On Estimating HIV Prevalence in India-Again by B.D.Aggarwala Department of Mathematics and Statistics, University of Calgary, Calgary, Alberta, T2N 1N4, Canada e-mail:aggarwal@math.ucalgary.ca

Abstract: The National AIDS Control Organization (NACO) of India had estimated. before this year, that there were 5.134 million HIV positive people in India at the end of 2004 and that they were increasing at the rate of more than quarter of a million people every year. In a recent publication, we estimated that, if the number of reported AIDS cases in India are only 50% efficient, i.e. if the number of actual AIDS cases in India is no more than twice the reported number, then the number of HIV positive people in India should have been no more than 2.5 million at the end of 2004. Many other people in the epidemiology community have the same point of view. Now, the government of India is also of the same view and "The latest data released by the government shows that the country has around 2 to 3 million people with HIV, much lower than last year's figure of 5.7 million". However, our assumption that the actual number of AIDS cases in India is only about twice the number reported, has been challenged, and it has been suggested that the Indian system of AIDS reporting is woefully inaccurate and the actual number of AIDS cases there could be three or even four times the reported number. In this paper, we consider this suggestion and show that, even if the actual number of AIDS cases was three, or even four, times the reported number, the number of HIV people in India, at the end of 2004, should still be no more than 2.5 million. This is because our previous estimate was an over estimate and had room to accommodate considerably more number of AIDS cases.

Keywords and phrases: HIV Prevalence, AIDS incidence, India. 2000 Mathematics Subject Classification: 92D25 1. **Introduction:** In the beginning, HIV infection manifests itself in the form of a mild fever, diarrhoea and/or rash which go away after a few weeks. After these initial symptoms, the patient stays asymptomatic for a number of years

after which some patients develop symptoms of persistent generalized lymphadenopathy (PGL)

and then they may progress to develop AIDS [1]. While an AIDS patient is very sick and goes to a doctor, the symptoms of primary HIV infection are so mild that a patient may simply take a Tylenol and hope that the symptoms will go away. They go away after a few weeks anyway. These facts require only a minor reluctance on the part of a patient for him/her not to report these symptoms to the doctor. His/her financial condition, hesitation to go to a doctor, lack of medical insurance, inaccessibility to a doctor or to a hospital (if the doctor is five miles away in the city for example, or as in a developing country, there is no hospital for twenty kilometres around) and many other factors may provide this 'minimum amount of hesitation' and the patient may not report these symptoms to a doctor. That a large number of HIV positive patients are unknown to the medical system is borne out by a survey report published by the Center for Disease Control and Prevention (CDC) in Atlanta, Georgia, U.S.A. in 2003 which stated that "the vast majority of young gays and bisexuals with the disease, 90 percent of blacks, 70 percent of Hispanics and 60 percent of whites, did not know that they were HIV positive" [2]. When after a number of years, an HIV positive person comes to the doctor with AIDS, the doctor wonders as to how many people like him/her there are in the community at large, and for how long has this patient been HIV positive. Counting of HIV positive people in a community is, therefore, more of a guesswork than a science. Our point of view is further substantiated by the fact that, in Canada as an example, in Dec. 2003, the number of HIV positive people was estimated to be somewhere between 26000 and 86000 by UNAIDS [3]. Such a large margin of error substantiates our point of view" [4].

The situation in India is no better where the estimates of HIV prevalence vary from the low of a few hundred thousand to a high of five million and more. Though there is some data on the number of AIDS incidence, this data is

