Level Set Framework for Curve Evolution and Image Segmentation

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Abstract:

The paper presents a framework for curve evolution and image segmentation,based on Level Set methods.A new variational formulation for geometric active contours that forces the level set function to be close to a signed distance function, and therefore completely eliminates the need of the costly re-initialization procedure. Our variational formulation consists of an internal energy term that penalizes the deviation of the level set function from a signed distance function, and an external energy term that drives the motion of the zero level set toward the desired image features, such as object boundaries. The resulting evolution of the level set function is the gradient flow that minimizes the overall energy functional. The proposed variational level set formulation has three main advantages over the traditional level set formulations.First, a significantly larger time step can be used for numerically solving the evolution partial differential equation and therefore speeds up the curve evolution. Second, the level set function can be initialized with general functions that are more efficient to construct and easier to use in practice than the widely used signed distance function.Third, the level set evolution in our formulation can be easily implemented by simple finite difference scheme and is computationally more efficient. The proposed algorithm has been applied to both simulated and real images with promising results.To detect objects in an image,active contour models evolve an initial curve subject to conatraints specified in the image.

Keywords **:***Curve Evolution,Geometric active contours,Re-initialization,Segmentation,Level Set methods.*

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1.Introduction:

In recent years, a large body of work on geometric active contours, i.e., active contours implemented via level set methods, has been proposed to address a wide range of image segmentation problems in image processing and computer vision . Level set methods were first introduced by Osher and Sethian for capturing moving fronts. Active contours were introduced by Kass, Witkins,and Terzopoulos for segmenting objects in images using dynamic curves. The existing active contour models can be broadly classified as either *parametric active contour* models or *geometric active contour* models according to their representation and implementation. In particular, the parametric active contours are represented explicitly as parameterized curves in a Lagrangian framework, while the geometric active contours are represented implicitly as level sets of a two-dimensional function that evolves in an Eulerian framework. fuzzy partition.

Geometric active contours are independently introduced by Caselles and Malladi respectively.These models are based on curve evolution theory and level set method . The basic idea is to represent contours as the zero level set of an implicit function defined in a higher dimension, usually referred as the *level set function*, and to evolve the level set function according to a partial differential equation (PDE). This approach presents several advantages over the traditional parametric active contours.First, the contours represented by the level set function may break or merge naturally during the evolution, and the topological changes are thus automatically handled. Second, the level set function always remains a function on a fixed grid, which allows efficient numerical schemes.

Early geometric active contour models are typically derived using a Lagrangian formulation that yields a certain evolution PDE of a parametrized curve. This PDE is then converted to an evolution PDE for a level set function using the related Eulerian formulation from level set methods. As an alternative, the evolution PDE of the level set function can be directly derived from the problem of minimizing a certain energy functional defined on the level set function. This type of variational methods are known as variational level set methods .

2. Level Set Formulation for Curve Evolution

2.1 Traditional Level Set Methods:

In level set formulation of moving fronts (or active contours),the fronts, denoted by *C*, are represented by the zero level set $C(t) = \{(x, y) \mid \phi(t, x, y) = 0\}$ of a level set function $\phi(t, x, y)$.. The evolution equation of the level set function *A* can be written in the following general form:

$$\frac{\partial \phi}{\partial t} + F|\nabla \phi| = 0 \qquad (1)$$

which is called *level set equation* . The function *F* is called the speed function. For image segmentation, the function *F* depends on the image data and the level set function ϕ.

In traditional level set methods , the level set function ϕ can develop shocks, very sharp and/or flat shape during the evolution, which makes further computation highly inaccurate. To avoid these problems, a common numerical scheme is to initialize the function ϕas a signed distance function before the evolution, and then "reshape" (or "re-initialize") the function ϕ to be a signed distance function periodically during the evolution. Indeed, the reinitialization process is crucial and cannot be avoided in using traditional level set methods.

