A Finite Element Study of the Transport Phenomena in MHD Micropolar Flow in a Darcy-Forchheimer Porous Medium

S. Rawat, R. Bhargava & O. Anwar Bég

Abstract—**We analyze the free convection magnetohydrodynamic micropolar flow, heat and species diffusion between vertical plates enclosing a non-Darcian porous medium with variable thermal conductivity and internal heat generation/absorption effects. The non-linear coupled partial differential conservation equations are transformed and solved using the finite element method subjected to appropriate boundary conditions. Numerical results for the velocity, angular velocity, temperature and concentration profiles as well as for the heat transfer rate and skin friction are plotted graphically and tabulated for the controlling thermophysical and hydrodynamic parameters, namely hydromagnetic number buoyancy ratio parameter, vortex viscosity parameter, Darcy number, Forchheimer number, thermal conductivity parameter and heat absorption/generation parameter to demonstrate the flow and transport phenomena behaviour. It is also shown that the volume flow rate, the total heat rate and the total species rate added to the fluid are decreased with a rise in vortex viscosity parameter. The flow scenario finds applications in Chemical processing, metallurgical transport modeling, aerodynamic heating and many geophysical processes e.g. crude oil recovery.**

Index Terms— **Fully developed, MHD, Non-darcy and Heat generation/absorption.**

I. Introduction

The study of convective flow, heat and mass transfer in porous media has been an active field of research as it play a crucial role in diverse applications, such as thermal insulation, extraction of crude oil and chemical catalytic reactors etc. Although considerable work has been reported on flow heat and mass transfer in porous media, a majority of porous studies have been on Darcy's law which states that the volume averaged velocity is proportional to the pressure gradient. Darcy's law however is valid only for slow flows through porous media with low permeability. At higher flow rates, there is a departure from the linear law and inertial effects become important. The Darcy-Forchheimer model describes the effect of inertia as well as viscous forces in porous media and was used by Poulikakos and Bejan [1], [2].

All the above-mentioned work has been based on the Newtonian i.e. Navier-Stokes fluid model, but the fluids used in most of the metallurgical and chemical engineering flows, exhibit strong non-Newtonian behaviour. To overcome the inadequacy of the Navier-Stokes equations to explain certain phenomena exhibited by fluids with suspended particles like colloidal suspension, exotic lubricants, animal blood etc, Eringen [3] developed the theory of micropolar fluids which take into account the local rotary inertia and couple stresses.

Recently, considerable attention has also been focused on new applications of magneto-hydrodynamics (MHD) and heat transfer in for e.g. metallurgical processing. Melt refining involves magnetic field application to control excessive heat transfer rates. Other applications of MHD heat transfer include MHD generators, plasma propulsion in astronautics, nuclear reactor thermal dynamics and ionized-geothermal energy systems etc.

But most of the MHD heat convection problem in porous media are done taking constant fluid properties, for example thermal conductivity. This considerably simplifies the analytical and experimental studies as the number of variables are reduced. However it has been strongly established that such thermophysical properties changes with temperature. To accurately predict flow and heat transfer rates, it is necessary to take into account this variation of physical properties with temperature.

Consideration of heat source and sink become important when dealing with chemical reactions and dissociating fluids. In such cases a source or sink term is added in the energy equation to include its effects.

Therefore, the purpose of this study is to analyze the steady, incompressible flow heat and mass transfer of an electrically conducting micropolar fluid in a Non-darcy porous medium with variable thermal conductivity and heat generation /absorption effects using the finite element method.

II. Formulation of the Problem

Consider the steady, laminar, incompressible, free-convection flow between two vertical plates embedded in a non-Darcy porous medium with heat- absorbing or generating thermo-micropolar fluid, subjected to a transverse magnetic field. It is assumed that the two walls are maintained at different temperatures and concentrations resulting in an *asymmetric* situation with respect to temperature and concentration

Manuscript received July 19, 2007.

Acknowledgment:

The author (S. Rawat) acknowledges and is grateful for the financial assistance provided to him by University Grant Commission, INDIA.

