a blank sheet of paper is fed by the first manipulator onto the printing table, and the second manipulator picks up the sheet after printing. This cyclical process occurs in a short period of time. Therefore, in such automatic machines, instead of two serial manipulators, it is advisable to use one manipulator (mechanism) with two end-effectors and one DOF. The parallel manipulators (PM) of a class RoboMech belong to such manipulators. PM having the property of manipulation robots such as a reproducing the specified laws of motions of the end-effectors, and the property of mechanisms such as a setting the laws of motions the actuators which simplify the control system and increase speed, are called PM of a class RoboMech [1-3].

In this paper, the methods of structural-parametric synthesis of a RoboMech class PM with two end-effectors are developed. There are many methods of structural and kinematic (parametric or dimensional) synthesis of mechanisms [4-6], where the kinematic synthesis of mechanisms is carried out for their given structural schemes. In this case, it is possible that a given structural scheme of the mechanism may not provide the specified laws of motions of the end-effectors. Therefore, it is necessary to carry out the kinematic synthesis together with the structural synthesis. The methods of structural-parametric synthesis allow to simultaneously determine the optimal structural schemes of PM and the geometrical parameters of their links according to the given laws of motions of the end-effectors and actuators.

II. STRUCTURAL SYNTHESIS

According to the developed principle of forming mechanisms and manipulators [1,2], the PM with two end-effectors is formed by connecting two output objects with a fixed base using closing kinematic chains (CKC), which can be active, passive and negative. If we connect these two output objects with the fixed base by two passive CKC *ABC* and *DEF*, having zero DOF, we obtain two serial manipulators (Fig. 2). In the paper [7], a PM of the fifth class with two end-effectors and two DOF (Fig. 2) was formed from these two serial manipulators by connecting the links 2 and 4 by the negative CKC *GH* of type **RR**, then by connecting the link *GH* with a fixed base by the negative CKC *IK* type **RR**, and by connecting the links *IK* and *DE* by the negative CKC *LM* of type **RR**, where **R** is a revolute kinematic pair. Each of the binary links of type **RR** has one negative DOF. The disadvantages of this PM is a small workspace because the links 2 and 4 of two serial manipulators *ABC* and *DEF* are connected by one link *GH*.

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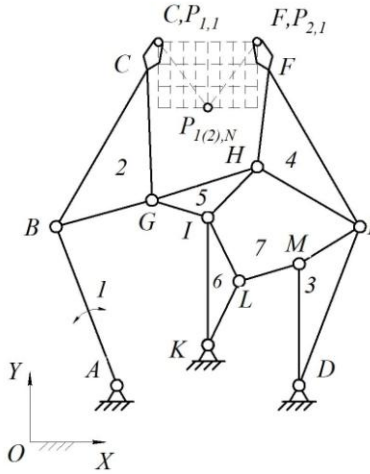

Fig. 3. PM with two end-effectors of the fifth class.

The workspace of the PM with two end-effectors can be increased by connecting the links 2 and 4 of the serial manipulators *ABC* and *DEF* by the active CKC *GHKI* with active kinematic pair *K*. As a result, we obtain PM *ABGHKIED* with three DOF, where the links *AB*, *KH* and *DE* are input links (Fig. 3). For formation of a RoboMech class PM with one DOF, we connect the links 1 and 5, as well as the links 3 and 6 by the negative CKC *ML* and *NQ* of type **RR**. As a result, we obtain a structural scheme of a RoboMech class PM with two end-effectors, which has the following structural formula

$$\mathrm{IV}(1,2,5,8) \leftarrow \mathrm{I}(0,8) \rightarrow \mathrm{IV}(3,4,6,9). \qquad (5)$$

Therefore, the formed RoboMech class PM consists of an input link 7 and two fourth class Assur groups, or two Stephenson II mechanisms with the common input link 7.

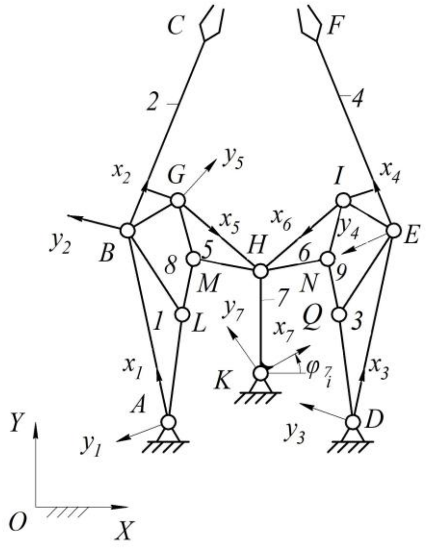

Fig. 2. PM of a class RoboMech with two end-effectors.

Thus, this RoboMech class PM with two end-effectors is formed by connecting of two output objects with a fixed base by two passive CKC *ABC* and *DEF*, one active CKC *GHKI* and two negatives CKC *LM* and *NQ*.

Since the active and negative CKC impose geometrical constraints on the motions of the output objects, then the formed PM of a RoboMech class with two end-effectors works at certain values of the geometrical parameters (synthesis parameters) of the links. Passive CKC doesn't impose geometrical constraints on the motions of output objects, therefore, their synthesis parameters vary taking into account the superimposed geometrical constraints of the connecting active and negative CKC. Consequently, the problem of parametric synthesis of the whole RoboMech class PM with two end-effectors is reduced to the subproblems of parametric synthesis of its structural modules (passive, active and negative CKC). Such a modular representation of structural - parametric synthesis simplifies the problem of designing of PM. Let consider the parametric synthesis of the structural modules of the RoboMech class PM with two end-effectors.