2.2. Drawbacks Associated with Reinitialization

Re-initialization has been extensively used as a numerical remedy in traditional level set methods . The standard re-initialization method is to solve the following *reinitialization equation*

$$\frac{\partial \phi}{\partial t} = \mathrm{sign}(\phi_0)(1 - |\nabla \phi|) \qquad (2)$$

where ϕ_0 is the function to be re-initialized, and sign(ϕ) is the sign function. There has been copious literature on re-initialization methods and most of them are the variants of the above PDE-based method. Unfortunately, if ϕ_0 is not smooth or ϕ_0 is much steeper on one side of the interface than the other, the zero level set of the resulting function ϕ can be moved incorrectly from that of the original function . Moreover, when the level set function is far away from a signed distance function, these methods may not be able to re-initialize the level set function to a signed distance function. In practice, the evolving level set function can deviate greatly from its value as signed distance in a small number of iteration steps, especially when the time step is not chosen small enough.

So far, re-initialization has been extensively used as a numerical remedy for maintaining stable curve evolution and ensuring desirable results. From the practical viewpoints, the re-initialization process can be quite complicated, expensive, and have subtle side effects. Moreover, most of the level set methods are fraught with their own problems, such as when and how to re-initialize the level set function to a signed distance function . There is no simple answer that applies generally to date. The variational level set formulation proposed in this paper can be easily implemented by simple finite difference scheme, without the need of re-initialization.

3. Variational Level Set Formulation Of Curve Evolution Without Re-Initialization.

3.1. GeneralVariational Level Set Formulation with Penalizing Energy

As discussed before, it is crucial to keep the evolving level set function as an approximate signed distance function during the evolution, especially in a neighborhood around the zero level set. It is well known that a signed distance function must satisfy a desirable property of $|\nabla \phi| = 1$|. Conversely, any function *A* satisfying $|\nabla \phi| = 1$ is the signed distance function plus a constant . Naturally, we propose the following integral

$$\mathcal{P}(\phi) = \int_\Omega \frac{1}{2}(|\nabla \phi| - 1)^2 dxdy \qquad (3)$$

as a metric to characterize how close a function ϕ is to a signed distance function in $\Omega \subset \Re^2$. This metric will play a key role in our variational level set formulation. With the above defined functional $\mathcal{P}(\phi)$, we propose the following variational formulation

$$\mathcal{E}(\phi) = \mu \mathcal{P}(\phi) + \mathcal{E}_m(\phi) \qquad (4)$$

where $\mu > 0$ is a parameter controlling the effect of penalizing the deviation of ϕ from a signed distance function, and $\mathcal{E}_m(\phi)$ is a certain energy that would drive the motion of the zero level curve of ϕ .

We denote by $\frac{\partial \mathcal{E}}{\partial \phi}$, the Gateaux derivative (or first variation) of the functional *E*, and the following evolution equation:

$$\frac{\partial \phi}{\partial t} = -\frac{\partial \mathcal{E}}{\partial \phi} \qquad (5)$$

is the *gradient flow* [18] that minimizes the functional $\mathcal{E}$. For a particular functional $\mathcal{E}(\phi)$defined explicitly in terms of ϕ, the Gateaux derivative can be computed and expressed in terms of the function ϕ and its derivatives .

We will focus on applying the variational formulation in (4) to active contours for image

ϕ

segmentation, so that the zero level curve of can evolve to the desired features in the image. For this purpose, the energy $\mathcal{E}_m$ will be defined as a functional that depends on image data , and therefore we call it the *external energy*. Accordingly, the energy is called the *internal energy* of the function ϕ,since it is a function of ϕ only.

During the evolution of ϕ according to the gradient flow (5) that minimizes the functional (4), the zero level curve will be moved by the external energy $\mathcal{E}_m$. Meanwhile, due to the penalizing effect of the internal energy, the evolving function ϕ will be automatically maintained as an approximate signed distance function during the evolution according to the evolution (5). Therefore the re-initialization procedure is completely eliminated in the proposed formulation.This concept is demonstrated further in the context of active contours next.

