An Algebraic State Estimation Approach for DC Motors

G. Mamani*, J. Becedas*, V. Feliu-Batlle * and H. Sira-Ramírez ** ‡

Abstract—**In this article a fast, non-asymptotic algebraic method is used for the estimation of state variables. The estimation is used to implement a position control scheme for DC motor. In addition the estimation of the Coulomb's friction coefficient of the servo motor model has been investigated. The approach is based on elementary algebraic manipulation that lead to specific formulae for the unmeasured states. The state estimation algorithm have been verified by simulation.**

Keywords: State estimation, Algebraic Identification, DC motors, State observer

1 INTRODUCTION

Control design method such as state feedback controls, which use the states in their control laws are designed under the assumption that all state variables are accessible for measurement. However, in many practical application it may not be economical or convenient to measure all of the state variables. An alternative approach is to use an estimation technique to provide estimates of the state variables which are not measured, for use in implementation of a feedback control law.

The foundation of linear state estimation was laid by Kalman in [1], who developed the Kalman filter. Later Luenberger in [2] introduced a deterministic version of the Kalman filter, known as Luenberger observer. The theorical properties of the Kalman filter and the Luenberger observer are well understood and can be found in estimation and/or system theory textbooks [3].

It is clear that for an observable system, represented in state space, the state estimation problem is intimately related to the computation of time derivates of the output signals, in a sufficient number. The main contribution of this article is that, we attempt an algebraic method, of non-asymptotic nature, for the estimation of states computing a finite number of time derivates of the output. The method is based on elementary algebraic manipulations that lead to specific formulae for the unmeasured states. Our approach uses the model of the system which is known almost most of times. In this work, we use an fast state estimation of continuous-time nature for the estimation of the state of a DC servomotor model in order to implement an PD control scheme. After the estimates of the state variables are obtained by algebraic method the Coulomb's friction coefficient is instantaneously estimated. The importance of this coefficient estimation is explained in [4] in order to appropriately control the system by compensating this non-linearity. The estimation method is based on elementary algebraic manipulations of the following mathematical tools: module theory, differential algebra and operational calculus. They were developed in [5]. A differential algebraic justification of this article follows similar lines to those encountered in [6].

2 MOTOR MODEL AND ESTIMATION PROCEDURE

This section is devoted to explain the linear model of the DC motor and the algebraic estimation method. We assume that the linear model is affected by unknown perturbation due to the Coulomb's friction effects.

2.1 DC Motor model

A common electromechanical actuator in many control systems is constituted by the DC motor [7]. The DC motor used is fed by a servo-amplifier with a current inner loop control. We can write the dynamic equation of the system by using Newton's Second Law:

$$kV = J\ddot{\hat{\theta}}_m + \nu\dot{\hat{\theta}}_m + \hat{\Gamma}_c(\dot{\hat{\theta}}_m) \quad (1)$$

where J is the inertia of the motor $\left[kg \cdot m^2\right]$, ν is the viscous friction coefficient $[N \cdot m \cdot s]$, $\hat{\Gamma}_c$ is the unknown Coulomb friction torque which affects the motor dynamics $[N \cdot m]$. This nonlinear friction term is considered as a perturbation, depending only on the sign of the angular velocity of the motor of the form $\mu sign(\dot{\hat{\theta}}_m)$ with μ constant. The parameter k is the electromechanical constant of the motor servo-amplifier system $[Nm/V]$. $\ddot{\hat{\theta}}_m$ and $\dot{\hat{\theta}}_m$ are the angular acceleration of the motor $\left[rad/s^2\right]$ and

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the angular velocity of the motor $[rad/s]$ respectively. The constant factor n is the reduction ratio of the motor gear; thus $\theta_m = \hat{\theta}_m/n$, where θ_m stands for the position of the motor gear and $\hat{\theta}_m$ for the position of the motor shaft. $\Gamma_c = \hat{\Gamma}_c n$, where Γ_c is the Coulomb friction torque in the motor gear. V is the motor input voltage $[V]$ acting as the control variable for the system. This is the input to a servo-amplifier which controls the input current to the motor by means of an internally PI current controller The electrical dynamics can be neglected because it is much faster than the mechanical dynamics of the motor. Thus, the servo-amplifier can be considered as a constant relation between the voltage and the current to the motor: $i_m = k_e V$ where i_m is the armature circuit current and k_e includes the gain of the amplifier, $\tilde{k}$, and the input resistance of the amplifier circuit R. The total torque delivered to the motor Γ_T is directly proportional to the armature circuit in the form $\Gamma_T = k_m i_m$, where k_m is the electromechanical constant of the motor. Thus, the electromechanical constant of the motor servo-amplifier system is $k = k_m k_e$. In order to obtain the transfer function of the system the following perturbation-free system is considered :

