

# Performance of DQPSK-OFDM Systems Over Slowly Nakagami-m Fading Channels

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**ABSTRACT:** This paper deals with the simulation of Nakagami fading in wireless channels using Orthogonal Frequency Division Multiplexing (OFDM) technique using Differential Quadrature Phase Shift Keying (DQPSK) modulation scheme. The paper proposes approximate methods for deriving Bit Error Rate (BER) in DQPSK-OFDM systems over frequency-selective Nakagami fading channels. The numerical results are plotted as BER Vs SNR for various values of Nakagami-m fading parameter 'm' and for various values of Doppler frequency shifts by using MATLAB software. The performance of Nakagami-m channel using DQPSK-OFDM technique is compared with that of the Nakagami-m channel without OFDM technique for various Doppler frequency shifts. It is observed that there is a decrease in the probability of error in Nakagami channels using OFDM technique compared to Nakagami-m channel without OFDM technique.

**KEYWORDS:** Nakagami-m channel, Orthogonal Frequency Division Multiplexing, DQPSK, Doppler frequency shift.

## INTRODUCTION

Fading is observed in wireless communication channels [1] due to multipath propagation. The Rayleigh distribution accurately models the fading effect for short-distance communications. Hence Rayleigh fading models are frequently utilized in simulating high frequency signals propagating in an ionospheric channel. However, Rayleigh fading falls short of describing long-distance fading effects with sufficient accuracy. This fact was first observed by Nakagami, who then formulated a parametric gamma distribution based density function. The model proposed by Nakagami provides a better explanation to less and more severe conditions than the Rayleigh model and provide a better fit to the mobile communication channel data.

This paper deals with the derivation of BER in DQPSK-OFDM [3] systems over frequency selective Nakagami channels. The BER equation includes the influence of Doppler frequency by introducing carrier to noise and interference power ratio (CNIR) which regards the influence caused by Doppler frequency shift as the Gaussian noise [2,3]. Furthermore differential coding/differential detection is used with the adoption of DQPSK as a method of sub carrier modulation [4]. The influence of Doppler frequency results in an increase in the BER

## II.DERIVATION OF BER

The desired approximate BER equation is derived by first analyzing the interference caused by Doppler frequency and then introducing an approximate equation of the BER in the single carrier systems of DQPSK [1] for the interference as equivalent additive Gaussian noise

### A. Effect caused by Doppler frequency shift

When the carrier  $\cos(2\pi f_c t)$  passes through the multipath channels, the received signal is affected by Doppler frequency shift  $f_D \cos \theta$ , where  $f_D$  is the arriving angle of the carrier with respect to the moving direction of the mobile terminal. The received signal  $r(t)$  can be expressed as [3]

$$r(t) = \cos 2\pi(f_c + f_D \cos \theta)t \\ = \cos(2\pi f_c t) \cos(2\pi f_D t \cos \theta) \\ - \sin(2\pi f_c t) \sin(2\pi f_D t \cos \theta) \quad \text{---(1)}$$

if  $2\pi f_D t \cos \theta \ll 1$ , Eq(1) is approximated as

$$r(t) \cong \cos(2\pi f_c t) - 2\pi f_D t \cos \theta \sin(2\pi f_c t) \quad \text{---(2)}$$

Therefore, the second term in Eq.(2) indicates the interference caused by Doppler frequency shift and affects the quadrature component by  $2\pi f_D t \cos \theta$ . If the duration of the symbol is T, then the total interference power I is given by

$$I = \frac{1}{T} \int_0^T (2\pi f_D t \cos \theta)^2 dt = \frac{1}{3} (2\pi f_D T \cos \theta)^2 \quad \text{---(3)}$$

since a phase fluctuation caused by fading has uniform distribution between 0 and  $2\pi$ , the average interference power I is derived by averaging I by as follows

$$I = \frac{1}{2\pi} \int_0^{2\pi} \frac{(2\pi f_D T \cos \theta)^2}{3} d\theta = \frac{(2\pi f_D T)^2}{6} \quad \text{---(4)}$$

The interference power I mentioned in eq (4) can be separated into the direct current (DC) and alternating current (AC) components. The DC Component  $I_{DC}$  and the average interference power of the DC Component  $I'_{DC}$  is calculated as