3.2. Variational Level Set Formulation of Active ContoursWithout Reinitialization

In image segmentation, active contours are dynamic curves that moves toward the object boundaries. To achieve this goal, we explicitly define an external energy that can move the zero level curve toward the object boundaries. Let *I* be an image, and *g* be the *edge indicator function* defined by

$$g = \frac{1}{1 + |\nabla G_\sigma * I|^2},$$

where G_σ is the Gaussian kernel with standard deviation σ. We define an external energy for a function $\phi(x, y)$ as below

$$\mathcal{E}_{g,\lambda,\nu}(\phi) = \lambda \mathcal{L}_g(\phi) + \nu \mathcal{A}_g(\phi) \qquad (6)$$

where $\lambda > 0$ and *v* are constants, and the terms $\mathcal{L}_g(\phi)$ and $\mathcal{A}_g(\phi)$ are defined by

$$\mathcal{L}_g(\phi) = \int_\Omega g\delta(\phi)|\nabla\phi| dxdy \qquad (7)$$

and

$$\mathcal{A}_g(\phi) = \int_\Omega gH(-\phi) dxdy, \qquad (8)$$

respectively, where δ is the univariate Dirac function, and *H* is the Heaviside function. Now, we define the following total energy functional

$$\mathcal{E}(\phi) = \mu\mathcal{P}(\phi) + \mathcal{E}_{g,\lambda,\nu}(\phi) \qquad (9)$$

The external energy $\mathcal{E}_{g,\lambda,\nu}$ drives the zero level set toward the object boundaries, while the internal energy $\mu\mathcal{P}(\phi)$ penalizes the deviation of ϕ from a signed distance function during its evolution.

To understand the geometric meaning of the energy $\mathcal{L}_g(\phi)$, we suppose that the zero level set of ϕ can be represented by a differentiable parameterized curve *C*(*p*), $p \in [0, 1]$. It is well known [9] that the energy functional $\mathcal{L}_g(\phi)$ in (7) computes the length of the zero level curve of ϕ in the conformal metric $g(C(p))|C'(p)|dp$. The energy functional $\mathcal{A}_g(\phi)$ in (8) is introduced to speed up curve evolution. Note that, when the function *g* is constant 1, the energy functional in (8) is the area of the region . The energy functional in (8) can be viewed as the weighted area of .
The coefficient *v* of can be positive or negative, depending on the relative position of the initial contour to the object of interest. For example, if the initial contours are placed outside the object, the coefficient *v* in the weighted area term should take positive value, so that the contours can shrink faster. If the initial contours are placed inside the object, the coefficient *v* should take negative value to speed up the expansion of the contours.

By calculus of variations , the Gateaux derivative (first variation) of the functional $\mathcal{E}$ in (9) can be written as

$$\frac{\partial \mathcal{E}}{\partial \phi} = -\mu[\Delta\phi - \mathrm{div}(\frac{\nabla\phi}{|\nabla\phi|})] - \lambda\delta(\phi)\mathrm{div}(g\frac{\nabla\phi}{|\nabla\phi|}) - \nu g\delta(\phi)$$

where Δ is the Laplacian operator. Therefore, the function ϕ that minimizes this functional satisfies the Euler-Lagrange equation $\frac{\partial\mathcal{E}}{\partial\phi} = 0$. The steepest descent process for minimization of the functional $\mathcal{E}$ is the following gradient flow:

$$\frac{\partial\phi}{\partial t} = \mu[\Delta\phi - \mathrm{div}(\frac{\nabla\phi}{|\nabla\phi|})] + \lambda\delta(\phi)\mathrm{div}(g\frac{\nabla\phi}{|\nabla\phi|}) + \nu g\delta(\phi) \qquad (10)$$

This gradient flow is the evolution equation of the level set function in the proposed method.

The second and the third term in the right hand side of (10) correspond to the gradient flows of the energy functional $\lambda\mathcal{L}_g(\phi)$ and $\nu\mathcal{A}_g(\phi)$, respectively, and are responsible of driving the zero level curve towards the object boundaries. To explain the effect of the first term, which is associated to the internal energy $\mu\mathcal{P}(\phi)$, we notice that the gradient flow

$$\Delta\phi - \mathrm{div}(\frac{\nabla\phi}{|\nabla\phi|}) = \mathrm{div}[(1 - \frac{1}{|\nabla\phi|})\nabla\phi]$$

has the factor $(1 - \frac{1}{|\nabla\phi|})$ as diffusion rate. If $|\nabla\phi| > 1$, the diffusion rate is positive and the effect of this term is

$|\nabla\phi|$. $|\nabla\phi| < 1$.

the usual diffusion, i.e. making ϕ more even and therefore reduce the gradient If the term has effect of reverse diffusion and therefore increase the gradient.