highly unreliable because of the difficulty of obtaining reliable data in rural areas where even the incidence of opportunistic infections due to HIV positivity may be relegated to the existence of some unknown disease, and where the accessibility to a hospital is simply not available in large areas of countryside. According to the National AIDS Control Organisation (NACO) of India, the number of HIV positive people in India was estimated at 3.5, 3.7, 3.86, 3.97, 4.58, 5.106 and 5.134 million at the end of the years 1998 to 2004 respectively [5]. This gives incremental HIV prevalence of 200000, 160000, 110000, 610000, 526000, and 28000 in the years 1999, 2000, 2001, 2002, 2003, and 2004 respectively. Instead of such wildly oscillating figures, it is perhaps more accurate to say that (disregarding the small number of deaths) HIV incidence in India was approximately 272000 per year over the last six years or 287000 per year over the last five years. We consider these figures, based on serological surveys in selected populations, to be very much on the high side but, in the absence of anything better, accept these as reliable. Also the cumulative number of AIDS cases at the end of May, 2005 was estimated at 109349 by the same organisation (NACO), so that, at the end of 2004, this number may be estimated at 100,000. We would think that the above figures for AIDS incidence are perhaps more reliable than the corresponding figures for HIV positive cases because of the difficulty of estimating HIV incidence due to the reasons outlined above. In a recent paper [4], because of the difficulty of getting accurate numbers of AIDS cases from the countryside, we assumed that the actual number of AIDS cases was twice the number reported and arrived at the conclusion that cumulative HIV incidence in India at the end of 2004 was in the neighbourhood of 2.5 million as against the expert (NACO) estimate of 5.134 million. Other people in the epidemiology community now have the same point of view [6]. Thus, according to one author, "there continues to be inadequate understanding of basic HIV epidemiology especially its transmission dynamics - along with gross ignorance of how current HIV/AIDS estimates and projections are "cooked" or made up. Most of the public, policy makers, and the news media have uncritically accepted high HIV prevalence estimates and refuse even to consider the possibility that lower prevalence estimates may be more accurate" [7]. It should be pointed out that now the government of India is also of the same view and "The latest data released by

the government shows that the country has around 2 to 3 million people with HIV, much lower than last year's figure of 5.7 million"[8]. However, our assumption that the actual number of AIDS cases in India is only about twice the actual number has been challenged and it has been suggested that the Indian system of AIDS reporting is woefully inadequate and the actual number of AIDS cases there could be three or even four times the reported number [9]. In this paper, we consider this suggestion and show that, even if the actual number of AIDS cases was three, or even four, times the reported number, the number of HIV people in India, at the end of 2004, should still be no more than 2.5 million. This is because our previous estimate was an over estimate and had room to accommodate considerably more number of AIDS cases. We shall also show that if the Indian system of AIDS reporting was only 20% efficient and consequently, the actual number of AIDS cases was five times the number reported, the cumulative HIV incidence at the end of 2004 could still not be much more than 2.5 million. As in the previous paper [4], since the antiretroviral drugs are being administered to only a tiny fraction of HIV positive people, we shall assume that they have no impact at all on the HIV situation in India.

The first cases of HIV/AIDS were diagnosed in India in 1986. However, in the beginning, the number of such cases is only in the single digits and, in a vast country like India, it is practically impossible for a case to come to light if the total number of such cases in the country is, let us say, ten. We shall therefore assume that the first cases of HIV in India appeared a few years earlier, say 1981, and the first cases of AIDS appeared in 1986 and were diagnosed either in 1986 or later.

As in the previous paper [4], we shall present cumulative HIV incidence estimates for India using the method of back calculation which calculates HIV incidence from the data on AIDS incidence, based on an estimate of the incubation period, the time it takes for an HIV positive person to develop AIDS. It has been estimated [7, p.117] that this incubation period in India may be as short as 8 years. Consequently, we take this period to be 7.98 years in what follows. It should be pointed out that this will bring down our HIV incidence estimate even lower compared with our earlier assumption of 9.18 years. We now write

$$\mathbf{F}[\mathsf{t}] = \mathbf{1} - \mathbf{Exp}[-(\lambda_1 \, \mathsf{t})^{\theta_1}],$$

where, $\lambda_1 = .115$ and $\theta_1 = 5$. The graph of F[t] is given in Fig.1.

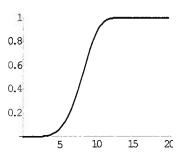


Fig. 1: Graph of $F[t] = 1 - Exp[-(\lambda_1 t)^{\Theta_1}]$, for $\lambda_1 = .115$ and $\theta_1 = 5$.

The first few values of F[t] are given by {F[0], F[2], F[4], F[6], F[8], F[10], F[12], F[20]}

{0, 0.000643427, 0.0203856, 0.144786, 0.482674, 0.866193, 0.993295, 1.}

The graph in Fig. 1 gives the probability distribution cumulative development to AIDS. The first few values say that out of a hundred patients say, most stay asymptotic for about four years, after which some patients start developing AIDS, and by about twelve years, most of them have done so. It follows that if g(s) is the number of people who become HIV positive at time s, and x(t) is the number of patients who have developed AIDS by the time t, then x(t), the total number of patients who have developed AIDS in the time interval (0,t) is given by the so called Back Calculation Method, namely,

$$x(t) = \int_0^t g(s) F(t-s) ds (1)$$

where F(t-s) is the probability that the patient has developed AIDS in t-s years or less.

