Hankinson's Criterion Investigation of Uniaxial Compressive Strength of Wood

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Abstract—**As oppose to tensile strength of wood, as an orthotropic material, its capacity to withstand applied force tending to reduce its size, is referred to as its compressive strength (CS). In this study, mathematical analysis of the CS of selected species of wood, was carried out. Exact solutions obtained from purely analytical approach, which is theoretical, is reported and graphical representation was achieved with the use of Microsoft excel and maple computer software. Comparison of the CS of all the different species of wood considered in this study was carried out. Specifically, the results show that Douglas fir has the highest and the lowest values of the uniaxial compressive strength (UCS), in direction at angle 90 and 0 to the grain respectively. On the average, in direction at any angle to the grain, Oak wood has the highest uniaxial compressive strength among the five wood species considered. Also the CS of all the wood species selected are never zero, no matter how close to zero they got.**

***Index Terms*— Uniaxial Compressive Strength, wood, Hankinson's criterion, investigation**

I. Introduction

WHEN a specimen of material extends as a result of applied force, it is said to be in tension. However, if it is said to be in compression if it compresses and shortens.
The maximum axial compression stress such materials can withstand before failing is known as uniaxial compressive strength (UCS) [1,2,3]. The tensile strength, on the other hands, resists tension

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In studying materials, especially as regards their strengths, it has been observed that some materials deform in such a way that they cannot be restored, while at the limit of their CS, others fracture [4,5]. Some material specimens exhibit high CS than others when compared with their tensile strength. For example, ceramics and concrete have high compressive strength and low tensile strength, However Metals have both tensile and compressive strengths that are very similar [6,7,8]. For structural design purposes, a safety factor is usually used to divide the compressive strength value, which in turn restricts it [8,9,10].

Orthotropic materials are also anisotropic materials, but not vice versa. Orthotropic materials differ properties change when measured from different directions [10,11,12]. An example of this is wood. The strengths of wood are a function of the grain orientation. The formula for wood's compressive strength first was proposed by Hankinson. His formula that evaluates the off-axis strength of wood is used in this study for different species of wood, both softwood and hardwood [9,10]. A mathematical investigation UCS of wood, an orthotropic material [16,17,18,19], was carried out in this study using Hankinson's criterion.

II. Hankinson's Criterion and Analysis

Hankinson's criterion is a formula for predicting the off-axis UCS of wood. Hankinson's criterion predicts that the UCS of wood in a direction at an angle θ to the grain [10], as given in equations (1) to (11), if the wood has UCS of P and Q parallel and perpendicular to the grain respectively.

The UCS of the wood in a direction at an angle 90 degrees and 0 degree are in Q perpendicular and P parallel to the grain respectively.

However, at other values of θ, both P and Q appear in the result, as shown in equation (7).

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Table 1: Wood species with their parallel and perpendicular to grain values

S/N	Wood species	P (MPa)	Q (MPa)
1	Loblolly Pine	49.2	5.4
2	Sitka Spruce	38.7	4.0
3	Red Oak	46.6	7.0
4	Yellow Poplar	38.2	3.4
5	Douglas Fir	87.6	2.0

Table 2: Values of UCS of selected species in different direction at different angles to the grain.

Angles	Loblolly Pine	Sitka Spruce	Red Oak	Yellow Poplar	Douglas Fir
0	49.2	38.7	46.6	38.2	87.6
10	39.54	30.65	39.83	29.19	38.25
20	25.25	19.21	28.05	17.39	14.58
30	16.25	12.21	19.30	10.73	7.49
40	11.31	8.45	13.97	7.31	4.69
50	8.54	6.35	10.78	5.45	3.35
60	6.95	5.15	8.89	4.40	2.65
70	6.03	4.47	7.77	3.81	2.26
80	5.55	4.11	7.18	3.50	2.06
90	5.40	4.00	7.00	3.40	2.00