S. Rawat is with the Department of Mathematics, Indian Institute of Technology, Roorkee, India (corresponding author: phone: +91- 9219873097; fax: +91-1332 - 273560 ; e-mail: sam.rawat@gmail.com).

R. Bhargava is with the Department of Mathematics, Indian Institute of Technology, Roorkee, India (e-mail: bhargava_iitr@rediffmail.com).

O. Anwar Beg is with the British Aerospace (BAE) Systems, Salwa Garden Village, Villa G09D, PO Box 1732, Riyadh 11441, Kingdom of Saudi Arabia (e-mail: docoabeg@hotmail.com)

respectively. The vertical plates are separated by a distance b with reference to an x, y coordinate system, where the x-axis is directed along the vertical plates and the y-axis is transverse to this. The flow is both hydrodynamically and thermally fully developed. Neglecting viscous heating and thermal dispersion effects, under the Boussinesq approximation, the conservation equations can be written as follows:

Velocity Equation:

$$(\mu+\kappa)\frac{d^2u}{dy^2}+\kappa\frac{dg}{dy}-\sigma B_o^2u+\rho\beta_T g_a(T-T_0) + \rho\beta_C g_a(C-C_0)-\frac{(\mu+\kappa)}{k_p}u-\frac{b_F\rho}{k_p}u^2=0 \tag{1}$$

Angular Momentum Equation:

$$\gamma\frac{d^2g}{dy^2}-\kappa\left(2g+\frac{du}{dy}\right)=0 \tag{2}$$

Energy Equation:

$$\frac{d}{dy}\left(k_f\frac{dT}{dy}\right)+Q_o(T-T_o)=0 \tag{3}$$

Diffusion Equation:

$$\frac{d^2C}{dy^2}=0 \tag{4}$$

with vertical walls boundary conditions as:

$$y=0:\ u=0,\ g=0,\ T=T_1,\ C=C_1 \tag{5a}$$

$$y=b:\ u=0,\ g=0,\ T=T_2,\ C=C_2 \tag{5b}$$

where σ is the electrical conductivity of the micropolar fluid, B_o is the strength of the transverse magnetic field, g_a is the gravity. μ and k_p designate respectively the Newtonian dynamic viscosity and permeability of the porous medium, g is the angular velocity of the micropolar fluid micro-elements, k_f is the thermal conductivity of the fluid, Q_o is the dimensional heat generation or absorption coefficient, T and C are the fluid temperature and concentration respectively, T_0 is the inlet temperature and C_0 is the inlet concentration. The left plate is kept at constant temperature T_1 and the right plate is maintained at a constant temperature T_2. Additionally, the concentration varies from C_1 on the left plate to C_2 on the right plate.

Considering the thermal conductivity as a linear function of temperature and defining as:

$$k_f=k_1\left[1+\alpha(T-T_0)\right]\ or\ \ k_f=k_1\left[1+S\theta\right] \quad where\ S=\alpha(T_1-T_0) \tag{6}$$

Here k_1 is the fluid thermal conductivity at temperature T_1 and α is a constant depending on the nature of the fluid. Proceeding with the analysis, we introduce the following similarity transformations:

$$u=\frac{\mu Gr}{b\rho}U,\quad y=Yb,\quad g=\frac{\mu Gr}{\rho b^2}H, \quad \theta=(T-T_0)/(T_1-T_0),\ \Phi=(C-C_0)/(C_1-C_0) \tag{7}$$

Substituting equation (7) into equations (1)-(5) leads to the following set of non-linear, ordinary differential equations:

Linear Momentum Equation:

$$(1+R)\frac{d^2U}{dY^2}+R\frac{dH}{dY}-MU+\theta+N_b\Phi -\frac{(1+R)}{Da}U-\frac{FsGr}{Da}U^2=0 \tag{8}$$

Angular Momentum Equation:

$$\left(1+\frac{R}{2}\right)\frac{d^2H}{dY^2}-BR\left(2H+\frac{dU}{dY}\right)=0 \tag{9}$$

Energy Equation:

$$\left(1+S\theta\right)\frac{d^2\theta}{dY^2}+S\left(\frac{d\theta}{dY}\right)^2+h_S\theta=0 \tag{10}$$