The details of the back calculation method are given in our previous paper [4] and will not be repeated here. However, for the sake of completeness, we shall give a very brief outline. We shall assume that the AIDS cases started appearing in 1986 and denote the number of these cases in the subsequent years by x_1 , x_2 , x_3etc. Assuming that the total number of HIV infected people up to the end of 2004 is N,

we try to maximise the expression (the likelihood function)

the year 2004 being the year number 19, and where we have assumed that the numbers x_1, x_2, \dots etc. have a multinomial distribution with sample size N and cell probabilities p_1, p_2 , etc., p_i being the probability that an individual infected sometime before the i-th year develops AIDS in the i-th year. After some algebra, this maximisation leads to an iterative process which successively improves upon these numbers p_i [4].

In this paper, we follow the procedure outlined in our previous paper [4] and give only the results. We consider three different cases, namely when the actual number of AIDS cases was three times, four times or five times the reported number. These reported numbers are given in our previous paper as also the appropriate exponential interpolation which gives these numbers as 1, 2, 4, 8, 14, 24, 43, 77, 137, 243, 434, 772, 1375, 2449, 4361, 7765, 13827, 24621 and 43843 in the years 1986 to 2004 respectively. Because of the excellent agreement of these numbers with the few reported numbers by NACO, we shall call these the 'reported numbers'. Further, assuming that the total number of HIV cases by 2004 was N, we calculate the yearly HIV incidence which would have produced these AIDS cases, through the back calculation method based on equation (1). The case when the actual numbers are two times the 'reported numbers' is discussed in the previous paper [4]. We now consider the case when the actual numbers are three times the reported numbers.

2. Case when the actual numbers are three times the reported numbers:

The method of Back Calculation, as explained in [4], is an iterative process which successively produces the HIV incidence profile which will produce the desired number of AIDS cases with the help of equation (1). In the beginning, we assume this incidence profile to be

flat and run the method taking N = 2,500,000. The results of the first 1000 iterations are given in Figs.2 and 3.

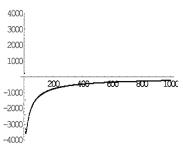


Fig. 2: Difference between the calculated number of AIDS cases and the assumed ones at the end of 2004 for the first 1000 iterations for N = 2.500,000.

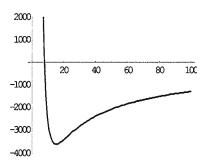


Fig 3: Detailed view of Fig.2 for the first 100 iterations. The results are given for the case when the curve first crosses the x-axis between eight and nine iterations. We give the results after nine iterations.

It is not clear from these figures whether the process converges. If it does, it must do so extremely slowly. Also, there does not seem to be any advantage in giving the results after 1,000 iterations as compared to only nine iterations, and the results reported when the curve crosses the x-axis after a small number of iterations have given useful information in other cases [10]. For this reason, we give the results here for the case when the curve in Fig.3 first crosses the x-axis, i.e. after nine iterations for N = 2,500,000 (and after 7 iterations for N = 2,300,000). The calculated yearly HIV incidence for this case is given in Fig.4 for N = 2,300,000, and for N = 2,500,000. For N = 2,300,000 and for the last five years preceding 2004, the yearly incidence was 285568., 289352., 290223., 290343., and 290350,(in 2004). For N =2,5000,000, as shown in Fig.4, it was considerably higher. If we therefore assume that the incidence figures given by NACO (average of 287,000 over these years) are correct, then the

cumulative HIV incidence up to 2004 must have been approximately 2,300,000. Subtract from this the number of AIDS patients that died and we conclude that HIV prevalence must have been even less than that.

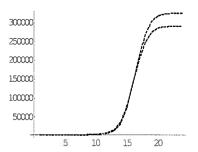


Fig.4: Calculated yearly HIV incidence up to 2004 when the cumulative HIV incidence is 2.3 or 2.5 million. The year 1981 is year one. The curve for 2.5 million is higher on the right

The comparison of the calculated cumulative number of AIDS cases with the 'actual' ones is given in Fig.5 for N = 2,300,000. Notice the excellent agreement.