III. PARAMETRIC SYNTHESIS OF STRUCTURAL MODULES

Given *N* discrete values of the grippers centers *C* and *F* coordinates X_{C_i}, Y_{C_i} and X_{F_i}, X_{F_i} $(i = 1, 2, ..., N)$.

The synthesis parameters of two passive CKC *ABC* and *DEF* (or serial manipulators) are X_A, Y_A, l_{AB}, l_{BC} and X_D, Y_D, l_{DE}, l_{EF}, where X_A, Y_A and X_D, Y_D are coordinates of the pivot joints *A* and *D* in the absolute coordinate system *OXY*; $l_{AB}, l_{BC}, l_{DE}, l_{EF}$ are lengths of the links *AB,BC,DE,EF*. The synthesis parameters of the passive CKC vary using the « LP_τ sequence» [8].

The synthesis parameters of the active CKC *GHKI* are $x_G^{(2)}, y_G^{(2)}, x_I^{(4)}, y_I^{(4)}, x_H^{(7)}, y_H^{(7)}, X_K, Y_K, l_{GH}, l_{HI}$, where $x_G^{(2)}, y_G^{(2)}, x_I^{(4)}, y_I^{(4)}, x_H^{(7)}, y_H^{(7)}$ are coordinates of the joints *G*, *I*, *H* in the moving coordinate systems $Bx_2y_2, Ex_4y_4, Kx_7y_7$, fixed to the links *BC*, *EF*, *KH*, respectively; X_K, Y_K are coordinates of the pivot joint *K* in the absolute coordinate system *OXY*; l_{GH}, l_{IH} are lengths of the links *GH*, *IH*.

Write the vector loop-closure equations of *OKHGBO* and *OKHIEO*

$$\mathbf{R}_K + \mathbf{\Gamma}(\varphi_{7i})\mathbf{r}_H^{(7)} + \mathbf{l}_{(HG)_i} = \mathbf{R}_{B_i} + \mathbf{\Gamma}(\varphi_{2i})\mathbf{r}_G^{(2)}, \qquad (2)$$

$$\mathbf{R}_K + \mathbf{\Gamma}(\varphi_{7i})\mathbf{r}_H^{(7)} + \mathbf{l}_{(HI)_i} = \mathbf{R}_{E_i} + \mathbf{\Gamma}(\varphi_{4i})\mathbf{r}_I^{(4)}, \qquad (3)$$

where $\mathbf{R}_K = \left[X_K, Y_K\right]^T$, $\mathbf{r}_H^{(7)} = \left[x_H^{(7)}, y_H^{(7)}\right]^T$,

$$\mathbf{l}_{(HG)_i} = \left[l_{HG}\cos\varphi_{(HG)_i}, l_{HG}\sin\varphi_{(HG)_i}\right]^T,$$

$$\mathbf{R}_{Bi} = \left[X_{Bi}, Y_{Bi}\right]^T,$$

$$\mathbf{r}_G^{(2)} = \left[x_G^{(2)}, y_G^{(2)}\right]^T, \mathbf{r}_H^{(7)} = \left[x_H^{(7)}, y_H^{(7)}\right]^T,$$

$$\mathbf{l}_{(HI)_i} == \left[l_{HI}\cos\varphi_{(HI)_i}, l_{HI}\sin\varphi_{(HI)_i}\right]^T,$$

$$\mathbf{R}_{E_i} = \left[X_{E_i}, Y_{E_i}\right]^T, \mathbf{r}_I^{(4)} = \left[x_I^{(4)}, y_I^{(4)}\right]^T,$$

$\mathbf{\Gamma}(\alpha)$ is an orthogonal rotation matrix

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$$\mathbf{\Gamma}(\alpha)=\begin{bmatrix}\cos(\alpha) & -\sin(\alpha)\\ \sin(\alpha) & \cos(\alpha)\end{bmatrix}.$$

The angles φ_{2i} and φ_{4i} in the Eqs (2) and (3), which determine the positions of the links BC and EF of the passive CKC ABC and DEF, are calculated from the analysis of positions of these CKC by the expressions

$$\varphi_{2i}=\mathrm{tg}^{-1}\frac{Y_{C_i}-Y_{B_i}}{X_{C_i}-X_{B_i}}, \tag{4}$$

$$\varphi_{4i}=\mathrm{tg}^{-1}\frac{Y_{F_i}-Y_{E_i}}{X_{F_i}-X_{E_i}}, \tag{5}$$

where

$$\begin{bmatrix}X_{Bi}\\ Y_{Bi}\end{bmatrix}=\begin{bmatrix}X_A\\ Y_A\end{bmatrix}+l_{AB}\begin{bmatrix}\cos\varphi_{1i}\\ \sin\varphi_{1i}\end{bmatrix}, \tag{6}$$

$$\begin{bmatrix}X_{Ei}\\ Y_{Ei}\end{bmatrix}=\begin{bmatrix}X_D\\ Y_D\end{bmatrix}+l_{DE}\begin{bmatrix}\cos\varphi_{3i}\\ \sin\varphi_{3i}\end{bmatrix}. \tag{7}$$