We use an image of a circular object, as shown in Fig.1 to show the evolution of ϕ according to Eq. (10). In Fig. 1, the first figure in the upper row shows the initial level set function, and its zero level curve is

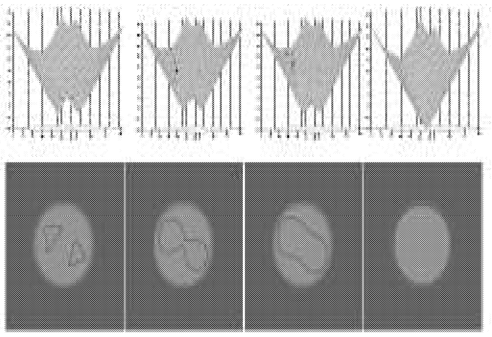


Figure 1. *Evolution of level set function ϕ is shown in Row 1 and Row 2 showsEvolution of zerolevel curve of the corresponding level set function ϕ in Row 1.*

plotted in first figure in the lower row. The upper row shows the evolution of the level set function ϕ, and the lower row shows the corresponding zero level curve of ϕ. The fourth column is the converged result of the evolution. As we can see from this figure, during the evolution, the evolving level set function ϕ is maintained very close to a signed distance function.

4. Implementation

4.1 Numerical Scheme

In practice, the Dirac function $\delta(x)$ in (10) is slightly smoothed as the following function $\delta_\varepsilon(x)$ defined by:

$$\delta_\varepsilon(x) = \begin{cases} 0, & |x| > \varepsilon \\ \frac{1}{2\varepsilon}[1 + \cos(\frac{\pi x}{\varepsilon})], & |x| \leq \varepsilon. \end{cases} \quad (11)$$

We use the regularized Dirac $\delta_\varepsilon(x)$ with ε = 1.5, for all the experiments in the project. Because of the diffusion term introduced by our penalizing energy, we no longer need the upwind scheme [4] as in the traditional level set methods.Instead, all the spatial partial derivatives $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ are approximated by the central difference, and the temporal partial derivative $\frac{\partial \phi}{\partial t}$ is approximated by the forward difference.The approximation of (10) by the above difference scheme can be simply written as

$$\frac{\phi_{i,j}^{k+1} - \phi_{i,j}^{k}}{\tau} = L(\phi_{i,j}^{k}) \quad (12)$$

where $L(\phi_{i,j})$ is the approximation of the right hand side in (10) by the above spatial difference scheme. The difference equation (12) can be expressed as the following iteration:

$$\phi_{i,j}^{k+1} = \phi_{i,j}^{k} + \tau L(\phi_{i,j}^{k}) \quad (13)$$

4.2. Selection of Time Step

In implementing the proposed level set method, the time step can be chosen significantly larger than the time step used in the traditional level set methods. We have tried a large range of the time step τ in our experiments, from 0.1 to 100.0. A natural question is: what is the range of the time step τ for which the iteration (13) is stable?It is found that the time step τ and the coefficient μ must satisfy $\tau\mu < \frac{1}{4}$ in the difference scheme described in Section 4.1, in order to maintain stable level set evolution. Using larger time step can speed up the evolution, but may cause error in the boundary location if the time step is chosen too large. There is a tradeoff between choosing larger time step and accuracy in boundary location.

4.3. Flexible Initialization of Level Set Function

In traditional level set methods, it is necessary to initialize the level set function ϕ as a signed distance function ϕ_0. If the initial level set function is significantly different from a signed distance function, then the re-initialization schemes are not able to re-initialize the function to a signed distance function. In our formulation, not only the reinitialization procedure is completely eliminated, but also the level set function ϕ is no longer required to be initialized as a signed distance function. Here, we propose the following functions as the initial function ϕ_0 . Let Ω_0 be a subset in the image domain ,a Ω $\partial\Omega_0$ be all the points on the boundaries of Ω_0 , which can be efficiently identified by some simple morphological operations. Then, the initial function ϕ_0 is defined as

$$\phi_0(x,y) = \begin{cases} -\rho, & (x,y) \in \Omega_0 - \partial\Omega_0 \\ 0 & (x,y) \in \partial\Omega_0 \\ \rho & \Omega - \Omega_0 \end{cases} \quad (14)$$

where ρ > 0 is a constant. We suggest to choose ρ larger than 2 ε, where ε is the width in the definition of the regularized Dirac function δ_ε in (11).