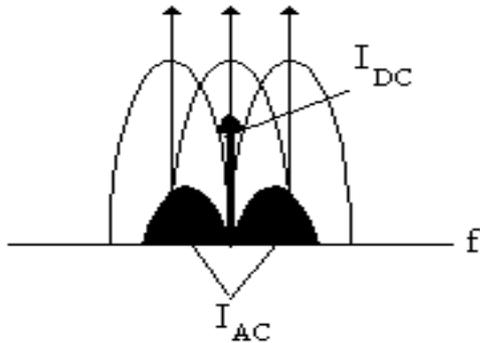
$$I_{DC} = \left( \frac{1}{T} \int_0^T (2\pi f_D t \cos \theta) dt \right)^2 = (\pi f_D T \cos \theta) \quad \text{---(5)}$$

$$I'_{DC} = \frac{1}{2\pi} \int_0^{2\pi} (\pi f_D T \cos \theta)^2 d\theta = \frac{(\pi f_D T)^2}{2} \quad \text{---(6)}$$

Consequently, the average interference power of the AC component  $I_{AC}$  is obtained as

$$I'_{AC} = I - I'_{DC} = \frac{(\pi f_D T)^2}{6} \quad \text{--- (7)}$$

Fig. 1 illustrates the frequency spectrum of the interference of the DC and AC components mentioned above. By considering the center carrier, the DC component  $I_{DC}$  is the interference to this carrier and the AC component  $I_{AC}$  is that the DC and AC components can be regarded as co-channel and interchannel interferences, respectively



**FIG. 1: SPECTRUM OF THE INTERFERENCE**  
Considering the IFFT/FFT duration as  $T_F$  and the guard interval as  $T_G$ , the symbol duration with OFDM is  $T_s = (T_F + T_G)$

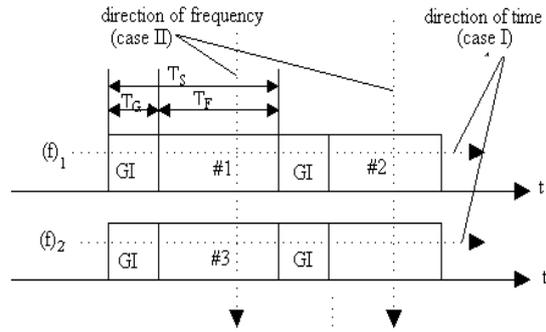
**B. Co-channel interference** With a differential detection, the symbol is demodulated according to the phase difference between two consecutive received symbols. Fig. 2 illustrates the directions of the differential detection in DQPSK-OFDM system. As shown in Fig.2, the symbol #2 is demodulated according to the symbol #1 with a differential detection in the time domain, which is considered as “case I”. On the other hand, the symbol #3 is demodulated according to the symbol #1 in the frequency domain, which is referred as “case II”. Fig.3 illustrates the phase shift caused by Doppler frequency shift. In case I, the energy per bit  $E_b$  of the signal in I-channel degrades and the interference to the signal in Q-channel occurs, because the phase shifts by Doppler frequency shift from #1 to #2 in a symbol duration  $T_s$  as shown in Fig.3. The latter is the co-channel interference  $I_{DC}$ , and then from Eq. (6), the average interference power  $I_{DC}$ , is given by

$$I_{DC} = \frac{(\pi f_D T_s)^2}{2} \quad \text{(case I)} \quad \text{---(8)}$$

On the other hand, in case II, as shown in Fig.3, the phase doesn't shift because symbols #1 and #3 are in the different carriers but are in the same time. Since the co-channel interference

doesn't occur,  $I_{DC}$  is given by

$$I_{DC} = 0 \quad \text{(case II)} \quad \text{---(9)}$$

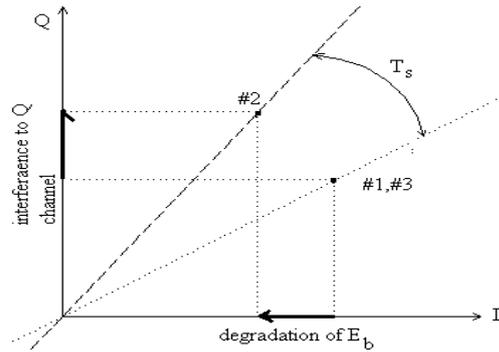


**FIG.2. DIRECTION OF DIFFERENTIAL DETECTION**

**C. Inter-channel interference**

The inter-channel interference  $I_{AC}$  occurs in both case I and case II because it is the interference which leaks from adjacent sub-carriers. Hence, defining the FFT duration as  $T_F$  and using Eq.(7), the average interference power  $I_{AC}$  is given by