$$\delta_\theta = \frac{PQ}{P\sin^2\theta + Q\cos^2\theta} \quad (1)$$

$$\delta_\theta = \frac{PQ}{P\sin^2\theta + Q(1-\sin^2\theta)} \quad (2)$$

$$\delta_\theta = \frac{PQ}{P\sin^2\theta + Q - Q\sin^2\theta)} \quad (3)$$

$$\delta_\theta = \frac{PQ}{Q + (P-Q)\sin^2\theta} \quad (4)$$

$when\ \theta = 90^0:$

$$\delta_{90^0} = \frac{PQ}{Q + (P-Q)} \quad (5)$$

$$= Q$$

$when\ \theta = 0^0:$

$$\delta_{0^0} = \frac{PQ}{Q} \quad (6)$$

$$= P$$

$when\ \theta = 30^0:$

$$\delta_{30^0} = \frac{PQ}{Q + \frac{1}{4}(P-Q)} \quad (7)$$

$$= \frac{4PQ}{P+3Q}$$

III. Results and Discussion

It can be seen from table 2, which summarizes the results of the analysis, that at angle 0^0, Douglas Fir has the highest UCS, while Yellow Poplar has the least. This implies that Douglas Fir has the highest maximum axial compressive stress, while Yellow Poplar has the least. At the other extreme, however, when the angle is 90^0, the Red Oak has the highest, while Douglas Fir has the least.

Generally, it can be seen from figure 1 that as the angle increases, the UCS of all the selected five species decrease. This implies that in a direction at angle of the grain, to a large extent, determines the value of the UCS of the species. There is a negative correlation. Hankinson's criterion shows that when the angle is equal to 0^0, the CS is same as the parallel to grain (P), but when the angle is 90^0, it is equivalent to perpendicular to the grain (Q).

Figure 3 depicts P and Q of the five species. Figures 4 to figure 8 show the CS of different species, at different angles to grain. The graphs are asymptotic to zero; this implies that the CS of these wood species is never zero, no matter how close to zero they get.

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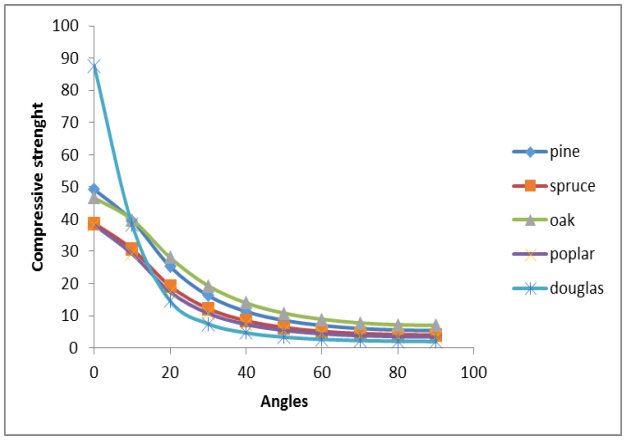

Figure 2: UCS of wood species in a direction at different angles to the grain.

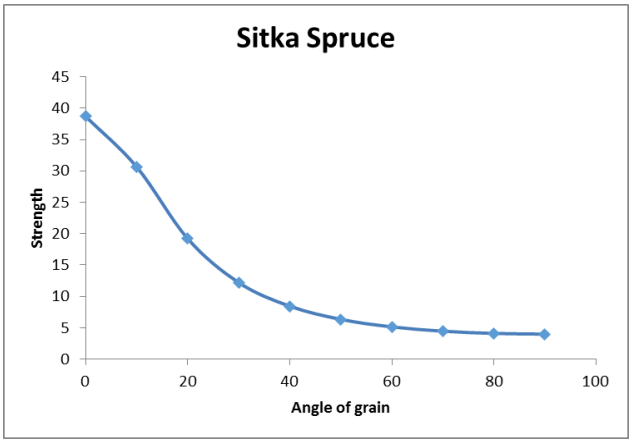

Figure 5: Sitka Spruce CS at different angles to grain

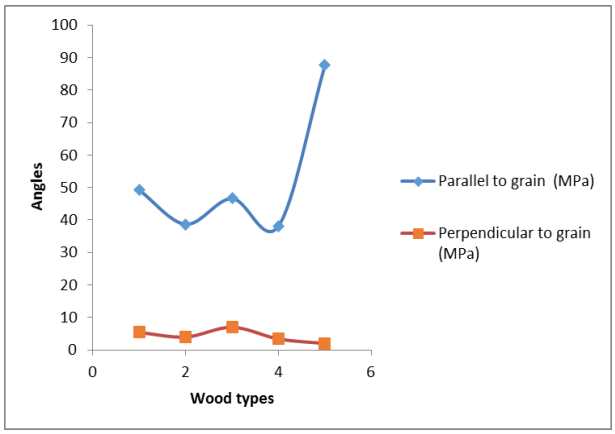

Figure 3: UCS of P parallel to the grain and Q perpendicular to the grain of wood specie