Unlike signed distance functions, which are computed from a contour, the proposed initial level set functions are computed from an arbitrary region Ω_0 in the image domain Ω . Such region-based initialization of level set function is not only computationally efficient, but also allows for flexible applications in some situations. For example, if the regions of interest can be roughly and automatically obtained in some way, such as thresholding, then we can use these roughly obtained regions as the region Ω_0 to construct the initial level set function ϕ_0 . Then, the initial level set function will evolve stably according to the evolution equation , with its zero level curve converged to the exact boundary of the region of interest.

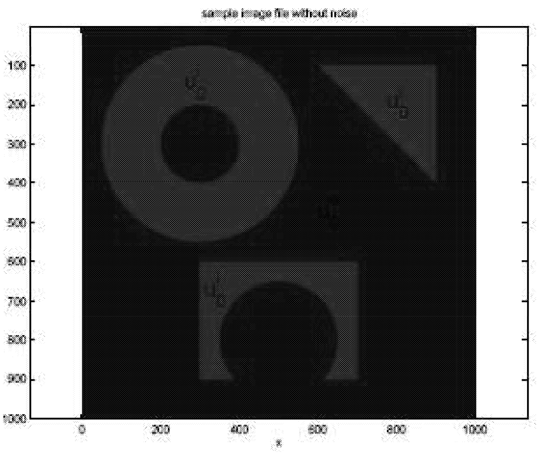
Note that this kind of initial function ϕ_0 significantly deviates from a signed distance function. During the evolution,the level set function Á may not be able to keep its profile globally as an approximate signed distance function in the entire image domain. But the evolution based on the proposed penalizing diffusion still maintains the level set function ϕ as an approximate signed distance function near the zero level set.

5.0 Level Set Framework for Image Segmentation

5. 1 Introduction

To detect objects in an image, active contour models evolve an initial curve subject to constraints specified in the image. T.F. Chan and L.A. Vese proposed an active contour model using an energy minimization technique.Their model works on noisy/blurred images, and does not rely on gradient values to find the boundary.

5.2. Model



Assume that image u_0 is formed by two regions of approximately constant intensities u_0^i and u_0^o , and the object to be detected is represented by the region with value u_0^i. If the boundary is given by C_0 , then $u_0 \approx u_0^i$ inside C_0 and $u_0 \approx u_0^o$ outside C_0. The following fitting energy

$$F_1(C) + F_2(C) = \int_{\text{inside } C} |u_0 - c_1|^2 dx + \int_{\text{outside } C} |u_0 - c_2|^2 dx \quad (1)$$

(where C is any variable curve, c1 and c2 are constants depending on C) is minimized when $C = C_0$. i.e.

$$\inf_C \{F_1(C) + F_2(C)\} \approx 0 \approx F_1(C_0) + F_2(C_0)$$

Adding some regularizing terms like the length of C and the area inside C, the energy function $F(C, c_1, c_2)$ is given by

$$F(C, c_1, c_2) = \mu \,(\text{length of } C)^p + \nu \,(\text{area inside } C) \quad (2)$$
$$+\lambda_1 \int_{\text{inside } C} |u_0 - c_1|^2 dx + \lambda_2 \int_{\text{outside } C} |u_0 - c_2|^2 dx$$

c_1 and c_2 are constant unknowns, $\mu \geq 0$, $\nu \geq 0$, $\lambda_1, \lambda_2 > 0 p > 0$ are fixed constants.

5.3. Level Set Formulation

In the level set method, C is represented by the zero level set of a Lipschitz function $\phi : \mathbf{R}^N \to \mathbf{R}$ such that

$$C = \{x \in \mathbf{R}^N : \phi(x) = 0\}$$
$$\text{inside } C = \{x \in \mathbf{R}^N : \phi(x) > 0\}$$
$$\text{outside } C = \{x \in \mathbf{R}^N : \phi(x) < 0\}$$