Diffusion Equation:

$$\left(\frac{d^2\Phi}{dY^2}\right)=0 \tag{11}$$

where $B=b^2/j$ and $R=\kappa/\mu$ are micropolar parameters (dimensionless material properties) and $N_b=\beta_C(C_1-C_0)/\beta_T(T_1-T_0)$ is the buoyancy ratio, $Gr=g_a\beta_T b^3\rho^2\left(T_1-T_0\right)/\mu^2$ is the Grashof number, $M=\sigma B_o^2b^2/\mu$ is the magnetic parameter, $Da=k_p/b^2$ is the Darcy number, $Fs=b_F/b$ is the Forchheimer (quadratic porous drag) number and $h_s=Q_ob^2/k_1$ is the heat absorption/generation parameter. The transformed boundary conditions now become:

$$\text{At}\quad Y=0:U=0,H=0,\theta=1,\Phi=1 \tag{12}$$

$$\text{At}\quad Y=1:U=0,H=0,\theta=m,\Phi=n \tag{13}$$

where $m=(T_2-T_0)/(T_1-T_0)$ is the wall temperature ratio and $n=(C_2-C_0)/(C_1-C_0)$ is the wall concentration ratio.

The shear stress at the left wall is given by:

$$\tau_1=\left[\left(\mu+\kappa\right)\left(\frac{du}{dy}\right)+\kappa g\right]_{y=0}=\frac{\left(\mu+\kappa\right)Gr\mu}{\rho b^2}U'(0) \tag{14}$$

The heat flux at the left wall may be written using Fourier's law as follows:

$$q_1=-k_f\left(\frac{dT}{dy}\right)_{y=0}=-k_f\frac{(T_1-T_0)}{b}\theta'(0) \tag{15}$$

The heat transfer coefficient at the left wall is given by:

$$h_1=\frac{q_1}{(T_1-T_0)}=-\frac{k_f}{b}\theta'(0) \tag{16}$$

The Nusselt number at the left wall can be defined thus:

$$Nu=\frac{h_1b}{k_f}=-\theta'(0) \tag{17}$$

The dimensionless volume flow rate is given by:

$$Q=\int_0^1 U\,dY \tag{18}$$

The dimensionless total heat rate added to the fluid is given by:

$$E = \int_0^1 U\theta dY \quad (19)$$

Finally the dimensionless total species rate added to the fluid is given by:

$$\varphi = \int_0^1 U\ \Phi\, dY \quad (20)$$

III. Numerical Solution by Finite Element method

The transformed two-point boundary value problem defined by equations (8-11) and (12-13) is solved using the finite element method. Details of the method are given in Reddy [4] and Bathe [5]. This technique has been employed extensively by the authors in many challenging heat transfer, biomechanics and metallurgical transport phenomena problems over the past few years. Bhargava et al [6] studied numerically the mixed micropolar heat transfer past a stretching surface with transpiration effects. Bég et al [7] also analyzed the two-dimensional micropolar convection in a porous medium enclosure using finite element and finite difference methods. More recently Bhargava et al [8] investigated the first order homogenous chemically-reactive heat and mass transfer in micropolar-saturated porous media. Other studies include third grade viscoelastic hydrodynamics in porous materials [9], natural convection boundary layers in geo-porous continua [10], pulsatile magneto-biofluid dynamics and mass transport in a channel [11].

The whole domain is subdivided into two noded elements. In a nutshell, the Finite element equation are written for all elements and then on assembly of all the element equations we obtain a matrix of order 328×328 . After applying the given boundary conditions a system of 320 equations remains for numerical solution, a process which is successfully discharged utilizing the Gauss-Seidel method maintaining an accuracy of 0.0005.

