Proceedings of the World Congress on Engineering and Computer Science 2007
WCECS 2007, October 24-26, 2007, San Francisco, USA

Approximation Based Adaptive Tracking Control of Uncertain Nonholonomic Mechanical Systems

Jing Wang [*], Zhihua Qu [†], Morrison S. Obeng [‡], and Xiaohe Wu [§]

Abstract—**In this paper, the trajectory tracking control problem of uncertain nonholonomic mechanical systems is investigated. By separately considering kinematic and dynamic models of a nonholonomic mechanical system, a new adaptive tracking control is proposed based on neural network approximation. The proposed design consists of two steps. First, the nonholonomic kinematic subsystem is transformed into a chained form, and the corresponding optimal control is derived. Second, an adaptive neural control is designed for the dynamic subsystem to make the outputs of the dynamic subsystem asymptotically track the optimal control signals chosen for the kinematic subsystem. The proposed control is simulated on a unicycle wheeled mobile robot.**

Keywords: Adaptive control, trajectory tracking, neural networks, nonholonomic mobile robots, uncertainties

1 Introduction

Control of nonholonomic systems has received considerable attention during the past decade [1]. The reason is that the class of nonholonomic systems with restricted mobility cannot be stabilized to a desired configuration (or posture) via smooth, or even continuous, pure-state feedback due to the celebrated Brockett's necessary condition [2]. The problem of trajectory tracking is generally different from the stabilization problem and thus the available control approaches for posture stabilization control are often not directly applicable. Based on the kinematic model or dynamic model of nonholonomic systems, the tracking problem can be classified as either kinematic tracking or dynamic tracking problem. For the kinematic tracking problem, where the systems are represented by their kinematic models and velocity acts as the control input, several methods have been proposed in [3][4]. In practice, however, it is more realistic to formulate the nonholonomic system control problem at the dynamic level, where the torque and force are taken as the control inputs. Recently, several researchers have investigated the dynamic tracking problem for nonholonomic systems. Using neural networks and backstepping, a method integrating a kinematic controller and a torque controller for the dynamic model of mobile robot has been presented in [5]. In [6], dynamic tracking problem of the nonholonomic systems with unknown inertia parameters was studied, where the controller ensuring partial states of the system to track the desired trajectories was proposed.

In this paper, as a natural extension to our recent results on globally stabilizing near-optimal kinematic tracking control for nonholonomic chained systems [7], we propose an adaptive tracking control for uncertain dynamic nonholonomic systems with the aid of neural network approximation. The proposed design consists of two steps. First, the nonholonomic kinematic subsystem is transformed into a chained form, by exploiting its special structure, the corresponding optimal control can be explicitly derived. Second, a new adaptive neural control is designed for the dynamic subsystem, which can guarantee the outputs of the dynamic subsystem (the inputs to the kinematic subsystem) to asymptotically track the designed optimal control signals for kinematic subsystem. Neural networks are used to parameterize the unknown system functions and their weights are adaptively tuned. A robust term is introduced to suppress the neural network approximation error and the bounded disturbances. It is rigorously proved that all the signals of the closed-loop system are bounded and the tracking errors converge to zero asymptotically. Simulation results verify the effectiveness of the proposed control.

2 Problem Formulation

2.1 Model of Nonholonomic System

In this paper, we consider a class of nonholonomic mechanical systems expressed in local coordinates (general-

[*] J.Wang is an Assistant Professor of Computer Engineering & Computer Science, School of Science, Engineering, and Mathematics, Bethune-Cookman University, 640 Dr. M.M.B. Blvd, Daytona Beach, FL 32114, USA. Phone: (386) 481-2671. Fax: (386) 481-2662. Email: wangj@cookman.edu.

[†] Z.Qu is a Professor of Electrical Engineering, School of Electrical and Computer Sciences, University of Central Florida, Orlando, FL 32816, USA. Email: qu@mail.ucf.edu.

[‡] M.Obeng is an Associate Professor of Computer Engineering & Computer Science, School of Science, Engineering, and Mathematics, Bethune-Cookman University, 640 Dr. M.M.B. Blvd, Daytona Beach, FL 32114, USA. Email: obengm@cookman.edu.