$$KV = J\ddot{\theta}_m + \nu\dot{\theta}_m \tag{2}$$

where $K = k/n$. To simplify the developments, let $A = K/J$, $B = \nu/J$.

2.2 The Procedure of fast state estimation

Consider the second order perturbed system given in (1). Taking it into account and also the fact that $K = k/n$ and after some rearrangements, we have,

$$\ddot{\theta}_m + B\dot{\theta}_m + \Gamma^* = AV \tag{3}$$

where $\Gamma^* = \frac{\hat{\Gamma}_c}{nJ}$. We consider this parameter as a constant perturbation input and it will be identified in the next stage.

We proceed to compute the unmeasured states, the motor velocity, $\frac{d\theta_m}{dt}$, and the motor acceleration, $\frac{d^2\theta_m}{dt^2}$, as follows:

Taking Laplace transforms, of (3) yields,

$$(s^2\theta_m(s) - s\theta_m(0) - \dot{\theta}_m(0)) + \tag{4}$$
$$+B(s\theta_m(s) - \theta_m(0)) + \frac{\Gamma^*}{s} = AV(s)$$

we obtain multiplying out by s,

$$(s^3\theta_m(s) - s^2\theta_m(0) - s\dot{\theta}_m(0)) + \tag{5}$$
$$+B(s^2\theta_m(s) - s\theta_m(0)) + \Gamma^* = AsV(s)$$

Taking the third derivative with respect to the complex variable s, we obtain independence of initial conditions. Then (5) results in an expression free of the initial conditions $\dot{\theta}_m(0)$, $\theta_m(0)$ and the Coulomb's friction coefficient Γ^*:

$$\frac{d^3}{ds^3}\left[s^3\theta_m(s)\right] + B\frac{d^3}{ds^3}\left[s^2\theta_m(s)\right] = A\frac{d^3}{ds^3}[sV(s)] \tag{6}$$

Recall that multiplication by s in the operational domain corresponds to derivation in the time domain. After rearrangements we multiply both sides of the resulting expression by s^{-2}. We obtain

$$s\frac{d^3\theta_m(s)}{ds^3} + 9\frac{d^2\theta_m(s)}{ds^2} + 18s^{-1}\frac{d\theta_m(s)}{ds} + 6s^{-2}\theta_m(s) \tag{7}$$
$$+B(\frac{d^3\theta_m(s)}{ds^3} + 6s^{-1}\frac{d^2\theta_m(s)}{ds^2} + 6s^{-2}\frac{d\theta_m(s)}{ds}) =$$
$$A(s^{-1}\frac{d^3V(s)}{ds^3} + 3s^{-2}\frac{d^2V(s)}{ds^2})$$

In the time domain, we have:

$$-\frac{d}{dt}(t^3\theta_m) + 9t^2\theta_m - 18\int_0^t \sigma\theta_m(\sigma)d\sigma$$
$$+6\int_0^t\int_0^\sigma \theta_m(\lambda)d\lambda d\sigma + B((-t^3\theta_m)$$
$$+6\int_0^t \sigma^2\theta_m(\sigma)d\sigma - 6\int_0^t\int_0^\sigma \lambda\theta_m(\lambda)d\lambda d\sigma)$$
$$= A(-\int_0^t \sigma^3 V(\sigma)d\sigma + 3\int_0^t\int_0^\sigma \lambda^2\theta_m(\lambda)d\lambda d\sigma) \tag{8}$$