IV. Results and Discussion

The numerical results so obtained are plotted for velocity, microrotation, temperature in figures 1-12 wherein the following default parameter values are used: *Da = 0.5, Fs = 5, Gr =0.5, B =1, R =1, S = 0.1,* $N_b = 2$, $h_s = 0.5$, *M = 1.0, m = 0.2 and n = 0.1.*

We observe from fig. 1 that an increase in Darcy number Da leads to an increase in dimensionless velocity U. This is due to the fact that larger values of Da correspond to higher permeability porous media (Da $\propto$ k_p), which implies less porous fiber resistance to the flow and therefore an acceleration in transport. Also the limit $Da \to \infty$ corresponds to the case of a vanishing porous medium and is associated naturally therefore with the maximum linear velocity scenario. Fig. 2 illustrates the effect of Darcy number Da on dimensionless angular velocity H. It is observed that microrotation continuously increases with an increase in the Darcy number Da which means that porous medium act as an hindrance in the rotary motions of the micro-elements.

Fig. 3. shows the dimensionless velocity distribution for different values of Forchheimer parameter *Fs*. Since *Fs* represents the inertial drag, thus an increase in the Forchheimer parameter increases the resistance to the flow and so a decrease in the fluid velocity ensues. Here Fs =0 represents the case when the flow is Darcian i.e. inertial effects are neglected and so the velocity is maximum in this case due to the total absence of inertial drag.

Figs. 4-5 show the effect of the vortex viscosity parameter *R* on velocity and microrotation distributions respectively. It is observed that as vortex viscosity parameter *R* increases, linear velocity decreases whereas the angular velocity increases which is in excellent concurrence with the results obtained by Cheng [12]. Since R is proportional to vortex viscosity of the fluid microstructure therefore an increase in the value of R leads to an increase in the angular velocity; as a result the increase in vortex viscosity increases the rotation of the micro-elements. Fig. 5 shows that maximum velocity is obtained when R = 0 (i.e. Newtonian fluid). This is due to the fact that the presence of micro-elements decelerates the flow and they are completely absent in the special case of Navier-Stokes fluids.

The effect of thermal conductivity parameter on temperature profile is shown in fig 6. Here it is observed that temperature profiles decreases with an increase in the thermal conductivity parameter S. In the case of constant thermal conductivity i.e. S = 0 (i.e thermal conductivity is independent of temperature) the temperature attain maximum values.

The presence of magnetic field in an electrically-conducting fluid tends to produce a body force which reduces the velocity of the fluid and angular velocity of microrotation as supported by profiles illustrated by figs. 7-8.

Figs. 9-10 depicts that increasing the value of h_s induces a rise in the angular velocity and temperature distributions of the fluid. This result qualitatively agrees with the expectation, since the effect of internal heat generation (h_s > 0) is to increase the rate of energy transport to the fluid, thereby increasing the temperature of the fluid. Also the angular velocity increases due to the increase in temperature. On the contrary, a heat sink ($h_s < 0$) has the opposite effect, namely cooling of the fluid.

Figs. 11, 12, 13 graphically illustrate the change of the dimensionless flow rate *Q* , the dimensionless total heat rate *E* added to the fluid and the dimensionless total species rate φ added to the fluid with the buoyancy ratio N_b for various vortex viscosity parameter in the presence as well as in the absence of the magnetic field.

Fig. 11 shows that an increase in buoyancy ratio boosts the fluid flow and thereby elevates the volume flow rate of the fluid between the two vertical plates. As expected it is observed that an increase in vortex viscosity parameter lead to

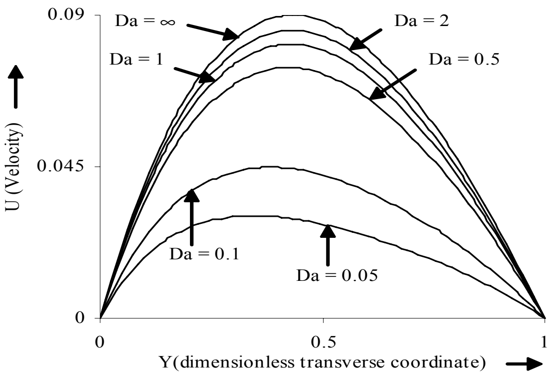

Fig. 1: U versus Y for various Da values.