$$I_{AC} = \frac{(\pi f_D T_F)^2}{6} \quad \text{---(10)}$$



**FIG.3 PHASE SHIFT CAUSED BY DOPPLER FREQUENCY SHIFT**

**III. SYSTEM MODEL**

Let us consider  $K$  transmitting stations accessing simultaneously to the same frequency band as shown in fig 4 and each receiving station recovers a desired information signal by demultiplexing the signal sent from all transmitting stations. The information signal  $b_{k2}(t)$  of each transmitting station  $\#k (1 \leq k \leq K)$  in the form of binary data sequence is transmitted by differential encoding and the band limiting in a modulation block. In the transmission block, the signal is delayed by an amount of  $\Delta t_k$  with phase offset  $\Delta \theta_k$  because respective stations access asynchronously to the system.  $z_i(t)$  be the signal obtained after filtering the received signal  $z(t)$  in the receiving station. Fig 5 shows the kinds of signals included in the output signal  $z_i(t)$  from the matched filter. As shown in the figure 4, signal  $z_i(t)$  is composed of desired signal, undesired signal and

thermal noise. The desired signal consists of direct wave  $d_{di}(t)$  and Rayleigh wave  $d_{ri}(t)$  related to the signal  $s_i(t)$  sent from the transmitting station #i, and the undesired signal consists of signals  $s_j(t)$  sent from the transmitting stations #j.

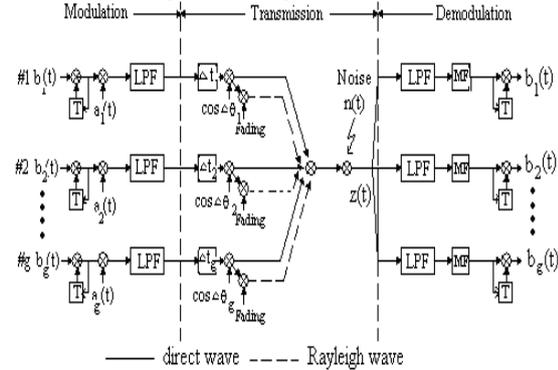


FIG.4. DS-CDMA SYSTEM TRANSMITTER

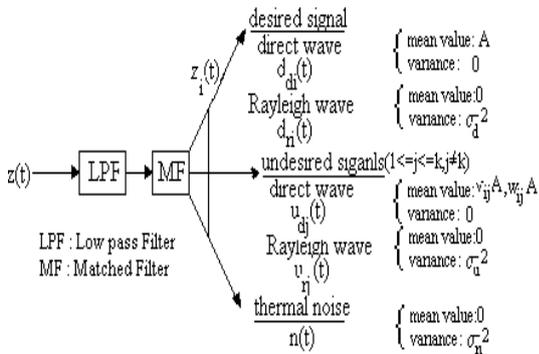


FIG.5 OUTPUT SIGNAL FROM MATCHED FILTER IN STATION # I.

The output signal from the differential decoder is obtained by detecting a phase difference between the signal  $z_i(t)|_{t=mt_b} = z_m$  and the previous signal  $z_i(t)|_{t=(m-1)t_b} = z_{m-1}$  [5]. let  $x_1 = z_m + z_{m-1}$  and  $x_2 = z_m - z_{m-1}$ . The PDFs of amplitude of  $x_1$  and  $x_2$  follow the Nakagami distribution [1]. Defining amplitude of  $x_1$  and  $x_2$  as  $r_1$  and  $r_2$  respectively, the mean values (B,C) and variances ( $\sigma_{r_1}^2, \sigma_{r_2}^2$ ) of  $r_1$  and  $r_2$  are obtained as[2]

$$\begin{aligned}
 B &= A\sqrt{I_{x_1}^2 + Q_{x_1}^2} = \sqrt{2\sigma_d^2 \xi (I_{x_1}^2 + Q_{x_1}^2)} \\
 C &= A\sqrt{I_{x_2}^2 + Q_{x_2}^2} = \sqrt{2\sigma_d^2 \xi (I_{x_2}^2 + Q_{x_2}^2)} \\
 \sigma_{r_1}^2 &= \bar{r}_1 = (S_{0+} + S_{i+})\sigma_d^2 + 2\sigma_n^2 \\
 \sigma_{r_2}^2 &= \bar{r}_2 = (S_{0-} + S_{i-})\sigma_d^2 + 2\sigma_n^2 \\
 S_{0+} &= 2 + 2J_0(2\pi f_D T_b) \\
 S_{0-} &= 2 - 2J_0(2\pi f_D T_b) \\
 S_{i+} &= \sum_{j=1, j \neq i}^K \left\{ \begin{aligned} &v_{ij}^2(n) + w_{ij}^2(n) \\ &+ 2v_{ij}(n)w_{ij}(n)J_0(2\pi f_D T_b) \end{aligned} \right\} \\
 S_{i-} &= \sum_{j=1, j \neq i}^K \left\{ \begin{aligned} &v_{ij}^2(n) + w_{ij}^2(n) \\ &- 2v_{ij}(n)w_{ij}(n)J_0(2\pi f_D T_b) \end{aligned} \right\} \quad (11)
 \end{aligned}$$