The angles φ_{1i} and φ_{3i} in Eqs (6) and (7) are determined by the expressions

$$\varphi_{1i}=\mathrm{tg}^{-1}\frac{Y_{C_i}-Y_A}{X_{C_i}-X_A}\pm\cos^{-1}\frac{l_{AC_i}^2+l_{AB}^2-l_{BC}^2}{2l_{AC_i}^2\cdot l_{AB}}, \tag{8}$$

$$\varphi_{3i}=\mathrm{tg}^{-1}\frac{Y_{F_i}-Y_D}{X_{F_i}-X_D}\pm\cos^{-1}\frac{l_{DF_i}^2+l_{DE}^2-l_{EF}^2}{2l_{DF_i}^2\cdot l_{DE}}, \tag{9}$$

where

$$l_{AC_i}=\left[\left(X_{C_i}-X_A\right)^2+\left(Y_{C_i}-Y_A\right)^2\right]^{\frac{1}{2}},$$

$$l_{DF_i}=\left[\left(X_{F_i}-X_D\right)^2+\left(Y_{F_i}-Y_D\right)^2\right]^{\frac{1}{2}}.$$

Eliminating the unknown angles $\varphi_{(HG)i}$ and $\varphi_{(HI)i}$, from Eqs (2) and (3) yields

$$\left[\mathbf{R}_K+\mathbf{\Gamma}(\varphi_{7i})\mathbf{r}_H^{(7)}-\mathbf{R}_{B_i}-\mathbf{\Gamma}(\varphi_{2i})\mathbf{r}_G^{(2)}\right]^2-l_{HG}^2=0, \tag{10}$$

$$\left[\mathbf{R}_K+\mathbf{\Gamma}(\varphi_{7i})\mathbf{r}_H^{(7)}-\mathbf{R}_{E_i}-\mathbf{\Gamma}(\varphi_{4i})\mathbf{r}_I^{(4)}\right]^2-l_{HI}^2=0, \tag{11}$$

Eqs (10) and (11) are the equations of geometrical constraints imposed on the motion of two output objects. The geometric meaning of Eqs (10) and (11) are the equations of two circles with radiuses l_{HG} and l_{HI} in relative motions of the planes Bx_2y_2 and Bx_4y_4 relative to the plane Kx_7y_7. The problem of determining the geometrical parameters of the links at which such geometrical constraints are approximately realized is the problem of parametric synthesis of the active CKC *GHKI.*

The left parts of Eqs (10) and (11) are denoted by $\Delta q_{1i}^{(1)}$ and $\Delta q_{2i}^{(2)}$, which are functions of weighted differences

$$\Delta q_{1i}^{(1)}=\left[\mathbf{R}_K+\mathbf{\Gamma}(\varphi_{7i})\mathbf{r}_H^{(7)}-\mathbf{R}_{B_i}-\mathbf{\Gamma}(\varphi_{2i})\mathbf{r}_G^{(2)}\right]^2-l_{HG}^2, \tag{12}$$

$$\Delta q_{2i}^{(2)}=\left[\mathbf{R}_K+\mathbf{\Gamma}(\varphi_{7i})\mathbf{r}_H^{(7)}-\mathbf{R}_{E_i}-\mathbf{\Gamma}(\varphi_{4i})\mathbf{r}_I^{(4)}\right]^2-l_{HI}^2=0. \tag{13}$$

After converting these equations and the following change of variables

$$\begin{bmatrix}p_1\\ p_2\end{bmatrix}=\begin{bmatrix}X_K\\ Y_K\end{bmatrix},\ \begin{bmatrix}p_4\\ p_5\end{bmatrix}=\begin{bmatrix}x_G^{(2)}\\ y_G^{(2)}\end{bmatrix},\ \begin{bmatrix}p_6\\ p_7\end{bmatrix}=\begin{bmatrix}x_H^{(7)}\\ y_H^{(7)}\end{bmatrix},$$

$$p_3=\frac{1}{2}(X_K^2+Y_K^2+x_H^{(7)^2}+y_H^{(7)^2}+x_G^{(2)^2}+y_G^{(2)^2}-l_{HG}^2),$$

$$\begin{bmatrix}p_8\\ p_9\end{bmatrix}=\begin{bmatrix}x_I^{(4)}\\ y_I^{(4)}\end{bmatrix},$$

$$p_{10}=\frac{1}{2}(X_K^2+Y_K^2+x_H^{(7)^2}+y_H^{(7)^2}+x_I^{(4)^2}+y_I^{(4)^2}-l_{HI}^2)$$

the functions Δq_{1i} and Δq_{2i} are represented as linear forms by groups $\mathbf{p}_1^{(j)}$ and $\mathbf{p}_2^{(k)}$ of synthesis parameters

$$\Delta q_{1i}^{(j)}=2\left(\mathbf{g}_{1i}^{(j)^T}\cdot\mathbf{p}_1^{(j)}-g_{01i}^{(j)}\right),\ (j=1,2,3), \tag{14}$$

$$\Delta q_{2i}^{(k)}=2\left(\mathbf{g}_{2i}^{(k)^T}\cdot\mathbf{p}_2^{(k)}-g_{02i}^{(k)}\right),\ (k=1,2,3), \tag{15}$$