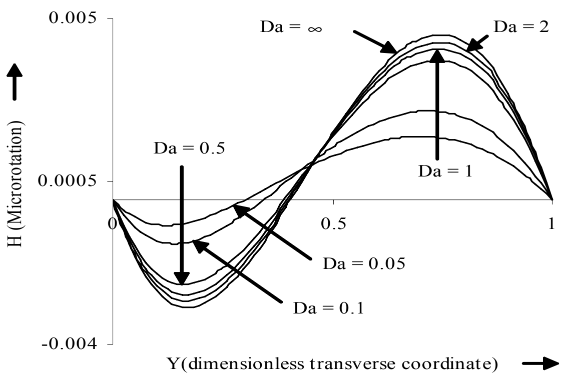

Fig. 2: H versus Y for various Da values.

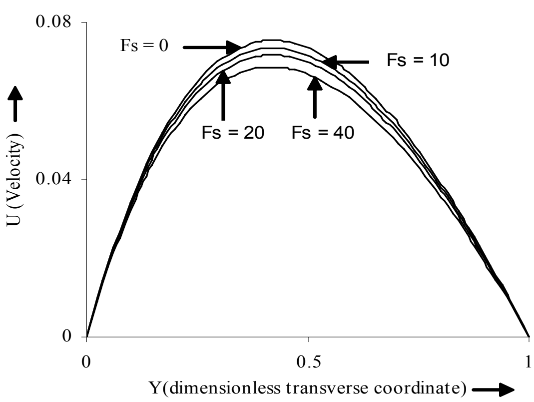

Fig. 3: U versus Y for various Fs values

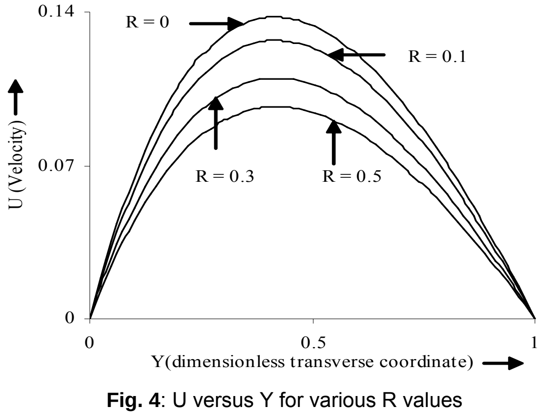

Fig. 4: U versus Y for various R values

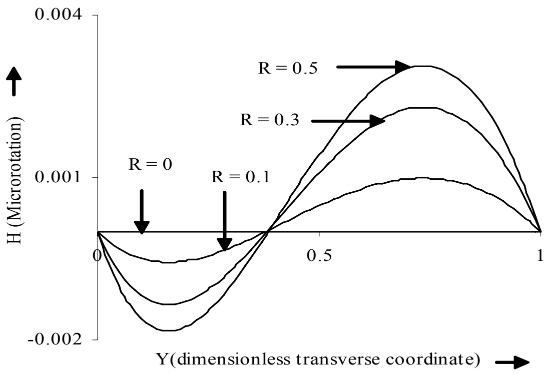

Fig. 5: H versus Y for various R values.

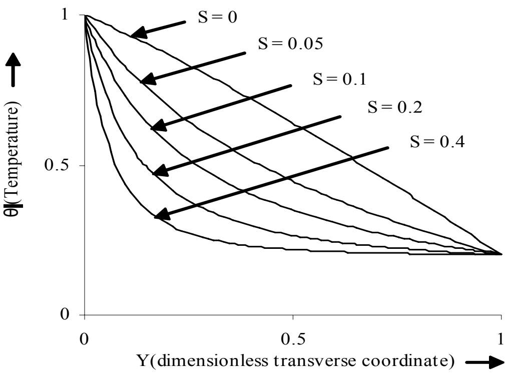

Fig. 6: θ versus Y for various S values.