Where A is the amplitude of the direct wave  $d_{di}(t)$  in the desired signal,  $v_{ij}$  and  $w_{ij}$  are interference

signal components from the undesired signal to the desired signal at  $t=(m-1)T_b$  and  $t=mT_b$ , respectively

The probability density function of  $r_1$  and  $r_2$  in Nakagami channel is given by [1]

$$\begin{aligned}
 p(r_1) &= \frac{m^m}{\Gamma(m)} r_1^{m-1} e^{-\frac{m}{r_1}} \\
 p(r_2) &= \frac{m^m}{\Gamma(m)} r_2^{m-1} e^{-\frac{m}{r_2}}
 \end{aligned}$$

If the number of simultaneously accessing stations are K, the BER in the receiving station # i is given by

$$\begin{aligned}
 P_e(i, k) &= \int_0^\infty p(r_1) dr_1 \int_0^\infty p(r_2) dr_2 \\
 P_e(i, K) &= \sum_{t=0}^{m-1} \frac{(\bar{r}_1)^t (\bar{r}_2)^m}{(m-1)!(r_1 + r_2)^{m+t}} \frac{(m+t-1)!}{t!} \quad (12)
 \end{aligned}$$

For simple case of one station i.e. K=1 the BER equation is obtained by setting  $v_{ijl}(n)$  and  $w_{ijl}(n)$  to zero in eq. (11) and eq.(12)

$$\begin{aligned}
 P_e &= \sum_{t=0}^{m-1} \frac{[(1 + j_0(2\pi f_D T_b)) \frac{CNIR}{2} + 1]^t (m+t-1)!}{[2(\frac{CNIR}{2} + 1)]^{m+t}} \\
 &\times \left\{ [(1 - j_0(2\pi f_D T_b)) \frac{CNIR}{2} + 1]^{-m} (m-1)! t! \right\}^{-1} \text{ Case-I}
 \end{aligned}$$

where  $CNIR = ((2E_b/N_0)^{-1} + I_{DC} + I_{AC})^{-1}$

$$P_e = \sum_{t=0}^{m-1} \frac{[CNIR + 1]^t (m+t-1)!}{[2(\frac{CNIR}{2} + 1)]^{m+t} (m-1)! t!} \text{ Case-II}$$

where  $CNIR = ((2E_b/N_0)^{-1} + I_{AC})^{-1}$

#### IV. CONCLUSIONS

A closed form expression is derived for the average bit error rate using DQPSK on frequency selective Nakagami fading channel using OFDM mitigation technique. The performance curves for Nakagami-m channel using DQPSK are obtained for various values of the Nakagami parameter 'm' and compared them with that employing the OFDM technique over the Nakagami Channel. It is observed that the probability of error decreases by implementing the OFDM technique. It is also observed that the average bit error rate increases with fading severity, which results in less gain at severe fading environments. In addition, the behavior of Nakagami-m channel (DQPSK-OFDM) for various Doppler frequency shifts is also observed. It is concluded that the BER increases with increase in Doppler frequency shift and it decreases with increase in the Nakagami fading parameter 'm'.