where

$$\mathbf{g}_{1i}^{(1)}=\begin{bmatrix}-X_{B_i}\\ -Y_{B_i}\\ 1\end{bmatrix}-\left[\begin{array}{c:c}\mathbf{\Gamma}(\varphi_{2i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline \begin{matrix}0 & 0\end{matrix} & 1\end{array}\right]\cdot\begin{bmatrix}p_4\\ p_5\\ 0\end{bmatrix}+\left[\begin{array}{c:c}\mathbf{\Gamma}(\varphi_{7i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline \begin{matrix}0 & 0\end{matrix} & 1\end{array}\right]\cdot\begin{bmatrix}p_6\\ p_7\\ 0\end{bmatrix},$$

$$\mathbf{g}_{2i}^{(2)}=\left[\begin{array}{c:c}\mathbf{\Gamma}^T(\varphi_{2i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline \begin{matrix}0 & 0\end{matrix} & 1\end{array}\right]\cdot\begin{bmatrix}X_{B_i}-p_1\\ Y_{B_i}-p_2\\ 1\end{bmatrix}-\left[\begin{array}{c:c}\mathbf{\Gamma}(\varphi_{7i}-\varphi_{2i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline \begin{matrix}0 & 0\end{matrix} & 1\end{array}\right]\cdot\begin{bmatrix}p_6\\ p_7\\ 0\end{bmatrix},$$

$$\mathbf{g}_{3i}^{(3)}=\left[\begin{array}{c:c}\mathbf{\Gamma}^T(\varphi_{7i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline \begin{matrix}0 & 0\end{matrix} & 1\end{array}\right]\cdot\begin{bmatrix}p_1-X_{B_i}\\ p_2-Y_{B_i}\\ 1\end{bmatrix}-\left[\begin{array}{c:c}\mathbf{\Gamma}(\varphi_{7i}-\varphi_{2i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline \begin{matrix}0 & 0\end{matrix} & 1\end{array}\right]\cdot\begin{bmatrix}p_4\\ p_5\\ 0\end{bmatrix},$$

$$g_{01i}^{(1)} = -\frac{1}{2}\left(X_{B_i}^2 + Y_{B_i}^2\right) - [X_{B_i}, Y_{B_i}] \cdot \mathbf{\Gamma}(\varphi_{2i}) \cdot \begin{bmatrix} p_4 \\ p_5 \end{bmatrix} +$$

$$+ [X_{B_i}, Y_{B_i}] \cdot \mathbf{\Gamma}(\varphi_{7i}) \cdot \begin{bmatrix} p_6 \\ p_7 \end{bmatrix} + \left[p_4, p_5\right] \cdot \mathbf{\Gamma}(\varphi_{7i} - \varphi_{2i}) \cdot \begin{bmatrix} p_6 \\ p_7 \end{bmatrix},$$

$$g_{01i}^{(2)} = -\frac{1}{2}\left(X_{B_i}^2 - Y_{B_i}^2\right) - [p_1 - X_{B_i}, p_2 - Y_{B_i}] \cdot \mathbf{\Gamma}(\varphi_{7i}) \cdot \begin{bmatrix} p_6 \\ p_7 \end{bmatrix},$$

$$g_{01i}^{(3)} = -\frac{1}{2}\left(X_{B_i}^2 - Y_{B_i}^2\right) - [X_{B_i} - p_1, Y_{B_i} - p_2] \cdot \mathbf{\Gamma}(\varphi_{7i}) \cdot \begin{bmatrix} p_4 \\ p_5 \end{bmatrix},$$

$$\mathbf{g}_{2i}^{(1)} = \begin{bmatrix} -X_{E_i} \\ -Y_{E_i} \\ 1 \end{bmatrix} - \left[\begin{array}{c:c} \mathbf{\Gamma}(\varphi_{4i}) & \begin{array}{c} 0 \\ 0 \end{array} \\ \hdashline 0 \quad 0 & 1 \end{array}\right] \cdot \begin{bmatrix} p_8 \\ p_9 \\ 0 \end{bmatrix} + \left[\begin{array}{c:c} \mathbf{\Gamma}(\varphi_{7i}) & \begin{array}{c} 0 \\ 0 \end{array} \\ \hdashline 0 \quad 0 & 1 \end{array}\right] \cdot \begin{bmatrix} p_6 \\ p_7 \\ 0 \end{bmatrix},$$

$$\mathbf{g}_{2i}^{(2)} = \left[\begin{array}{c:c} \mathbf{\Gamma}^T(\varphi_{4i}) & \begin{array}{c} 0 \\ 0 \end{array} \\ \hdashline 0 \quad 0 & 1 \end{array}\right] \cdot \begin{bmatrix} X_{E_i} - p_1 \\ Y_{E_i} - p_2 \\ 1 \end{bmatrix} - \left[\begin{array}{c:c} \mathbf{\Gamma}(\varphi_{7i} - \varphi_{4i}) & \begin{array}{c} 0 \\ 0 \end{array} \\ \hdashline 0 \quad 0 & 1 \end{array}\right] \cdot \begin{bmatrix} p_6 \\ p_7 \\ 0 \end{bmatrix},$$