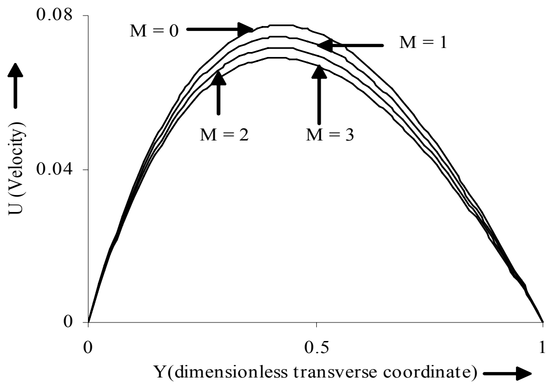

Fig. 7: U versus Y for various M values.

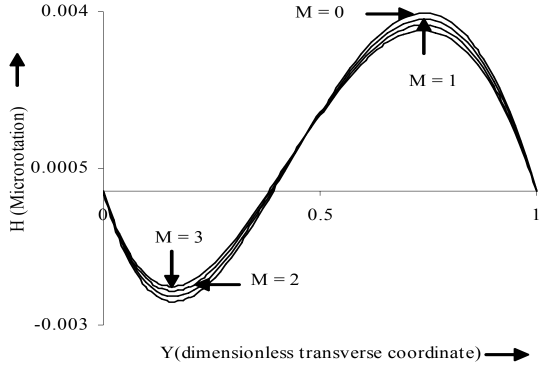

Fig. 8: H versus Y for various M values.

ISBN:978-988-98671-6-4

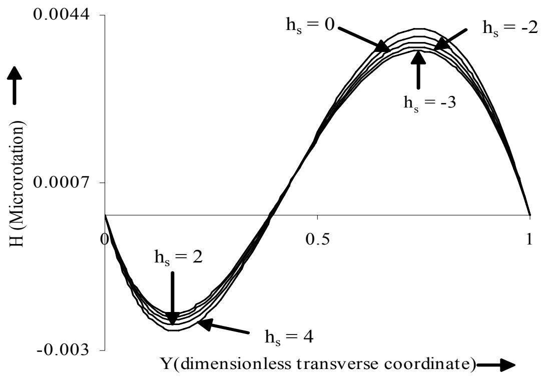

Fig. 9: H versus Y for various h_s values.

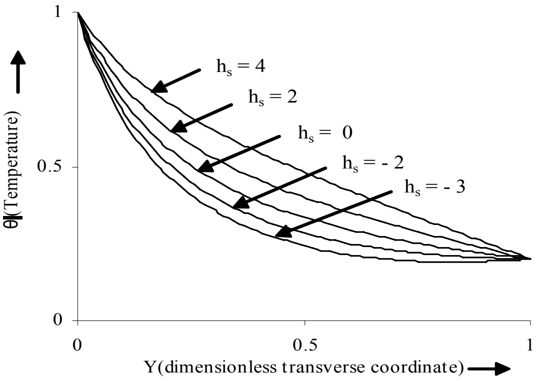

Fig. 10: θ versus Y for various h_s values.

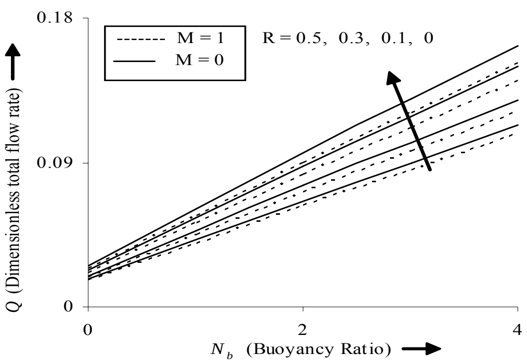

Fig. 11: *Q* versus N_b for various R values.

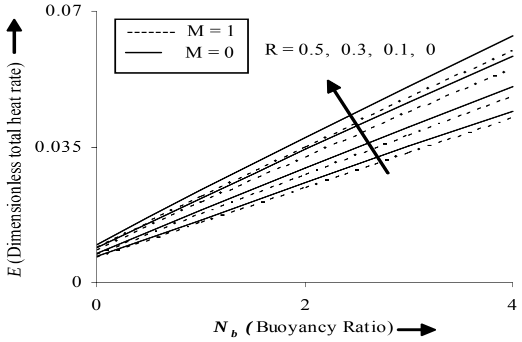

Fig. 12: *E* versus N_b for various R values.