[§] X.Wu is an Assistant Professor of Computer Engineering & Computer Science, School of Science, Engineering, and Mathematics, Bethune-Cookman University, 640 Dr. M.M.B. Blvd, Daytona Beach, FL 32114, USA. Email:wux@cookman.edu.

ized coordinates), and by the following form [8]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + d(t) = B(q)\tau + J^T(q)\lambda \quad (1)$$
$$J(q)\dot{q} = 0 \quad (2)$$

where $q = \begin{bmatrix} q_1 & \cdots & q_n \end{bmatrix}^T \in \Re^n$ is the generalized coordinates, $M(q) \in \Re^{n\times n}$ is a bounded positive-definite symmetric inertia matrix, $C(q,\dot{q}) \in \Re^{n\times n}$ is the centripetal and coriolis matrix, $G(q) \in \Re^n$ is the gravitation force vector, $B(q) \in \Re^{n\times r}$ is the input transformation matrix, $\tau \in \Re^r$ is the input vector of forces and torques, $J(q) \in \Re^{(n-m)\times n}$ is the matrix associated with the constraints, $\lambda \in \Re^{n-m}$ is the vector of constraint forces on the contact point between the rigid body and the surface, and $d(t) \in \Re^n$ denotes bounded unknown disturbances including unstructured unmodeled dynamics. Assume that constraints in (2) belong to the so-called nonholonomic constraints [9]. Dynamic system (1) has the following properties [10]:

Property 1 *$M(q)$, $C(q)$ and $G(q)$ are bounded in the sense that there exist constant scalars c_1, c_2, and positive functions $k_c(q)$ and $k_g(q)$ such that $c_1 I \le M(q) \le c_2 I$, $||C(q,\dot{q})|| \le k_c(q)||\dot{q}||$ and $||G(q)|| \le k_g(q)$.*

Property 2 *$\dot{M} - 2C$ is skew-symmetric, i.e., $X^T(\dot{M} - 2C)X = 0,\ \forall X \neq 0$.*

The control objective is to construct a real-time feedback control law τ such that asymptotical stability of $(q - q_d)$ can be achieved for systems (1) and (2), where $q_d(t)$ is a given n-dimensional desired trajectory. The following assumption is standard.

Assumption 1 *Desired trajectory $q_d(t)$ is differentiable and uniformly bounded, and satisfies the nonholonomic constraints in (2), that is, $J(q_d)\dot{q}_d = 0$.*

To start, we do the following model reduction by using the methods in [11, 6]. Specifically, let vector fields $s_1(q), \cdots, s_m(q)$ form a basis in the null space of $J(q)$, that is,

$$S^T(q)J^T(q) = 0, \quad (3)$$

where $S(q) = [s_1(q), \cdots, s_m(q)]^T$. It then follows from (2) that there exists a vector $v = [v_1, \cdots, v_m]^T \in R^m$ such that

$$\dot{q} = S(q)v(t) = s_1(q)v_1 + \cdots + s_m(q)v_m, \quad (4)$$

Equation (4) is the so-called kinematic model of nonholonomic systems [11]. Differentiating both sides of (4) yields $\ddot{q} = \dot{S}(q)v + S(q)\dot{v}$, and substituting it into equation (1) leads to

$$M_1(q)\dot{v} + C_1(q,\dot{q})v + G_1(q) + d_1(t,q) = B_1(q)\tau, \quad (5)$$

where $M_1(q) = S^T(q)M(q)S(q) \in \Re^{m\times m}$, $C_1(q,\dot{q}) = S^T(q)\left[M(q)\dot{S}(q) + C(q,\dot{q})S(q)\right] \in \Re^{m\times m}$, $G_1(q) = S^T(q)G(q) \in \Re^m$, $B_1(q) = S^T(q)B(q) \in \Re^{m\times r}$, and $d_1(t,q) = S^T(q)d_1(t)$.

To further facilitate the trajectory tracking control design, we proceed with transforming the kinematic subsystem (4) into its nonholonomic chained form. It is well known that many mechanical systems with nonholonomic constraints can be either locally or globally converted to the chained form under a coordinate change and a control mapping [9]. Interesting examples of such mechanical systems include tricycle-type mobile robots, car-like robots, cars towing several trailers, the knife edge, a vertical rolling wheel, and a rigid spacecraft with two torque actuators, and so on [1]. This canonical form allows us to design controls for a general class of nonholonomic systems. In this paper, we adopt the chained form for our design and thus make the following assumption.