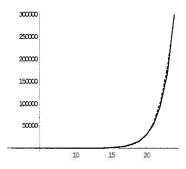


Fig. 5: Comparison of calculated cumulative number of AIDS cases (dotted line) with the 'actual' ones in our model for N = 2,300,000 for the case when the actual number of AIDS cases is supposed to be three times the 'reported' ones.

We shall now consider the case when the actual number of AIDS cases was four times the 'reported' ones.

3. Case when the actual numbers are four times the reported numbers:

The counterparts of Figs.2 and 3 in this case were similar and we give the results for the case when the curves crossed the x-axis after five iterations for N = 2,500,000 and after seven iterations for N=3,000,000.

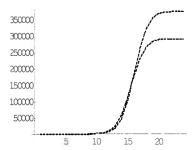


Fig.6: Calculated yearly HIV incidence up to 2004 when the cumulative HIV incidence is 2.5 or 3.0 million. The year 1981 is year one. The curve for 3.0 million is higher on the right

For N = 2500,000 the yearly incidence for five years preceding 2004 was 291041, 292798, 293091, 293110, and 293110 9 (in 2004). Since the average of these numbers is already higher than the corresponding average of NACO figures, we conclude that in this case as well, the cumulative HIV incidence was slightly less than 2,500,000. Subtract from this, the number of AIDS patients who died, and the HIV prevalence at the end of 2004 was even less. The comparison of the calculated cumulative number of AIDS cases with the 'actual' ones is given Fig.7 for N = 2,500,000.

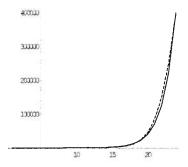


Fig. 7: Comparison of calculated cumulative number of AIDS cases with the 'actual' ones (solid line) in our model for N=2,500,000 for the case when the actual number of AIDS cases is supposed to be four times the 'reported' ones. The results for the case when the actual number of AIDS cases was five times the 'reported' ones are given next.

4. Case when the actual numbers are five times the reported numbers:

The counterparts of Figs. 2 and 3 in this case were similar and we give the results for the case when the curves crossed the x-axis after four iterations for N=2,500,000 and after five iterations for N=3,000,000.

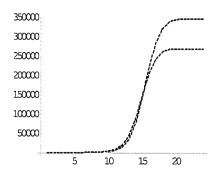


Fig.8: Calculated yearly HIV incidence up to 2004 when the cumulative HIV incidence is 2.5 or 3.0 million for this case. The year 1981 is year one. The curve for 3.0 million is higher on the right.

For N = 2,500,000, the yearly incidence for the last five years preceding 2004 was 269249, 268971, 268686, 268606, and 268599 (in 2004). Since the average of these numbers is only slightly lower than the corresponding average of NACO figures, we conclude that in this case as well, the cumulative HIV incidence was only slightly more than 2,500,000. However, if the NACO figures for HIV incidence are on the high side, then 2.5 million may be about right or slightly higher. Subtract from this the number of AIDS patients who died, and the HIV prevalence at the end of 2004 was even less. The yearly incidence for the last five years preceding 2004 for N = 3,000,000 was 344847, 346273, 346415, 346400, 346397 (in 2004) which is considerably higher than the NACO figures. We conclude that HIV prevalence at the end of 2004 was closer to 2.5 million than to 3.0 million. The comparison of the calculated cumulative number of AIDS cases with the 'actual' ones is given Fig.9 for N = 2,500,000.

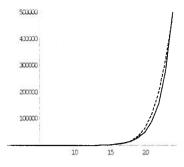


Fig. 9: Comparison of calculated cumulative number of AIDS cases with the 'actual' ones (solid line) in our model for N = 2,500,000 for the case when the actual number of AIDS cases is supposed to be five times the 'reported' ones.

5. Discussion: Our conclusion is that cumulative HIV incidence in India at the end of 2004 was close to 2.5 million or slightly less. With the number of HIV positive people increasing at the rate of roughly quarter of a million per year, this number should be close to 3 million at the end of 2006. Take away from it the number of AIDS deaths, and the HIV prevalence today should be slightly less. This helps shed light on the question "Were the number of HIV positive people in India at the end of 2006, three million or six million?" This question has been asked before [7, p.192]. Our estimate would be "less than 3 million", which contrasts with the previous NACO estimate of more than 6 million, which estimate was the one reported before the new serological surveys in 2007. This supports the opinion that "Unrealistically high HIV prevalence estimates or projections in countries such as China and India will within a few years be exposed as, at best, naive efforts of well meaning AIDS program advocates and at worst as the work of AIDS "experts" who deny the levelling or decreasing global HIV prevalence trends to support their own political, social, economic or personal agendas"[7, p.138].