$$\mathbf{g}_{3i}^{(3)} = \left[\begin{array}{c:c} \mathbf{\Gamma}^T(\varphi_{7i}) & \begin{array}{c} 0 \\ 0 \end{array} \\ \hdashline 0 \quad 0 & 1 \end{array}\right] \cdot \begin{bmatrix} p_1 - X_{E_i} \\ p_2 - Y_{E_i} \\ 1 \end{bmatrix} - \left[\begin{array}{c:c} \mathbf{\Gamma}(\varphi_{7i} - \varphi_{4i}) & \begin{array}{c} 0 \\ 0 \end{array} \\ \hdashline 0 \quad 0 & 1 \end{array}\right] \cdot \begin{bmatrix} p_8 \\ p_9 \\ 0 \end{bmatrix},$$

$$g_{02i}^{(1)} = -\frac{1}{2}\left(X_{E_i}^2 + Y_{E_i}^2\right) - [X_{E_i}, Y_{E_i}] \cdot \mathbf{\Gamma}(\varphi_{4i}) \cdot \begin{bmatrix} p_8 \\ p_9 \end{bmatrix} +$$

$$+ [X_{E_i}, Y_{E_i}] \cdot \mathbf{\Gamma}(\varphi_{7i}) \cdot \begin{bmatrix} p_6 \\ p_7 \end{bmatrix} + \left[p_8, p_9\right] \cdot \mathbf{\Gamma}(\varphi_{7i} - \varphi_{4i}) \cdot \begin{bmatrix} p_6 \\ p_7 \end{bmatrix},$$

$$g_{02i}^{(2)} = -\frac{1}{2}\left(X_{E_i}^2 - Y_{E_i}^2\right) - [p_1 - X_{E_i}, p_2 - Y_{E_i}] \cdot \mathbf{\Gamma}(\varphi_{7i}) \cdot \begin{bmatrix} p_6 \\ p_7 \end{bmatrix},$$

$$g_{02i}^{(3)} = -\frac{1}{2}\left(X_{E_i}^2 - Y_{E_i}^2\right) - [X_{E_i} - p_1, Y_{E_i} - p_2] \cdot \mathbf{\Gamma}(\varphi_{7i}) \cdot \begin{bmatrix} p_8 \\ p_9 \end{bmatrix}.$$

The linear representability of the geometrical constraints Eqs (14) and (15) with respect to the groups $\mathbf{p}_1^{(j)}$ and $\mathbf{p}_2^{(k)}$ of synthesis parameters allows to formulate the following approximation problems of parametric synthesis:

-Chebyshev approximation,

- least-square approximation

to determine the groups $\mathbf{p}_1^{(j)}$ and $\mathbf{p}_2^{(k)}$ of synthesis parameters.

In the Chebyshev approximation problem, the vectors of synthesis parameters are determined from the minimum of the functionals

$$S_1^{(j)}(\mathbf{p}_1^{(j)}) = \max_{i=1,N} \left| \Delta q_{1i}^{(j)}(\mathbf{p}_1^{(j)}) \right| \to \min_{\mathbf{p}_1^{(j)}} S_1^{(j)}(\mathbf{p}_1^{(j)}), \tag{16}$$

$$S_2^{(k)}(\mathbf{p}_2^{(k)}) = \max_{i=1,N} \left| \Delta q_{2i}^{(k)}(\mathbf{p}_2^{(k)}) \right| \to \min_{\mathbf{p}_2^{(k)}} S_2^{(k)}(\mathbf{p}_2^{(k)}). \tag{17}$$

In the least-square approximation problem, the vectors of synthesis parameters are determined from the minimum of the functionals

$$S_1^{(j)}(\mathbf{p}_1^{(j)}) = \sum_{i=1}^{N} \left(\Delta q_{1i}^{(j)} \right) \to \min_{\mathbf{p}_1^{(j)}} S_1^{(j)}(\mathbf{p}_1^{(j)}), \tag{18}$$

$$S_2^{(k)}(\mathbf{p}_2^{(k)}) = \sum_{i=1}^{N} \left(\Delta q_{2i}^{(k)} \right) \to \min_{\mathbf{p}_2^{(k)}} S_2^{(k)}(\mathbf{p}_2^{(k)}), \tag{19}$$

Since the synthesis parameters of the active CKC *GHKI* are simultaneously included in functionals (16-19), their values are determined by joint consideration of the functionals (16) and (17), and also (18) and (19).

The linear representability of Eqs (12) and (13) in the forms (14) and (15) allows for solving the Chebyshev approximation problem (16) and (17), to apply the kinematic inversion method, which is an iterative process, at each step of which one group of synthesis of the parameters $\mathbf{p}_1^{(j)}$ and $\mathbf{p}_2^{(k)}$ is defined. In this case, the problem of linear programming is solved by four parameters. To do this, we introduce a new variable $\mathbf{p}_{11} = \varepsilon$, where ε is a required accuracy of the approximation. Then the minimax problems (16) and (17) are reduced to the following linear programming problem: determine the minimum of the sum

$$\sigma = \mathbf{c}^T \cdot \mathbf{x} \to \min_{\mathbf{x}} \sigma, \tag{20}$$

where $\mathbf{c} = \left[0,...,0,1\right]^T$, $\mathbf{x} = \left[\mathbf{p}^{j(k)}, \mathbf{p}_{11}\right]^T$ with the following restrictions

$$\left.\begin{array}{l} \left[\mathbf{g}_{1(2)i}^{(j(k))}, -\frac{1}{2}\right] \cdot \begin{bmatrix} \mathbf{p}^{j(k)} \\ p_{11} \end{bmatrix} = \mathbf{g}_{01(2)i}^{(j(k))} \\ \left[\mathbf{g}_{1(2)i}^{(j(k))}, \frac{1}{2}\right] \cdot \begin{bmatrix} \mathbf{p}^{j(k)} \\ p_{11} \end{bmatrix} = \mathbf{g}_{01(2)i}^{(j(k))} \end{array}\right\} \tag{21}$$

The sequence of the obtained values of the functions $S_{1(2)}^{(j(k))}(\mathbf{p}^{(j(k))})$ will decrease and have a limit as a sequence bounded below, because $S_{1(2)}^{(j(k))}(\mathbf{p}^{(j(k))}) \geq 0$ for any $\mathbf{p}^{(j(k))}$.