Assumption 2 *There exist a diffeomorphic coordinate transformation $X = T_1(q)$ and a control mapping $v = T_2(q)u$, such that kinematic model (4) can be converted into a m-input, $(m-1)$-chain, single-generator chained form [12] given by:*

$$\begin{array}{rcl} \dot{x}_1 &=& u_1, \\ \dot{x}_{j,i} &=& x_{j,i+1}u_1, \quad 2 \le i \le n_j - 1,\ 1 \le j \le m-1 \\ \dot{x}_{j,n_j} &=& u_{j+1}, \end{array} \quad (6)$$

where $X = [x_1, X_2, \ldots, X_m]^T \in R^n$ with $X_j = [x_{j-1,2}, \ldots, x_{j-1,n_{j-1}}]$ $(2 \le j \le m)$ are the sub-states, and $u = [u_1, u_2, \ldots, u_m]^T$ are the inputs of the kinematic subsystem.

Upon having transformations T_1 and T_2, dynamic subsystem (5) can also be converted into the space of the new variables as:

$$M_2(X)\dot{u} + C_2(X,\dot{X})u + G_2(X) + d_2(t,X) = B_2(X)\tau, \quad (7)$$

where $M_2(X) = T_2^T(q)M_1(q)T_2(q)|_{q=T_1^{-1}(X)} \in R^{2\times 2}$, $C_2(X,\dot{X}) = T_2^T(q)\left[C_1(q,\dot{q})T_2 + M_1\dot{T}_2(q)\right]|_{q=T_1^{-1}(X)} \in R^{2\times 2}$, $G_2(X) = T_2^T(q)G_1(q)|_{q=T_1^{-1}(X)} \in R^2$, $B_2(X) = T_2^T(q)B_1(q)|_{q=T_1^{-1}(X)} \in R^{2\times r}$, and $d_2(t,X) = T_2^T(q)d_1(t,q)|_{q=T_1^{-1}(X)}$.

The same transformations yielding the chained form will also be applied to the given desired trajectory q_d. Since $q_d(t)$ satisfies (2), it can be easily verified that

$$\dot{q}_d = S(q_d)v_d. \quad (8)$$

Under the same transformations, that is, $X_d = T_1(q_d)$ and $v_d = T_2(q_d)u_d$, equation (8) can also be transformed into the chained form as (6). Therefore, the dynamic tracking problem can be recast as the problem of constructing a control τ for system (6) and (7) such that $\lim_{t\to\infty}(X - X_d) = 0$.

Assumption 3 *[7] The desired trajectory $q_d(t)$ to be followed is persistent such that the $u_{1d}(t)$ in the chained form is uniformly right continuous, uniformly bounded, and uniformly nonvanishing.*

Assumption 4 *There exists a known finite positive constant $c_3 > 0$, such that $\sup_{t\geq 0} \|d_2(t, X)\| \leq c_3$.*

2.2 Linearly Parameterized Neural Approximator

A linearly parameterized neural approximator will be used to approximate the unknown bounding functions $\phi_i(\cdot)$. Several function approximators can be applied for this purpose, such as, radial basis function (RBF) neural networks [13, 14], high-order neural networks [15] and fuzzy systems [16], which can be described as $W^T S(z)$ with input vector $z \in R^n$, weight vector $W \in R^l$, node number l, and basis function vector $S(z) \in R^l$. Universal approximation results indicate that, if l is chosen sufficiently large, then $W^T S(z)$ can approximate any continuous function to any desired accuracy over a compact set [15, 14]. In this paper, we use the RBF NN to approximate a smooth function. That is, for the unknown nonlinear functions $\phi(x)$, we have the following approximation over the compact sets Ω

$$\phi(x) = W^{*T}\psi(x) + \omega(x), \quad \forall x \in \Omega \subset R^l \tag{9}$$

where $W^* \in R^l$ is an unknown constant parameter vector, the NN node number $l > 1$, $\omega(x)$ is the approximation error, and $\psi(x) = [\psi_1(x), \cdots, \psi_l(x)]^T$ is the basis function vector, with $\psi_i(x)$ being chosen as the commonly used Gaussian functions, which have the form

$$\psi_i(x) = \exp\left[\frac{-(x-\mu_i)^T(x-\mu_i)}{\eta_i^2}\right], \quad i = 1, 2, ..., l, \tag{10}$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \cdots, \mu_{ii}]^T$ is the center of the receptive field and η_i is the width of the Gaussian function.

Remark 1 *The optimal weight vector W^* in (9) is an "artificial" quantity required only for analytical purposes. Typically, W^* is chosen as the value of W that minimizes $\omega(x)$ for all $x \in \Omega$, i.e., $W^* := \arg\min_{W\in R^l}\left\{\sup_{x\in\Omega}|\phi(x) - W^T\psi(x)|\right\}$.*

According to the universal approximation theorem [14], approximation error $\omega(x)$ must be bounded upon having the expression of (9).