Using the standard definition for the Heaviside function Hand the dirac measure δ ,

$$H(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$$

$$\delta(z) = \frac{d}{dz} H(z) \quad \text{(in the sense of distributions)}$$

We have the following results:

$$\text{length of } C = \int_\Omega |\nabla H(\phi)| dx = \int_\Omega \delta(\phi) |\nabla \phi| dx \quad (3)$$

$$\text{Area inside } C = \int_\Omega H(\phi) dx \text{ , thus} \quad (4)$$

$$\int_{\text{inside } C} |u_0 - c_1|^2 dx = \int_\Omega |u_0 - c_1|^2 H(\phi) dx \quad (5)$$

$$\int_{\text{outside } C} |u_0 - c_2|^2 dx = \int_\Omega |u_0 - c_1|^2 \{1 - H(\phi)\} dx \quad (6)$$

Therefore,

$$F(\phi, c_1, c_2) = \mu \left(\int_\Omega \delta(\phi)|\nabla \phi| dx \right)^p + \nu \int_\Omega H(\phi) dx + \quad (7)$$
$$\lambda_1 \int_\Omega |u_0 - c_1|^2 H(\phi) dx + \lambda_2 \int_\Omega |u_0 - c_1|^2 \{1 - H(\phi)\} dx$$

Minimizing the energy functional with respect to c_1 and c_2 gives

$$c_1(\phi) = \frac{\int_\Omega u_0 H(\phi) dx}{\int_\Omega H(\phi) dx} \quad (8)$$

$$c_2(\phi) = \frac{\int_\Omega u_0 \{1 - H(\phi)\} dx}{\int_\Omega \{1 - H(\phi)\} dx} \quad (9)$$

which correspond to the average value of u_0 inside C and outside C respectively. (note, the curve must have a non-empty interior and exterior).

5.4 Euler-Lagrange Equations

To compute the associated Euler-Lagrange equations for ϕ, we need regularized versions of H and δ such that $\delta_\epsilon = H'_\epsilon$ In particular, we use

$$H_\epsilon(z) = \frac{1}{2}\left(1 + \frac{2}{\pi} \arctan\left(\frac{z}{\epsilon}\right)\right) \quad (10)$$
$$\delta_\epsilon(z) = H'_\epsilon(z) = \frac{1}{\pi}\left(\frac{\epsilon}{\epsilon^2 + z^2}\right)$$

The associated regularized functional F_ϵ of F will be

$$F_\epsilon(\phi, c_1, c_2) = \mu \left(\int_\Omega \delta_\epsilon(\phi)|\nabla \phi| dx \right)^p + \nu \int_\Omega H_\epsilon(\phi) dx \quad (11)$$
$$+\lambda_1 \int_\Omega |u_0 - c_1|^2 H_\epsilon(\phi) dx + \lambda_2 \int_\Omega |u_0 - c_1|^2 \{1 - H_\epsilon(\phi)\} dx$$

Keeping c_1 and c_2 fixed we compute

$$\lim_{t \to 0} \frac{1}{t} [F_\epsilon(\phi + t\psi, c_1, c_2) - F_\epsilon(\phi, c_1, c_2)] \quad (12)$$

(where ψ is a test function) to obtain the Euler Lagrange equations for ϕ.

$$\delta_\epsilon(\phi)\Big[\mu\, p \left(\int_\Omega \delta_\epsilon(\phi)|\nabla \phi| \right)^{p-1} \nabla \cdot \left(\frac{\nabla \phi}{|\nabla \phi|} \right) \quad (13)$$
$$-\nu - \lambda_1 (u_0 - c_1)^2 + \lambda_2 (u_0 - c_2)^2 \Big] = 0 \text{ on } \Omega$$

$$p \left(\int_\Omega \delta_\epsilon(\phi)|\nabla \phi| dx \right)^{p-1} \frac{\delta_\epsilon(\phi)}{|\nabla \phi|} \frac{\partial \phi}{\partial n} = 0 \text{ on } \partial\Omega \quad (14)$$