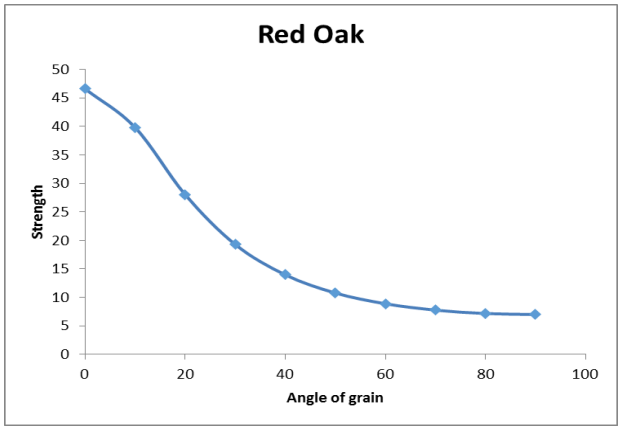

Figure 6: Red Oak CS at different angles to grain

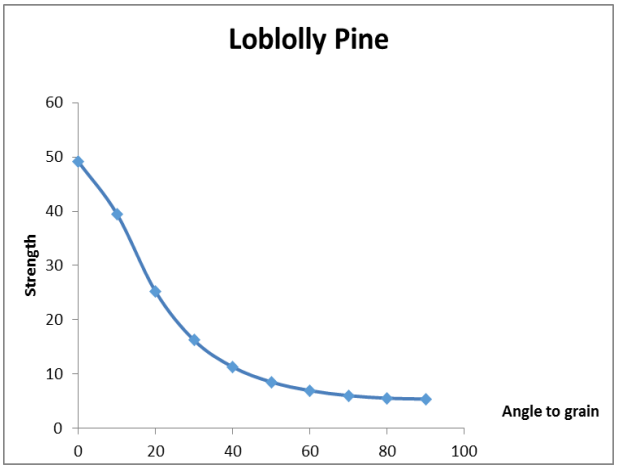

Figure 4: Loblolly Pine CS at different angles to grain

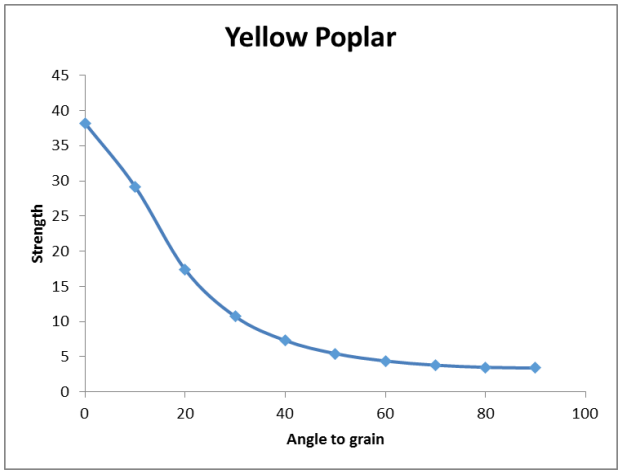

Figure 7: Yellow Poplar CS at different angles to grain

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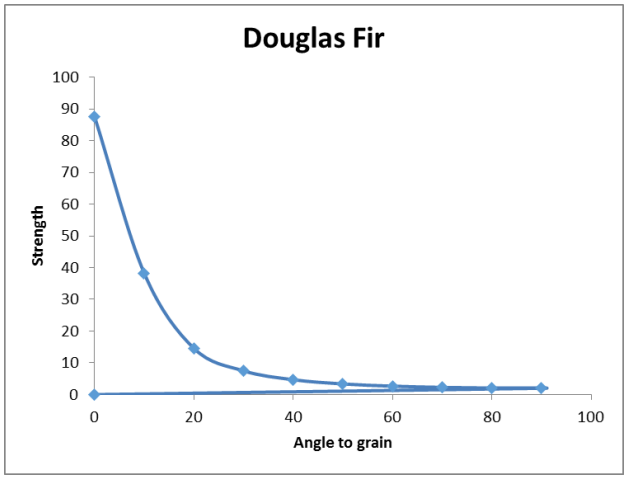

Figure 8: Douglas fir CS at different angles to grain

IV. CONCLUSION

Mathematical investigation of uniaxial compressive strength of selected species of wood, was carried out analytically in this study, using Hankinson's criterion. For brevity, five wood species were selected with their P and Q. Their CS were shown in table 2. However figure 2 shows the UCS of species in a direction at different angles of the grain, and figure 3 shows the difference between P and Q of the species.

On the average, in direction at any angle to the grain, Oak has the highest UCS among the five species considered in this paper. Douglas fir has the highest value of the UCS in direction at angle 0 to the grain, but the least in direction at angle 90 to the grain. Oak, therefore, has the highest maximum axial compressive stress among the five wood species selected.

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