Assumption 5 *Over a compact region $\Omega \subset R^l$ $|\omega(x)| \leq c_4, \quad \forall x \in \Omega$, where $c_4 \geq 0$ is a known constant.*

3 Main Results

In this section, we will proceed with the tracking control design for dynamic nonholonomic system (6) and (7).

3.1 Design of Tracking Control for Kinematic Subsystem

Without loss of generality, we consider the chained system (6) with $m = 2$, that is, consider the class of nonholonomic chained systems of the form:

$$\dot{x}_1 = u_1, \ \ \dot{x}_2 = x_3 u_1, \ \ \cdots, \ \ \dot{x}_{n-1} = x_n u_1, \ \ \dot{x}_n = u_2, \tag{11}$$

where $x = [x_1, \cdots, x_n]^T \in \Re^n$ is the state, and $u = [u_1, u_2]^T \in \Re^2$ is the control input. For trajectory tracking, the desired trajectory to be followed is given by:

$$\dot{x}_{1d} = u_{1d}, \ \ \dot{x}_{(i-1)d} = x_{id}u_{1d}, \ \ \dot{x}_{nd} = u_{2d}, \tag{12}$$

where $x_d = [x_{1d}, \cdots, x_{nd}]^T \in \Re^n$, $u_d(t) = [u_{1d}(t), u_{2d}(t)]^T \in \Re^2$ is the time-varying reference input (i.e., open-loop steering control), and x_{3d} up to x_{nd} are assumed to be uniformly bounded. Let $x_e = [x_{1e}, \cdots, x_{ne}]^T \triangleq x - x_d$ denote the state tracking error. Then, the error dynamics between (11) and (12) can be expressed as

$$\dot{x}_e = F(u_{1d}(t))x_e + [H + G(x_d, x_e)](u - u_d), \tag{13}$$

where

$$F(u_{1d}(t)) = \mathrm{diag}\{F_1, F_2(u_{1d}(t))\}, \ H = \mathrm{diag}\{H_1, H_2\},$$

$$F_1 = 0, \ H_1 = 1, \ F_2(u_{1d}(t)) = u_{1d}(t)F_2^*, G = \begin{bmatrix} 0 & 0 \\ G_2 & 0 \end{bmatrix},$$

$$F_2^* \triangleq \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & & 1 \\ 0 & 0 & \cdots & & 0 \end{bmatrix}, \ H_2 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} z_2 + x_{3d} & z_3 + x_{4d} & \cdots & z_{n-1} + x_{nd} & 0 \end{bmatrix}^T.$$

Therefore, error dynamics in (13) can be partitioned into the following two subsystems:

$$\dot{x}_{1e} = F_1 x_{1e} + H_1(u_1 - u_{1d}), \tag{14}$$

$$\dot{z} = F_2 z + H_2(u_2 - u_{2d}) + G_2(u_1 - u_{1d}), \tag{15}$$

where $z = [z_1, \cdots, z_{n-1}]^T \triangleq [x_{2e}, \cdots, x_{ne}]^T \in \Re^{n-1}$. The decomposition into subsystems (14) and (15) yields two useful properties. First, subsystem (14) is of first order, linear, time-invariant, and independent of subsystem (15). Subsystem (15) is nonlinear but has a linear time varying nominal system defined by

$$\dot{z} = F_2(u_{1d}(t))z + H_2(u_2 - u_{2d}). \tag{16}$$

Second, coupling from subsystems (14) to (15) is through $G_2(x_d, z)v_1$, the only nonlinear term in the system. To this end, the kinematic tracking control design is stated in the following theorem.

Theorem 1 *[7] Consider nonlinear tracking error system (13) under assumption 3. Then under the control $u(x_e,t)=u^*(x_e,t)$, where*

$$u^*(x_e,t) \triangleq \begin{bmatrix} u_1^* & u_2^* \end{bmatrix}^T = -R^{-1}(t)H^T P(t)x_e + u_d, \tag{17}$$

with $u_1^*(x_{1e},t) = -r_1^{-1}p_1x_{1e}+u_{1d}(t)$, and $u_2^*(x_{2e},t) = -r_2^{-1}(t)H_2^TP_2(t)z+u_{2d}(t)$, where $P(t)=diag[p_1,P_2(t)]$, $p_1=\sqrt{q_1r_1}$ for positive constants q_1 and r_1, and matrix $P_2(t)$ is the solution to the following reduced-order differential Riccati equation: for some $P_2(\infty)>0$,

$$\begin{aligned} 0 &= \dot{P}_2(t)+P_2(t)F_2(t)+F_2^T(t)P_2(t) \\ &\quad -P_2(t)H_2r_2^{-1}(t)H_2^TP_2(t)+Q_2, \end{aligned} \tag{18}$$

where $R=diag\{r_1,r_2\}$, and $Q=diag\{q_1,Q_2\}>0$, the closed loop system is globally and exponentially stable.