From here we obtain the estimation of the motor velocity

$$\frac{d\theta_m}{dt} = \frac{1}{t^3}(6t^2\theta_m - 18\int_0^t \sigma\theta_m(\sigma)d\sigma$$
$$+6\int_0^t\int_0^\sigma \theta_m(\lambda)d\lambda d\sigma) + \frac{1}{t^3}(-Bt^3\theta_m +$$
$$6B\int_0^t \sigma^2\theta_m(\sigma)d\sigma - 6B\int_0^t\int_0^\sigma \lambda\theta_m(\lambda)d\lambda d\sigma)$$
$$+\frac{1}{t^3}(A\int_0^t \sigma^3 V(\sigma)d\sigma - 3A\int_0^t\int_0^\sigma \lambda^2 V(\lambda)d\lambda d\sigma) \tag{9}$$

by multiply both sides of the resulting expression in (7) by s, we have:

$$(s^2\frac{d^3\theta_m(s)}{ds^3} + 9s\frac{d^2\theta_m(s)}{ds^2} + 18\frac{d\theta_m(s)}{ds} + 6s^{-1}\theta_m(s))$$
$$+B(s\frac{d^3\theta_m(s)}{ds^3} + 6\frac{d^2\theta(s)}{ds^2} + 6s^{-1}\frac{d\theta_m(s)}{ds})$$
$$= A(\frac{d^3V(s)}{ds^3} + 3s^{-1}\frac{d^2V(s)}{ds^2}) \tag{10}$$

which may be written in the time domain as:

$$-\frac{d^2}{dt^2}(t^3\theta_m) + 9\frac{d}{dt}(t^2\theta) - 18t\theta_m + 6\int_0^t \theta_m(\sigma)d\sigma$$

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$$+B(\frac{d}{dt}(-t^3\theta_m) + 6t^2\theta_m - 6\int_0^t \sigma\theta_m(\sigma)d\sigma) - At^3V$$

$$+3A\int_0^t \sigma^2 V(\lambda)d\sigma$$

We obtain the following expression for the motor acceleration, $\frac{d^2\theta_m}{dt^2}$:

$$\frac{d^2\theta_m}{dt^2} = \frac{1}{t^3}(3t^2\frac{d\theta_m}{dt} - 6t\theta_m + 6\int_0^t \theta_m(\sigma)d\sigma + 3Bt^2\theta_m)$$

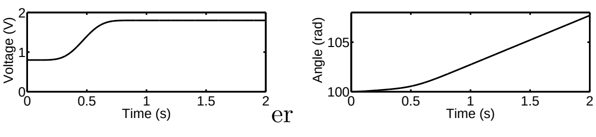
$$+\frac{1}{t^3}(-Bt^3\frac{d\theta_m}{dt} - 6B\int_0^t \sigma\theta_m(\sigma)d\sigma + At^3V$$

$$-3A\int_0^t \sigma^2 V(\lambda)d\sigma) \qquad (11)$$

This expression may now be evaluated with the help of the already computed estimate of $\frac{d\theta_m}{dt}$.

3 Simulation

This section is devoted to show the good performance of the proposed state estimation method. The values of the motor parameters used in simulations are $A = 61.13\ (N/(V\cdot Kg\cdot s)$, $B = 15.15\ (N\cdot s/(Kg\cdot m)$, $k = 0.21\ (N\cdot m/V)$, $n = 50$, $\mu = 34.74\ (N\cdot m)/(kg\cdot m^2)$. The differential equation of the closed loop system is solved by using a $1\cdot 10^{-3}$ $[s]$ fixed step fifth order Dormand-Prince method. We consider that there exists a servo amplifier used to supply voltage to the DC motor; this amplifier accepts control inputs from the computer in the range of $[-10, 10]$ $[V]$. The signal used in the on-line estimation of the motor velocity are the input voltaje to the DC motor and the motor position as a result of that input. In this case we have chosen the input to be a Bezier's eighth order polynomial with an offset of 0.8 (V). We have considered the following initial conditions for the motor in order to show the robustness of the method to every initial conditions: $\theta_m(0) = 100, \dot{\theta}_m(0) = 0$. Both signals are depicted in Fig.1(a) and Fig.1(b) respectively. The results are compared with fifth order error numerical



er

(a) (b)

Figure 1: Input and response of the motor. (a) Input to the DC motor. (b) Response of the DC motor.