Let consider the solution of the least-square approximation problem (18) and (19) for the synthesis of the considered active CKC *GHKI*. From the necessary conditions for the minimum of functions $S_{1(2)}^{(j(k))}$ by groups $\mathbf{p}_{1(2)}^{(j(k))}$ of synthesis parameters

$$\frac{\partial S_{1(2)}^{(j(k))}}{\partial \mathbf{p}_{1(2)}^{(j(k))}} = 0 \tag{22}$$

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we obtain the systems of linear equations in the forms

$$\mathbf{H}_{1(2)}^{(j(k))} \cdot \mathbf{p}_{1(2)}^{(j(k))} = \mathbf{h}_{1(2)}^{(j(k))}, (j,k=1,2,3). \qquad (23)$$

Solving the systems of equations (23) for each group of synthesis parameters for given values of the remaining parameter groups, we determine their values

$$\mathbf{p}_{1(2)}^{(j(k))} = \mathbf{H}_{1(2)}^{(j(k))-1} \cdot \mathbf{h}_{1(2)}^{(j(k))} \qquad (24).$$

It is not difficult to show that the Hessian $\mathbf{H}_{1(2)}^{(j(k))}$ is positively defined together with the main minors. Then the solutions of the systems (23) correspond to the minimum of the functions $S_{1(2)}^{(j(k))}$. Consequently, the least-square approximation problem for parametric synthesis is reduced to the linear iteration method, at each step of which the systems of linear equations are solved.

Let consider the solution of parametric synthesis problem of the negative CKC *LM* and *NQ*. For this, we preliminarily determine the positions of the synthesized active CKC links of the *GH* and *IH*

$$\varphi_{5i} = \mathrm{tg}^{-1}\frac{Y_{I_i} - Y_{G_i}}{X_{I_i} - X_{G_i}} - \cos^{-1}\frac{l_{(GI)_i}^2 + l_{GH}^2 - l_{IH}^2}{2 l_{(GI)_i} \cdot l_{GH}}, \qquad (25)$$

$$\varphi_{6i} = \mathrm{tg}^{-1}\frac{Y_{H_i} - Y_{I_i}}{X_{H_i} - X_{I_i}}, \qquad (26)$$

where

$$\begin{bmatrix} X_{G_i} \\ Y_{G_i} \end{bmatrix} = \begin{bmatrix} X_{B_i} \\ Y_{B_i} \end{bmatrix} + \begin{bmatrix} \cos\varphi_{2i} & -\sin\varphi_{2i} \\ \sin\varphi_{2i} & \cos\varphi_{2i} \end{bmatrix} \cdot \begin{bmatrix} x_G^{(2)} \\ y_G^{(2)} \end{bmatrix},$$

$$\begin{bmatrix} X_{I_i} \\ Y_{I_i} \end{bmatrix} = \begin{bmatrix} X_{E_i} \\ Y_{E_i} \end{bmatrix} + \begin{bmatrix} \cos\varphi_{4i} & -\sin\varphi_{4i} \\ \sin\varphi_{4i} & \cos\varphi_{4i} \end{bmatrix} \cdot \begin{bmatrix} x_I^{(4)} \\ y_I^{(4)} \end{bmatrix},$$

$$l_{(G_I)} = \left[\left(X_{I_i} - X_{G_i} \right)^2 + \left(Y_{I_i} - Y_{G_i} \right)^2 \right]^{\frac{1}{2}},$$

$$\begin{bmatrix} X_{H_i} \\ Y_{H_i} \end{bmatrix} = \begin{bmatrix} X_{G_i} \\ Y_{G_i} \end{bmatrix} + l_{GH} \begin{bmatrix} \cos\varphi_{5i} \\ \sin\varphi_{5i} \end{bmatrix}.$$

Write the vector loop-closure equations of *OBGMLAO* and *OEINQDO*

$$\mathbf{R}_{G_i} + \mathbf{\Gamma}(\varphi_{5i})\mathbf{r}_M^{(5)} + \mathbf{l}_{ML} = \mathbf{R}_{A_i} + \mathbf{\Gamma}(\varphi_{1i})\mathbf{r}_L^{(1)}, \qquad (27)$$

$$\mathbf{R}_{I_i} + \mathbf{\Gamma}(\varphi_{6i})\mathbf{r}_N^{(6)} + \mathbf{l}_{NQ} = \mathbf{R}_{D_i} + \mathbf{\Gamma}(\varphi_{3i})\mathbf{r}_Q^{(3)}, \qquad (28)$$

where $\mathbf{R}_{G_i} = \left[X_{G_i}, Y_{G_i} \right]^T, \mathbf{r}_M^{(5)} = \left[x_M^{(5)}, y_M^{(5)} \right]^T,$