The proposed kinematic tracking control design is directly applicable to general chained form (6). The only difference is that, analogous to the decomposition to two subsystems, the resulting error system of the (m,n) chained model contains m subsystems.

3.2 Design of Adaptive Neural Control for Dynamic Subsystem

In this subsection, we design the realistic control law τ for the dynamic nonholonomic system (7), which will make the outputs of the dynamic subsystem (the inputs of the kinematic subsystem) u tend to the optimal control $u^*=[u_1^*,u_2^*]^T$. Define the auxiliary tracking error $u_e=u-u^*$. By differentiating u_e and using (7), the system dynamic model can be written in terms of the tracking error u_e as

$$M_2(X)\dot{u}_e+C_2(X,\dot{X})u_e = -h(X,\dot{X},u^*,\dot{u}^*)+B_2(X)\tau-d_2, \tag{19}$$

where $h(X,\dot{X},u^*,\dot{u}^*) = M_2(X)\dot{u}^*+C_2(X,\dot{X})u^*+G_2(X)$. Due to the fact that system inertia parameters are unknown, the exact expression of nonlinear term $h(X,\dot{X},u^*,\dot{u}^*)$ may not be obtained. In what follows, control is designed according to the neural approximation of unknown nonlinear term $h(X,\dot{X},u^*,\dot{u}^*)$. That is, over a compact set $\Omega\subset\Re^{2(n+m)}$, we have the following approximation

$$h(X,\dot{X},u^*,\dot{u}^*) = W^{*T}\psi(X,\dot{X},u^*,\dot{u}^*)+\omega, \tag{20}$$

where $W^*\in\Re^{l\times n}$ is the unknown optimal weight matrix, $\psi(X,\dot{X},u^*,\dot{u}^*)\in\Re^l$ are the basis function for the neural networks and approximation error ω is bounded on the compact set Ω based on assumption 5, i.e, $|\omega|\le c_4$. Using (20), dynamic system (19) can be rewritten as

$$\begin{aligned} M_2(X)\dot{u}_e+C_2(X,\dot{X})u_e &= -W^{*T}\psi(X,\dot{X},u^*,\dot{u}^*)-\omega \\ &\quad +B_2(X)\tau-d_2(t,X). \end{aligned} \tag{21}$$

Now define the adaptive control as

$$\begin{aligned} \tau(t) &= B_2^+[W^T\psi-k_1u_e-k_2\mathrm{sgn}(u_e) \\ &\quad -2(B+G)^TPx_e], \end{aligned} \tag{22}$$

$$\dot{W}_i = -\Gamma^{-1}\psi(X,\dot{X},\gamma,\dot{\gamma})u_{e,i}, \tag{23}$$

where $W\triangleq[W_1,W_2,\cdots,W_m]\in\Re^{l\times m}$ is the estimate of W^*, $u_{e,i}$ is the ith element of vector u_e, B_2^+ is the left inverse of B_2 defined as $B_2^+=B_2^T(B_2B_2^T)^{-1}$, $\Gamma\in\Re^{l\times l}$ is a symmetric positive definite constant matrix, k_1 and $k_2\ge c_3+c_4$ are positive constants to be designed. The main result of this paper is stated as follows.

Theorem 2 *Consider the mechanical system described by (1) and (2). Then the control law (22) and adaptation law (23), all the closed-loop system signals are uniformly bounded, and tracking errors x_e and u_e converge to zero asymptotically.*

Proof: It follows from substituting (22) into (21), the closed-loop system equation becomes

$$\begin{aligned} M_2(X)\dot{u}_e+C_2(X,\dot{X})u_e &= -k_1u_e+\tilde{W}^T\psi \\ &\quad -k_2\mathrm{sgn}(u_e)-\omega-d_2-2(B+G)^TPx_e, \end{aligned} \tag{24}$$

where $\tilde{W}=W-W^*$. Now consider the Lyapunov function candidate $V=V_1+V_2$, where $V_1=\frac{1}{2}\left[u_e^TM_2(X)u_e+\sum_{i=1}^n\tilde{W}_i^T\Gamma\tilde{W}_i\right]$, and $V_2=x_e^TPx_e$. The time derivative of V_1 along the trajectory of (24) is