derivative. Fig.2(a) depicts the output of the velocity motor observer represented by $\frac{d\theta_t}{dt}$. We can compare it with that of the numerical derivative here represented by $\frac{d\theta_{tn}}{dt}$. Note that the two signals are superimposed. The difference between them is depicted in Fig.2(b) and this is with 10^{-3} order. In Fig.2(c) the estimation of the motor acceleration $\frac{d^2\theta_t}{dt^2}$ is depicted in addition to the numerical estimation $\frac{d^2\theta_{tn}}{dt^2}$. The difference $\frac{d^2\theta_t}{dt^2} - \frac{d^2\theta_{tn}}{dt^2}$ between them is depicted in Fig.2(d). Now, the difference is more noticeable because is required the first derivative of the signal to obtain the second one and in the case of the numerical estimation not knowledge of the system is used, this is the reason of the increasingly difference. On the other hand, the observer proposed take all the information of the system as posible providing more exacts estimations. This premise will be demonstrated in the application of Coulomb's friction estimation where more accuracy state estimation provides better parameter estimation.

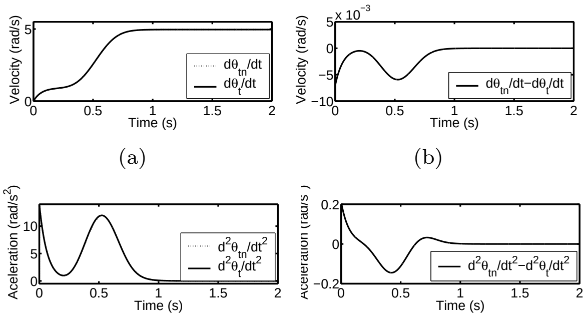


Figure 2: Comparison between the estimations of the velocity and acceleration of the DC motor with the numerical method and with the algebraic state estimation. (a) Velocities estimation. (b) Difference between the two velocity estimations $\dot{\theta}_{tn} - \dot{\theta}_t$. (c) Accelerations estimation. (d)Difference between the two acceleration estimations $\ddot{\theta}_{tn} - \ddot{\theta}_t$

In the new simulations robustness with respect noise of the algebraic state estimation is demonstrated. We consider noise in the measure (i.e in the motor position measure) with zero mean and 10^{-3} standard deviation. When noise appears in a measure, numerical estimation of derivatives of a signal is very imprecise and the estimation of bounded derivatives amplify the noise level. These signals are, customarily, quite noisy and the use of low pass filters become necessary to smooth them causing the well known dynamic delays that affect the performance of the obtained signals as a result. A solution to this problems may be the use of algebraic state estimator, which present robustness to the noise. In Fig.3(a) the numerical estimation of the motor velocity is depicted. It is obvious the effect that the noise produces in the estimation. Fig.3(b) depicts the velocity estimation with the algebraic state estimator. In Fig.3(c) the second derivative of the motor position is represented. Note that in this signal the noise level has been increased. At last, Fig.3(d) depicts the second derivative of the motor estimated with the algebraic state estimator. Let us recall that in no one of the estimations filters have been used. An scheme of the observers implementation is depicted in Fig.4. This

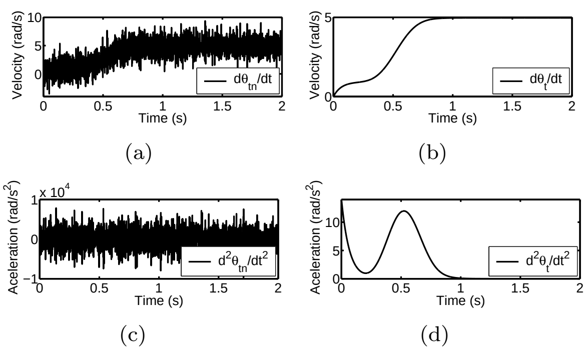


Figure 3: Velocity estimations. (a) Numerical velocity estimation. (b) Algebraic velocity estimation. (c) Numerical acceleration estimation. (d) Algebraic acceleration estimation

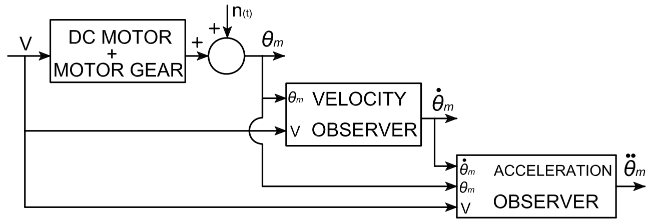


Figure 4: Scheme of the algebraic observers implementation

technique may be used in many applications such as estimation of parameters in which estimation of states are required and control of a feedback system because the estimators can be used in both open and closed loop due to the method does not require dependence between the system input and output.