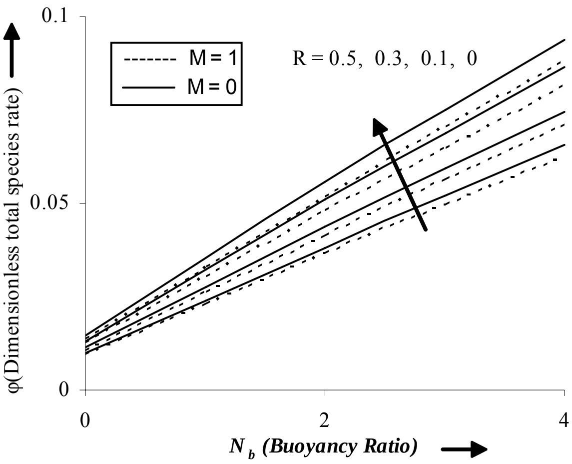

Fig. 13: φ versus N_b for various R values.

Table 1: Values of $U'(0)$ and $-\theta'(0)$ for different values of *S*

Da = 0.5, Fs = 5, Gr = 0.5, B = 1, R = 1, N_b = 2.0, m = 0.2, n = 0.1, M = 1				
S	$h_s = -1$		$h_s = 1$	
	$U'(0)$	$-\theta'(0)$	$U'(0)$	$-\theta'(0)$
0	0.435827	1.13671	0.456516	0.410581
0.05	0.411499	2.50368	0.424817	1.82458
0.1	0.395268	4.01456	0.404712	3.43191
0.05	0.384192	5.51302	0.391645	5.00457
0.2	0.376229	6.94741	0.382414	6.49539
0.25	0.370241	8.30639	0.375691	7.89902
0.3	0.365645	9.58883	0.370585	9.21819

a increase in the rotation of microelements which decelerates the fluid flow resulting in a decrease in dimensionless volume flow rate of the fluid flowing through the vertical channel. Besides this, Fig. 11 also reveals that in the presence of magnetic field (M = 1), there is a reduction in the dimensionless volume flow rate as compared to the electrically non-conducting case when there is no magnetic field (i.e. M = 0).

Fig. 12 demonstrates that an increase in buoyancy ratio boosts the fluid flow resulting in an increment in the heat transfer rate between the vertical plate and the fluid, thus increasing the dimensionless total heat rate *E* added to the fluid. It is found that there is a decrease in the total heat transfer rate added to the fluid as the vortex viscosity parameter R increases from 0.1 to 0.5. Moreover, the presence of magnetic field (M = 1) substantially reduces the total heat transfer rate added to the fluid as compared to the case when there is no magnetic field (i.e. M = 0).

From equation (20), we see that, a direct linear relationship exists between φ and Φ. As expected, an increase in the buoyancy ratio increases the fluid flow which leads to an increase in the total species transfer rate between the wall and the fluid flowing through the vertical channel. It is also observed that an increase in the vortex viscosity parameter *R*, clearly reduces the total species rate added to the fluid in the vertical channel. Besides this, Fig. 13 demonstrates that the presence of magnetic field (M =1) also decelerates the total species rate added to the fluid in the channel.

Table 1. elucidates the effect of thermal conductivity parameter *S* and heat source parameter h_s on $U'(0)$ and $-\theta'(0)$. We observe that an increase in thermal conductivity parameter causes a reduction in skin friction coefficient (at the left wall) whereas heat transfer rate increases as expected. From this table, we also notice that as heat source parameter increases, $U'(0)$ increases while $-\theta'(0)$ decreases.

In order to check the accuracy of our present computations, computations were executed with finite difference solutions and demonstrated excellent correlation. These have not been reproduced for brevity.