$$\begin{aligned} \dot{V}_1 &= u_e^TM_2\dot{u}_e+\frac{1}{2}u_e^T\dot{M}_2u_e+\sum_{i=1}^n\tilde{W}_i^T\Gamma\dot{\tilde{W}}_i \\ &= -k_1u_e^Tu_e+u_e^T\tilde{W}^T\psi-u_e^T[k_2\mathrm{sgn}(u_e)+\omega+d_2] \\ &\quad -2u_e^T(B+G)^TPx_e-u_e^TC_2u_e+\frac{1}{2}u_e^T\dot{M}_2u_2 \\ &\quad +\sum_{i=1}^n\tilde{W}_i^T\Gamma\dot{\tilde{W}}_i. \end{aligned} \tag{25}$$

It follows from Property 2 and the expressions of M_2 and C_2 that $\dot{M}_2-2C_2$ is also skew-symmetric. Then using (23), (25) can be simplified as

$$\begin{aligned} \dot{V}_1 &= -k_1u_e^Tu_e-u_e^T[k_2\mathrm{sgn}(u_e)+\omega+d_2] \\ &\quad -2u_e^T(B+G)^TPx_e. \end{aligned} \tag{26}$$

The time derivative of V_2 along the trajectory (13) is $\dot{V}_2=x_e^T\dot{P}x_e+2x_e^TP[Ax_e+B(u_e+u^*-u_d)]+2x_e^TPG(u_e+u^*-u_d)$. Thus noting the expression of u^* in (17) and (18), we have

$$\dot{V}_2\le -x_e^TQx_e+2x_e^TP(B+G)u_e. \tag{27}$$

To this end, combining (26) and (27) yields

$$\dot{V}\le -k_1u_e^Tu_e-u_e^T[k_2\mathrm{sgn}(u_e)+\omega+d_2]-x_e^TQx_e. \tag{28}$$

Noting the choice of k_2, we know that $-u_e^T[k_2\mathrm{sgn}(u_e)+\omega+d_2]\le 0$, and it follows from (28) that

$$\dot{V}\le -k_1u_e^Tu_e-x_e^TQx_e<0,$$

from which the bounded of closed-loop system signals and the convergence of u_e and x_e can be concluded by using Barbalat's lemma [17]. □

4 Simulation

Consider the unicycle wheeled mobile robot moving on a horizontal plane, which has three wheels, two are differential drive fixed wheels, one is a caster wheel, and is characterized by the configuration $q = [x, y, \theta]^T$. We assume that the robot does not contain flexible parts, all steering axes are perpendicular to the ground, the contact between wheels and the ground satisfies the condition of pure rolling and non-slipping. Then constraint of the non-slipping condition can be written as $\dot{x}\sin\theta - \dot{y}\cos\theta = 0$. From the constraint, we have $J(q) = \begin{bmatrix} \sin\theta & -\cos\theta & 0 \end{bmatrix}$, which leads to $S(q) = [\cos\theta, \sin\theta, 0; 0, 0, 1]^T$. Lagrange formulation can be used to derive the dynamic equations of the mobile robot. Following the description in section 2, dynamics of the unicycle robot can be written as

$$\begin{array}{l} \dot{x} = v_1\cos\theta,\ \dot{y} = v_1\sin\theta,\ \dot{\theta} = v_2 \\ M_1(q)\dot{v} + C_1(q)v + G_1 = B_1\tau, \end{array} \tag{29}$$

where $M_1 = \begin{bmatrix} m_0 & 0 \\ 0 & I_0 \end{bmatrix}$, $C_1 = 0$, $G_1 = 0$, $B_1 = 1/R\begin{bmatrix} 1 & 1 \\ L & -L \end{bmatrix}$, $v = [v_1, v_2]^T$ with v_1 and v_2 linear and angular velocities, m_0 is the mass of the mobile robot, I_0 is its inertia moment around the vertical axis at point Q, R is the radius of the wheels and $2L$ is the length of the axis of the fixed wheels, and $\tau = [\tau^1, \tau^2]^T$ is the torque provided by the motors. Using the coordinate transformations $X = T_1(q)$ and state feedback $u = T_2^{-1}(q)v$ given by

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \sin\theta & -\cos\theta & 0 \\ \cos\theta & \sin\theta & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

and $u_1 = v_2$, $\quad u_2 = v_1 - v_2x_2$, system (29) is converted to

$$\begin{array}{l} \dot{x}_1 = u_1,\ \dot{x}_2 = x_3u_1,\ \dot{x}_3 = u_2, \\ M_2(X)\dot{u} + C_2(X, \dot{X})u + G_2(X) = B_2(X)\tau, \end{array}$$

where $M_2(X) = \begin{bmatrix} x_2^2m_0 + I_0 & x_2m_0 \\ x_2m_0 & m_0 \end{bmatrix}$, $C_2(X, \dot{X}) = \begin{bmatrix} x_2\dot{x}_2m_0 & 0 \\ m_0\dot{x}_2 & 0 \end{bmatrix}$, $B_2(X) = 1/R\begin{bmatrix} x_2 + L & x_2 - L \\ 1 & 1 \end{bmatrix}$, $G_2 = 0$.