$$\mathbf{l}_{(ML)_i} = \left[l_{ML}\cos\varphi_{(ML)_i}, l_{ML}\sin\varphi_{(ML)_i} \right]^T, \mathbf{R}_A = \left[X_A, Y_A \right]^T,$$

$$\mathbf{r}_L^{(1)} = \left[x_L^{(1)}, y_L^{(1)} \right]^T, \mathbf{R}_{I_i} = \left[X_{I_i}, Y_{I_i} \right]^T, \mathbf{r}_N^{(6)} = \left[x_N^{(6)}, y_N^{(6)} \right]^T,$$

$$\mathbf{l}_{(NQ)i} \left[l_{NQ}\cos\varphi_{(NQ)_i}, l_{NQ}\sin\varphi_{(NQ)_i} \right]^T,$$

$$\mathbf{R}_D = \left[X_D, Y_D \right]^T, \mathbf{r}_Q^{(3)} = \left[x_Q^{(3)}, y_Q^{(3)} \right]^T.$$

Eliminating the unknown angles $\varphi_{(ML)_i}$ and $\varphi_{(NQ)_i}$ from Eqs (27) and (28) yields

$$\left[\mathbf{R}_{G_i} + \mathbf{\Gamma}(\varphi_{5i})\mathbf{r}_M^{(5)} - \mathbf{R}_A - \mathbf{\Gamma}(\varphi_{1i})\mathbf{r}_I^{(1)} \right]^2 - l_{ML}^2 = 0, \qquad (29)$$

$$\left[\mathbf{R}_{I_i} + \mathbf{\Gamma}(\varphi_{6i})\mathbf{r}_N^{(6)} - \mathbf{R}_D - \mathbf{\Gamma}(\varphi_{3i})\mathbf{r}_Q^{(3)} \right]^2 - l_{NQ}^2 = 0. \qquad (30)$$

Eqs (29) and (30) are the equations of geometrical constraints imposed on the motions of links 1 and 5, 3 and 6 by the negative CKC *ML* and *NQ*. The geometrical meanings of these constraints are the equations of two circles in the relative motions of the planes of links 1 and 5, 3 and 6 with radiuses l_{ML} and l_{NQ}. The problem of determining the geometrical parameters of the links, at which such geometric constraints are approximately realized, is the problem of parametric synthesis of two negative CKC *ML* and *NQ*.

The left parts of Eqs (29) and (30) are denoted by Δq_{3i} and Δq_{4i}, which are functions of weighted differences

$$\Delta q_{3i} = \left[\mathbf{R}_{G_i} + \mathbf{\Gamma}(\varphi_{5i})\mathbf{r}_M^{(5)} - \mathbf{R}_A - \mathbf{\Gamma}(\varphi_{1i})\mathbf{r}_{1i}^{(1)} \right]^2 - l_{ML}^2, \quad (31)$$

$$\Delta q_{4i} = \left[\mathbf{R}_{I_i} + \mathbf{\Gamma}(\varphi_{6i})\mathbf{r}_H^{(6)} - \mathbf{R}_D - \mathbf{\Gamma}(\varphi_{3i})\mathbf{r}_Q^{(3)} \right]^2 - l_{NQ}^2. \quad (32)$$

After converting these equations and the following change of variables

$$\begin{bmatrix} p_{11} \\ p_{12} \end{bmatrix} = \begin{bmatrix} x_L^{(1)} \\ \mathrm{y}_L^{(1)} \end{bmatrix}, \begin{bmatrix} p_{14} \\ p_{15} \end{bmatrix} = \begin{bmatrix} x_M^{(5)} \\ \mathrm{y}_M^{(5)} \end{bmatrix},$$

$$p_{13} = \frac{1}{2}\left(x_L^{(1)^2} + \mathrm{y}_L^{(1)^2} + x_M^{(5)^2} + \mathrm{y}_M^{(5)^2} - l_{LM}^2\right),$$

$$\begin{bmatrix} p_{16} \\ p_{17} \end{bmatrix} = \begin{bmatrix} x_Q^{(3)} \\ \mathrm{y}_Q^{(3)} \end{bmatrix}, \begin{bmatrix} p_{19} \\ p_{20} \end{bmatrix} = \begin{bmatrix} x_N^{(5)} \\ \mathrm{y}_N^{(5)} \end{bmatrix},$$

$$p_{18} = \frac{1}{2}\left(x_Q^{(3)^2} + \mathrm{y}_Q^{(3)^2} + x_N^{(6)^2} + \mathrm{y}_N^{(6)^2} - l_{QN}^2\right)$$

the functions (31) and (32) are expressed linearly by two groups of synthesis parameters