V. Conclusion

The numerical results indicate that:

a) Increasing the Darcy number (Da) increases both the velocity and microrotation function throughout the flow regime.
b) Increasing the Forchheimer (quadratic drag) number (Fs) has an impeding effect on the flow.
c) Increasing vortex viscosity parameter (R) decelerates the flow but increases microrotation in the flow regime.
d) Increasing the thermal conductivity parameter (S) substantially decreases the velocity, microrotation and also temperature functions throughout the vertical channel.
e) Increasing the magnetic field parameter (M) significantly decreases the velocity and microrotation functions. Thus magnetic field parameter M can be used for controlling the velocity as well as microrotation profiles as required in many industrial processes including metallurgical refining, alloy flows etc.
f) Increasing the heat source parameter (h_s) boosts the microrotation and temperature profile throughout the micropolar fluid saturated domain. This can therefore be used to great effect in drawing of filaments through metallic baths, customization of steel flows etc.
g) It is found that as the thermal conductivity parameter (S) increases, the skin friction coefficient decreases while the heat transfer rate increases. Moreover the Skin friction coefficient is elevated and the heat transfer rate decreased with a rise in heat source parameter. Hence the thermal conductivity parameter, the magnetic field parameter and the heat source parameter can be used effectively for controlling not only the velocity and temperature profiles but also the skin friction and the heat transfer rate, in metallurgical processing.
h) Finally our numerical computations also indicate that volumetric flow rate, total heat rate added to the fluid, and the total species rate added to the fluid is lower for the case of micropolar fluids (R > 0) as compared to Newtonian fluids $(R = 0)$.

The solutions presented in this work for various thermophysical effects would be useful for subsequent analysis in heat and mass transfer in metallic materials processing and also geophysical processes.

References

[1] D. Poulikakos, A Bejan, "The departure from Darcy flow in natural convection in a vertical porous layer", *Phys. Fluids J.*, 28 1985, pp. 3477-3485
[2] D. Poulikakos, "A departure from the Darcy model in boundary layer natural convection in a vertical porous layer with uniform heat flux", *Trans. ASME J. Heat Transfer*, 107, 1985, pp.716-720.
[3] A. C. Eringen, "Theory of Micropolar Fluids," J. Mathematics Mechanics 16, 1966, pp.1-18.
[4] J. N. Reddy, "An Introduction to the Finite Element Method," MacGraw-Hill. New York 1985.
[5] K. J. Bathe, "Finite Element Procedures," Prentice – Hall, New Jersey 1996.
[6] R. Bhargava, L. Kumar and H. S. Takhar, "Finite element solution of mixed convection micropolar flow driven by a porous stretching sheet," Int. J. Engineering Science, 41, 2003, pp. 2161-2178.
[7] O. A. Beg, R. Bhargava, S. Rawat, H. S. Takhar, P. Bhargava, "Double diffusive micropolar flow, heat and species transfer in porous enclosure," Int. J. Heat Mass Transfer, submitted for publication.
[8] R. Bhargava, S. Rawat, H. S. Takhar, T. A. Bég, and O. A. Bég, "A study of buoyancy-driven micropolar heat and mass transfer in a Darcian porous regime with chemical reaction," Nonlinear Analysis: Modelling and Control 24, 1, 2007, pp.1-24.
[9] O. A. Beg, R. Bhargava, S. Rawat, H. S. Takhar, P. Bhargava, 2006b. "Convective heat transfer of a third grade viscoelastic fluid in a non-Darcian porous medium: numerical simulation," Applied Mathematical Modeling J., UK , submitted for publication.
[10] O. A. Bég, R. Bhargava, P. Bhargava, T. A. Bég, H. S. Takhar,. "Numerical Analysis of Grashof and Darcy Number Effects on Dissipative Natural Convection Boundary Layers in a Micropolar fluid-saturated Geological Porous Medium," Int. J. Fluid Mechanics Research, 34, 4, 2007, pp. 287-307.
[11] O. A. Bég, R. Bhargava, S. Rawat, H. S. Takhar, "Pulsatile magneto-biofluid flow and mass transfer in a non-Darcian porous medium channel," Meccanica, 42, 2007, pp.247–262.
[12] Ching - Yang Cheng, "Fully developed natural convection heat and mass transfer of a micropolar fluid in a vertical channel with asymmetric wall temperatures and concentrations," Int. Comm. in Heat Mass Transfer 33, 2006, pp. 627-635.

ISBN:978-988-98671-6-4