It should be pointed out that we arbitrarily took the value of θ_1 in our model to be five. This value has been estimated variously in the medical literature and some researchers have estimated it at as low as 2.3 [11]. However, with a lower value of θ_1 , AIDS will progress faster in the beginning, and if we keep the same value of incubation period, this will result in an even lower estimate of HIV prevalence. Keeping the same value of incubation period, we tried a lower value of θ_1 , and indeed our estimate of HIV prevalence was lower than above. Thus we tried the case of $\theta = 2.3$ and $\lambda = .111$ which gives an incubation period of 7.98 as before. When the actual number of AIDS cases is five times the reported ones and for N = 2,500,000 the HIV incidence in the last five years before 2004 was 328006, 342061, 340998, 334307, 331178. These numbers are higher than the estimates of NACO implying that a lower value of N would be more appropriate for this case. The comparison of calculated AIDS cases with actual ones is given in Fig.10.

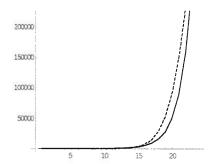


Fig.10: Comparison of calculated AIDS cases (dotted line) with the actual ones for $\theta_1 = 2.3$ and $\lambda = .111$ implying an incubation period of 7.98 years, for the case when the actual number of AIDS cases was five times the actual ones.

In this figure, for a given value of t, the calculated number of AIDS cases (dotted line) is much higher than the actual ones, implying that HIV positive people are contracting AIDS much too fast in the model. Comparing with the previous figures (Fig. 5, or 7, or 9), we would think that $\theta_1 = 5$ is a more reasonable value of θ_1 .

We have pointed out that with cumulative HIV incidence of 2.5 million at the end of 2004, the cumulative number of AIDS cases may be 2, 3, 4, or 5 hundred thousand. It follows that if we only know the HIV incidence (or HIV prevalence) at one point in time, it is not possible to estimate historical HIV incidence without additional assumptions. Such assumptions have been made [12], and historical and future trends of HIV and AIDS estimated. Thus "assuming 5 million HIV infected adults in Sub Saharan Africa in 1990, the WHO estimated there would be 700,000 cumulative AIDS cases by the end of 1990." [13, p.270].

In this connection, it is interesting to compare the above infection curves with the model used by World Health Organisation (WHO). Their model of an infection curve is the family of Gamma curves [13, p.270] $F(t) = A t^p e^{-t/\mu}$ for appropriate constants A, p and μ . The local maxima of this curve occurs at $t = p\mu$ and in the next figure, we compare these curves with the infection curves of figures 4, 6, and 8 for N = 2500,000.

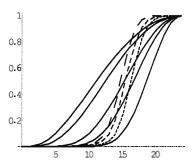


Fig11: The infection curves of figures 4, 6, and 8 above compared with Gamma curves for various values of the parameters in the gamma curves. In each case, the local max of the Gamma curve $F(t) = A t^p e^{-t/\mu}$ occurs at $t = L = p\mu = 24$ and the constant A in the curve is adjusted so that this max is one. The infection curves of figures 4, 6, and 8 are for N = 2500,000 and are scaled so that the max of each one is one. These Gamma curves (solid lines) are for p = 3, 4, 8, 12 and 24, starting from the left. The infection curves are for the cumulative AIDS incidence of 300 (short dashes), 400(medium dashes), and 500 (long dashes) thousand up to the end of 2004.