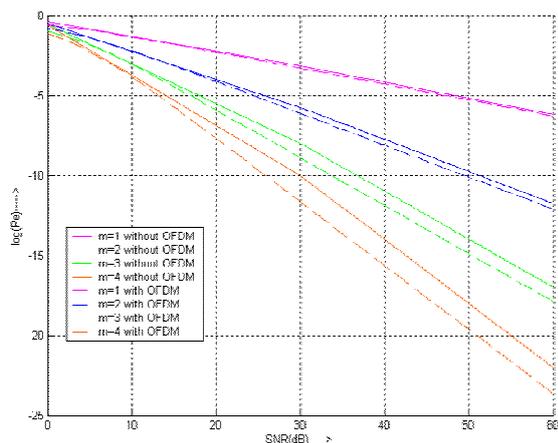


FIG 1 PERFORMANCE OF DQPSK OVER NAKAGAMI-m CHANNEL WITH AND WITHOUT OFDM

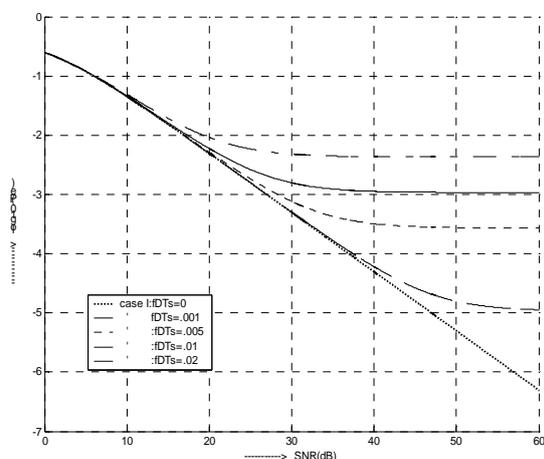


FIG 2. PERFORMANCE OF DQPSK/OFDM OVER NAKAGAMI-m FADING CHANNEL FOR VARIOUS DOPPLER FREQUENCY SHIFTS (CASE I) (m=1)

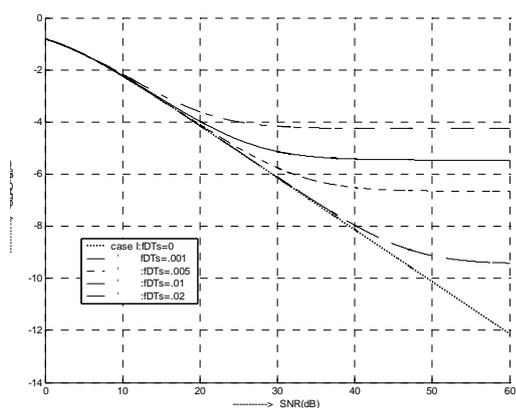


FIG 3. PERFORMANCE OF DQPSK/OFDM OVER NAKAGAMI-m FADING CHANNEL FOR VARIOUS DOPPLER FREQUENCY SHIFTS (CASE I) (m=2)

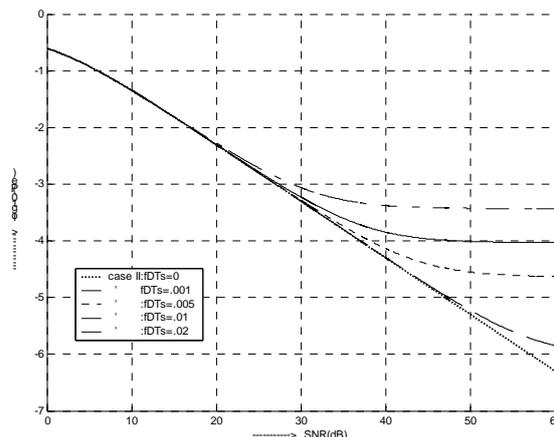


FIG 4. PERFORMANCE OF DQPSK/OFDM OVER NAKAGAMI-m FADING CHANNEL FOR VARIOUS DOPPLER FREQUENCY SHIFTS (CASE II) (m=1)

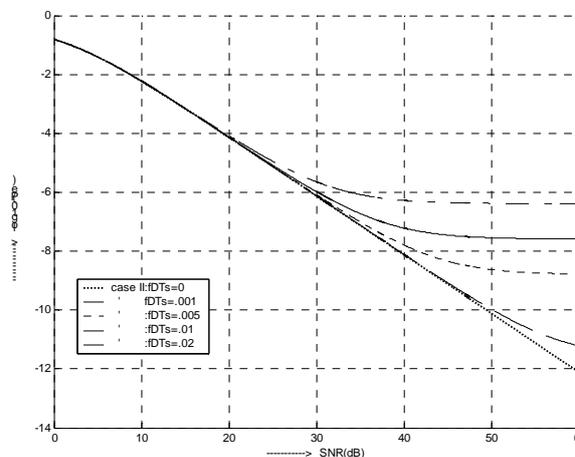


FIG 5. PERFORMANCE OF DQPSK/OFDM OVER NAKAGAMI-m FADING CHANNEL FOR VARIOUS DOPPLER FREQUENCY SHIFTS (CASE II) (m=2)

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