For the tracking control, the desired trajectory is chosen to be

$$q_{d1} = 2\sin t, q_{d2} = -2\cos t, q_{d3} = t,$$

with $v_{1d} = 2$ and $v_{2d} = 1$. Using the above diffeomorphism transformation, the desired trajectory in the chained form is $x_{1d} = t, x_{2d} = 2, x_{3d} = 0$, with $u_{1d} = 1$ and $u_{2d} = 0$. In the simulation, the parameters of the system are chosen to be $m_0 = I_0 = 0.5$, $R = 0.1$, $L = 1.0$, $c_1 = c_2 = 2$, $\lambda = 1$, $Q = 10I_{2\times2}$, and $R = 0.01I_{2\times2}$. The size of RBF neural network is chosen to $l = 9$, variances $\sigma = 1$, centers $\nu_i = [\nu_{i_1}, \cdots, \nu_{i_{12}}]^T, i = 1, \cdots, l$ with $\nu_{1_j} = -0.8, \nu_{2_j} = -0.5, \nu_{3_j} = -0.2, \nu_{4_j} = -0.1, \nu_{5_j} = 0, \nu_{6_j} = 0.1, \nu_{7_j} = 0.2, \nu_{8_j} = 0.5, \nu_{9_j} = 0.8$, where $j = 1, \cdots, 12$. The initial weighs of NNs are chosen as 0. The control parameters are chosen as $k_1 = 10$, $k_2 = 10$ and $\Gamma = \mathrm{diag}\{10\}$.

In the simulation, initial positions and velocities of the robot are set to be $q(0) = [3, 0, 0.5]$ and $\dot{q}(0) = [0, 0, 0]$. Simulation results are shown in figures 1 up to 4. Tracking errors $(q - q_d)$ and $(u - u^*)$ are shown in figure 1 and 2, respectively. Figure 3 contains the physical control input τ. The boundedness of neural network weights is shown by figure 4.

5 Conclusion

In this paper, a new adaptive trajectory tracking control is proposed for a class of uncertain dynamic nonholonomic systems. The control is synthesized at the dynamic level of system model. Neural networks are applied to approximate the unknown system functions. The stability of the closed-loop system is proved by using Lyapunov direct method. Simulation results illustrated the effectiveness of the proposed control.

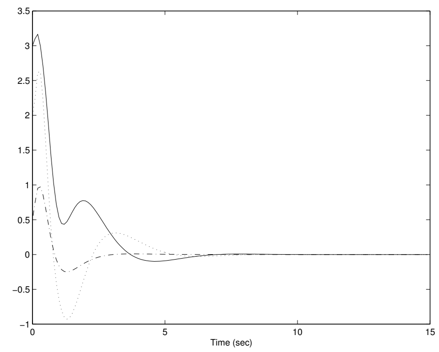

Figure 1: Tracking errors ("solid line": $x - q_{d1}$, "dotted line": $y - q_{d2}$, "dashdot line": $\theta - q_{d3}$)

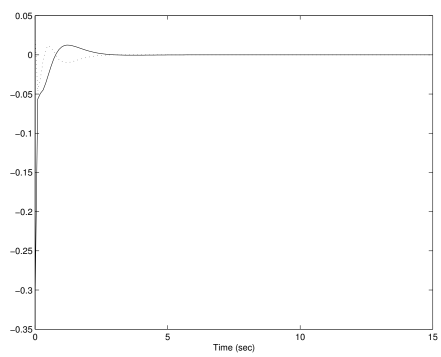

Figure 2: Tracking errors ("solid line": $u_1 - u_1^*$ and "dotted line": $u_2 - u_2^*$)