5.5 Implementation

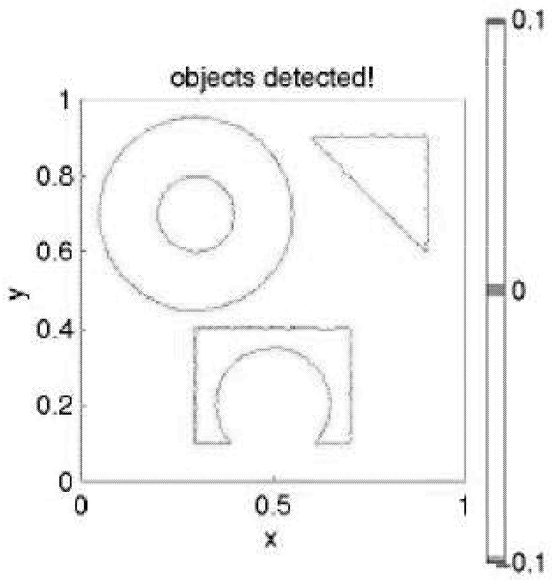
1. Intialize ϕ^0
2. Calculate $c_1(\phi^0)$, $c_2(\phi^0)$ and $L = \int_\Omega \delta_\epsilon(\phi)|\nabla \phi|\, dx$
3. Solve PDE

$$\frac{\partial \phi}{\partial t} = \delta_\epsilon(\phi)\Big[\mu\, p(L)^{p-1} \nabla \cdot \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - \nu \quad (15)$$
$$-\lambda_1 (u_0 - c_1)^2 + \lambda_2 (u_0 - c_2)^2 \Big] = 0 \text{ on } \Omega$$

$$\frac{\partial \phi}{\partial n} = 0 \text{ on } \partial\Omega \quad (16)$$

5.6 Results

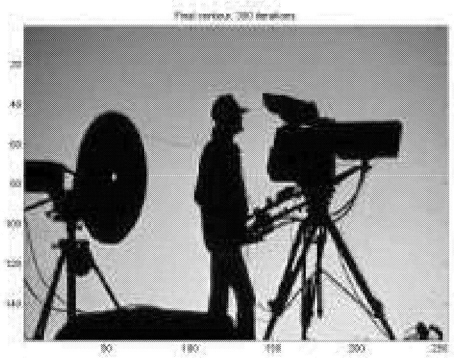
Starting with an initial contours centered at $(0.5, 0.5)$ with radii 0.1, 0.3 and 0.4, the algorithm converged correctly to the the test image. It was impressive to watch the contour deal with topological changes, the sharp edges and the convex shape.



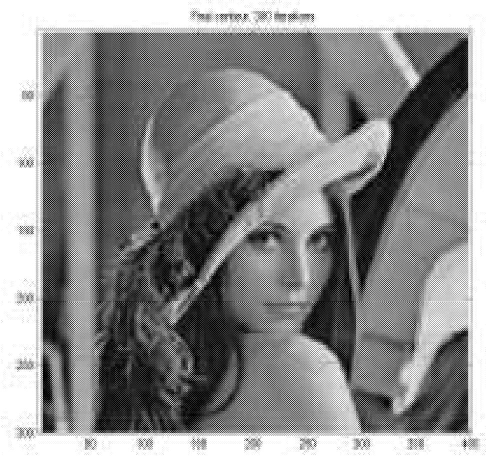
6 Simulation Results

A1. Original Image

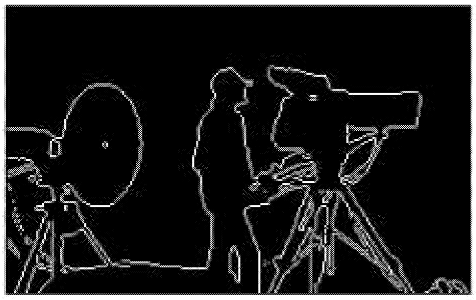
A2. Original Image



B1. Image Contour



B2. Image Contour



C1. Edge Image

C2. Edge Image

7.Conclusions

The paper presents a new variational level setformulation that completely eliminates the need for reinitialization.The proposed level set method can be easily implemented by using simple finite difference scheme and is computationally more efficient than the traditional level set methods. In our method, significantly larger time step can be used to speed up the curve evolution, while maintaining stable evolution of the level set function. Moreover, the level set function is no longer required to be initialized as a signed distance function. We propose a region-based initialization of level set function, which is not only computationally more efficient than computing signed distance function, but also allows for more flexible applications. We demonstrate the performance of the proposed algorithm using both simulated and real images, and in particular its robustness to the presence of weak boundaries and strong noise.

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