It is to be noted that for the given values of the parameters, the WHO curves do not give satisfactory approximation to the infection curves that we have calculated. To get a better approximation, we go to the Weibull distribution and write

$$H(t) = 0 \text{ in } 0 \le t < \alpha$$
, and $H(t) = 1 - \text{Exp}[-(\lambda_1 (t-\alpha))^{\theta 1}] \text{ in } t \ge \alpha$

for appropriate values of α , λ_1 , and θ_1 . In each case we take $\theta_1 = 7$ and $\lambda_1 = .1$ and compare the curves for the cumulative AIDS incidence of 300,000 and 500,000 with the ones for H(t) above for $\alpha = 7$ and for $\alpha = 5.3$ respectively. The comparison is given in figures 12 and 13. The comparison for the cumulative AIDS incidence of 400,000 with the same curve for $\alpha = 6.2$ is similar and is not shown here. It can be seen that the area under the assumed infection curve for the case of AIDS incidence being 500,000 (dotted line in Fig.12) is .4% more than the area under the calculated curve (solid line). In comparison, the approximation by any of the gamma curves is vastly more in error. This suggests that gamma curves, which are widely used in the EPI Model [12] may not be the most suitable curves for modeling HIV infection curves and that the Weibull curves, which have just one more parameter, may be more suitable.

Adopting of these curves by modeling agencies like WHO may give much better estimates, particularly in countries like India and China, where because of the very large population base, yearly hiv incidence is not likely to come down anytime soon. These curves also suggest that if the hiv infection in India started in 1986, and if we accept the NACO estimate of current yearly HIV incidence of more than quarter of a million, then the cumulative number of AIDS cases at the end of 2004 should be 500,000. If it started in 1987, then it should be 400,000 and if it started in 1988, then it should be 300,000. The first cases of HIV positivity in India were diagnosed in 1986.

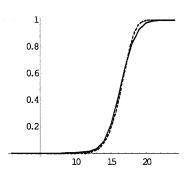


Fig. 12: Comparison of the infection curve calculated above for the case of 300,000 AIDS incidence at the end of 2004 (solid line) with H(t) given in the text for $\alpha = 7$.

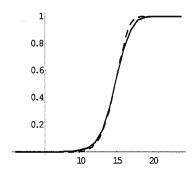


Fig. 13: Comparison of the infection curve calculated above for the case of 500,000 AIDS incidence at the end of 2004 (solid line) with H(t) given in the text for $\alpha = 5.3$.

Once you decide upon the infection curve, it is easy to calculate the number of AIDS deaths till the end of 2004 and then HIV prevalence at the end of that year. It should be pointed out that in any serological survey, we only estimate the HIV prevalence, so that it is important for us to estimate the number of AIDS deaths in this paper. AIDS deaths have been estimated before [14]. As in that paper, AD(t),

the cumulative number of AIDS deaths at any time t, is given by

$$AD(t) = \int_0^t g(s) k(t-s) ds$$

where k (t) =
$$\int_{0}^{1} tG[(1-w) t] F'[wt] dw$$

where g(s) is the infection curve, and F[t] and G[t] define the progression from HIV infection to AIDS and from AIDS to death respectively. In our case, we have just calculated the infection curves g(s), F(t) is given in Fig.1 and G(t) is the same function as in Fig.1 but with $\lambda_1 = .9$, which gives the time from development of AIDS to death as 1.02019 years, a figure which is suited to a developing country like India [7, p.117]. It turns out that for the case of AIDS incidence being 500,000 to the end of 2004, the number of AIDS deaths up to the end of 2004, would be 307060, slightly over 300,000 (the numbers AD(t) are also seen to closely follow a Weibull curve). This puts the HIV prevalence at the end of 2004 at well below 2.5 million, a figure which this writer was the first to advocate [4] and which is now accepted by all people working with HIV/AIDS in India [8].

The cumulative number of AIDS deaths at the end of 2003 turns out to be 184063 implying that a total of 122997 AIDS patients died in the year 2004. This is about a quarter of the total AIDS incidence up to the end of 2004, and may provide some guidance when calculating this number for other countries. Since we are assuming HIV incidence of approximately 250,000, this further implies that HIV prevalence in India is on the increase these days.

Conclusion: In a previous paper [4], we argued that the cumulative HIV incidence in India, at the end of year 2004, must not have been greater than 2.5 million in India, as against the estimates of WHO and NACO of 5.134 million at that time. This figure of 2.5 million has now been agreed upon by all people concerned [6]. In this paper, we have suggested that Weibull functions, and not gamma curves, should be used for modeling HIV infection incidence and also argued that the system of AIDS reporting in India is seriously deficient and the number of

AIDS cases there, by the end of 2004, was most probably 500,000 and not 100,000 as reported by NACO, provided we accept the (NACO) estimate of yearly HIV incidence of more than quarter of a million per year [5].

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