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$$\mathbf{p}_3^{(1)}=[p_{11},p_{12},p_{13}]^T,\ \ \mathbf{p}_3^{(2)}=[p_{14},p_{15},p_{13}]^T,$$
$$\mathbf{p}_4^{(1)}=[p_{16},p_{17},p_{18}]^T,\ \ \mathbf{p}_4^{(2)}=[p_{19},p_{20},p_{18}]^T$$

in the forms

$$\Delta q_{3i}^{(j)}=2\left(\mathbf{g}_{3i}^{(j)}\cdot\mathbf{p}_3^{(j)}-g_{03i}^{(j)}\right),(j=1,2), \tag{33}$$

$$\Delta q_{4i}^{(k)}=2\left(\mathbf{g}_{4i}^{(k)^T}\cdot\mathbf{p}_4^{(k)}-g_{04i}^{(k)}\right),(k=1,2), \tag{34}$$

where

$$\mathbf{g}_{3i}^{(1)}=\left[\begin{array}{c:c}\mathbf{\Gamma}^{-1}(\varphi_{1i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline \begin{matrix}0 & 0\end{matrix} & 1\end{array}\right]\cdot\begin{bmatrix}X_{G_i}-X_A\\ Y_{G_i}-Y_A\\ 1\end{bmatrix}-\left[\begin{array}{c:c}\mathbf{\Gamma}(\varphi_{2i}-\varphi_{1i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline \begin{matrix}0 & 0\end{matrix} & 1\end{array}\right]\cdot\begin{bmatrix}p_{14}\\ p_{15}\\ 0\end{bmatrix},$$

$$\mathbf{g}_{3i}^{(2)}=\left[\begin{array}{c:c}\mathbf{\Gamma}^{-1}(\varphi_{2i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline \begin{matrix}0 & 0\end{matrix} & 1\end{array}\right]\cdot\begin{bmatrix}X_{G_i}-X_A\\ Y_{G_i}-Y_A\\ 1\end{bmatrix}-\left[\begin{array}{c:c}\mathbf{\Gamma}^{-1}(\varphi_{2i}-\varphi_{1i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline \begin{matrix}0 & 0\end{matrix} & 1\end{array}\right]\cdot\begin{bmatrix}p_{11}\\ p_{12}\\ 0\end{bmatrix},$$

$$g_{03i}^{(1)}=-\frac{1}{2}\left[\left(X_{G_i}-X_A\right)^2+\left(Y_{G_i}-Y_A\right)^2\right]+$$
$$+\left[X_{G_i}-X_A,Y_{G_i}-Y_A\right]\cdot\mathbf{\Gamma}(\varphi_{2i})\cdot\begin{bmatrix}p_{14}\\ p_{15}\end{bmatrix},$$

$$g_{03i}^{(2)}=-\frac{1}{2}\left[\left(X_{G_i}-X_A\right)^2+\left(Y_{G_i}-Y_A\right)^2\right]-$$
$$+\left[X_{G_i}-X_A,Y_{G_i}-Y_A\right]\cdot\mathbf{\Gamma}(\varphi_{1i})\cdot\begin{bmatrix}p_{11}\\ p_{12}\end{bmatrix},$$

$$\mathbf{g}_{4i}^{(1)}=-\left[\begin{array}{c:c}\mathbf{\Gamma}^{-1}(\varphi_{3i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline \begin{matrix}0 & 0\end{matrix} & 1\end{array}\right]\cdot\begin{bmatrix}X_{I_i}-X_D\\ Y_{I_i}-Y_D\\ 1\end{bmatrix}-\left[\begin{array}{c:c}\mathbf{\Gamma}(\varphi_{4i}-\varphi_{3i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline \begin{matrix}0 & 0\end{matrix} & 1\end{array}\right]\cdot\begin{bmatrix}p_{19}\\ p_{20}\\ 0\end{bmatrix},$$

$$\mathbf{g}_{4i}^{(2)}=\left[\begin{array}{c:c}\mathbf{\Gamma}^{-1}(\varphi_{4i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline \begin{matrix}0 & 0\end{matrix} & 1\end{array}\right]\cdot\begin{bmatrix}X_{I_i}-X_D\\ Y_{I_i}-Y_D\\ 1\end{bmatrix}-\left[\begin{array}{c:c}\mathbf{\Gamma}^{-1}(\varphi_{4i}-\varphi_{3i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline \begin{matrix}0 & 0\end{matrix} & 1\end{array}\right]\cdot\begin{bmatrix}p_{16}\\ p_{17}\\ 0\end{bmatrix},$$

$$g_{04i}^{(1)}=-\frac{1}{2}\left[\left(X_{I_i}-X_D\right)^2+\left(Y_{I_i}-Y_D\right)^2\right]+$$
$$+\left[X_{I_i}-X_D,Y_{I_i}-Y_D\right]\cdot\mathbf{\Gamma}(\varphi_{4i})\cdot\begin{bmatrix}p_{19}\\ p_{20}\end{bmatrix},$$

$$g_{04i}^{(2)}=-\frac{1}{2}\left[\left(X_{I_i}-X_D\right)^2+\left(Y_{I_i}-Y_D\right)^2\right]-$$
$$+\left[X_{I_i}-X_D,Y_{I_i}-Y_D\right]\cdot\mathbf{\Gamma}(\varphi_{3i})\cdot\begin{bmatrix}p_{16}\\ p_{17}\end{bmatrix}.$$

Further, on the basis of the approximation problems of the Chebyshev and least-square approximations, outlined above, the parametric synthesis of the considered CKC *LM* and *QN* separately is carried out.

V. CONCLUSION

The methods of structural-parametric synthesis of a novel RoboMech class PM with two end-effectors are developed. The investigated PM is formed by connecting the two moving output objects with the fixed base by two passive, one active and two negative CKC. The active and negative CKC impose the geometrical constraints on the motions of the output objects, and they work with certain geometrical parameters of links. Geometrical parameters of the active and negative CKC links are determined on the base of Chebyshev and least-square approximations.

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