ISBN:978-988-98671-6-4

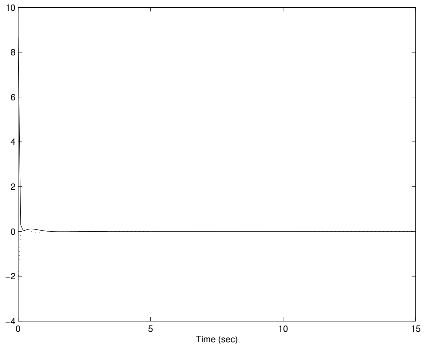

Figure 3: The real controls ("solid line": τ_1 and "dotted line": τ_2)

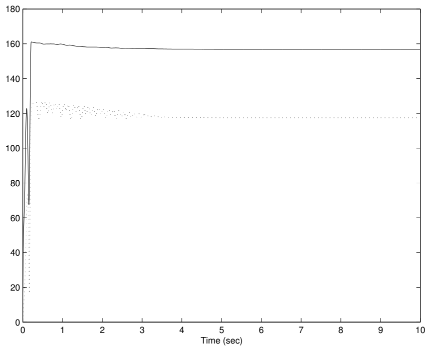

Figure 4: The boundedness of NNs weights (W_1: solid line; W_2: dotted line)

References

[1] I. Kolmanovsky and N. H. McClamroch, "Developments in nonholonomic control problems," *IEEE Contr. Syst. Mag.*, vol. 15, pp. 20–36, 1995.

[2] R. W. Brockett, "Asymptotic stability and feedback stabilization," in *Differential Geometric Control Theory*, 1983, vol. R. W. Brockett, R. S. Millman, and H. J. Sussmann, Eds, pp. 181–191.

[3] Z. P. Jiang and H. Nijmeijer, "A recursive technique for tracking control of nonholonomic systems in chained form," *IEEE Transaction on Automatic Control*, vol. 265-279, pp. 44, 1999.

[4] E. Lefeber, A. Robertsson, and H. Nijmeijer, "Linear controllers for exponential tracking of systems in chained form," *International Journal of Robust and Nonlinear Control*, vol. 10, pp. 243–263, 2000.

[5] R. Fierro and F. L. Lewis, "Control of a nonholonomic mobile robot using neural networks," *IEEE Transactions on Neural Networks*, vol. 9, pp. 589–600, 1998.

[6] C. Y. Su and Y. Stepanenko, "Robust motion/force control of mechanical systems with classical nonholonomic constraints," *IEEE Transactions on Automatic Contr.*, vol. 39, pp. 609–614, 1994.

[7] Z. Qu, J. Wang, C. E. Plaisted, and R. A. Hull, "A global-stabilizing near-optimal control design for real-time trajectory tracking and regulation of nonholonomic chained systems," *IEEE Transactions on Automatic Control*, vol. 51, pp. 1440–1456, 2006.

[8] A. Bloch and S. Drakunov, "Stabilization and tracking in the nonholonomic integrator via sliding modes," *System and contr. lett.*, vol. 29, pp. 91–99, 1996.

[9] R. M. Murray and S. S. Sastry, "Nonholonomic motion planning: Steering using sinusoids," *IEEE Trans. on Auto. Contr.*, vol. 38, pp. 700–716, 1993.

[10] F.L. Lewis, C.T. Abdallah, and D.M. Dawson, *Control of Robot Manipulators*, Macmillan, New York, 1993.

[11] M. Reyhanoglu A. Bloch and N.H. McClamroch, "Control and stabilization of nonholonomic dynamic systems," *IEEE Trans. on Auto Contr.*, vol. 37, pp. 1746–1757, 1992.

[12] G. C. Walsh and L. G. Bushnell, "Stabilization of multiple input chained form control systems," *System and Control Letters*, pp. 227–234, 1995.

[13] S. S. Ge, C. C. Hang, T. H. Lee, and T. Zhang, *Stable Adaptive Neural Network Control*, Kluwer Academic Publisher, Norwell, USA, 2001.

[14] R. M. Sanner and J. E. Slotine, "Gaussian networks for direct adaptive control," *IEEE Trans. Neural Networks*, vol. 3, pp. 837–863, 1992.

[15] E. B. Kosmatopoulos, M. M. Polycarpou, M. A. Christodoulou, and P. A. Ioannou, "High-order neural network structures for identification of dynamical systems," *IEEE Trans. Neural Networks*, vol. 6, pp. 422–431, 1995.

[16] L. X. Wang, *Adaptive Fuzzy Systems and Control: Design and Analysis*, Prentice-Hall, Englewood Cliffs, NJ, 1994.

[17] M. Krstic, I. Kanellakopoulos, and P. V. Kokotovic, *Nonlinear and Adaptive Control Design*, Wiley, New York, 1995.

ISBN:978-988-98671-6-4