3.1 Estimation application

From the 1990's decade interest in controlling systems with gear reduction coupled in the motor shaft has increased. Researchers had to deal with non linearities which strongly affected the motor dynamics and which were produced by the friction torque [4]. In order solve that problem researchers used many techniques such as robust control schemes with high gain that minimized this effect [9] or the most modern techniques such as neural networks [8] which delay the obtaining of the non linearity parameters. We here propose a new and precise technique to obtain such parameters by using algebraic state estimators, which can be used in real time and in continuous time without the used of any sort of filter. It is well established that for a system operating at relatively high speed, the Coulomb's friction torque is a function of the angular velocity. For those systems, the Coulomb's friction is often expressed as a signum function dependent on the rotational speed.Consider system (3) with $\Gamma^* = \mu sign(\dot{\theta}_m)$ [1] From this equation, and due to the fact that the angular velocity and acceleration of the motor are obtained with the fast state estimation method, A and B are known, and we have

$$\mu sign(\dot{\theta}_m) = AV - (\ddot{\theta}_m)_e - B(\dot{\theta}_m)_e \qquad (12)$$

The term: $\mu sign(\dot{\theta}_m)$ is a perturbation produced by the Coulomb's friction torque, where μ is the scaled Coulomb's friction amplitude, or coefficient [2]. With the motor spinning only in one direction, Coulomb's friction coefficient will not change its sign, and can be considered as a constant. When the motor angular velocity is close to zero, the Coulomb's friction effect is that of a chattering high frequency signal.

$$\Gamma^* = \mu sign(V) = \left\{ \begin{array}{c} \mu \ (V > 0), \\ -\mu (V < 0) \end{array} \right\} \qquad (13)$$

Then, if the motor spins always in the same direction, in the identification time interval we have that $\Gamma^* = \mu$ and

$$\mu = AV - (\ddot{\theta}_m)_e - B(\dot{\theta}_m)_e \qquad (14)$$

Fig.5(a) depicts the estimation of the Coulomb's friction coefficient by using numerical state estimation (μ_n signal) and by using algebraic state estimation (μ_s signal). Note that the estimation μ_s is obtained from the beginning at time $t \approx 0$ and this value is maintained while the estimator works. Nevertheless, the estimation μ_n which uses numerical state estimations introduce an error until $t = 1$ (s), time at which the Bezier's trajectory finishes. The error of the two estimates with respect the real value of the Coulomb's parameter μ is depicted in Fig.5(b). $\varepsilon_{\mu_n} = \mu_n - \mu$ represents the error in the estimation with the numerical method and $\varepsilon_{\mu_s} = \mu_s - \mu$ represents the error in the estimation with the algebraic method. Note the error in the estimation with the algebraic state estimators has null error. This is because the state estimation with the proposed method provides an exact estimation of the bounded derivatives of the motor position due to the estimator uses all the information as posible from the system to estimate. Fig.6 depicts the

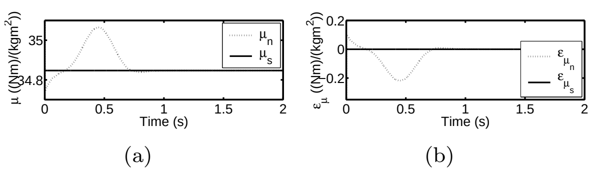


Figure 5: Coulomb estimation with both numerical and algebraic methods, μ_n, μ_s numerical and algebraic estimates respectively. (a) Estimations of coulomb. (b) Estimation errors.

[1]The model $sign(\dot{\theta}_m)$ is defined as 1 if $\dot{\theta}_m > 0$ and as -1 if $\dot{\theta}_m < 0$

[2]Note that $\Gamma^* = \frac{\hat{\Gamma}_c}{nJ} = \mu sign(\dot{\theta}_m)$ then, the Coulomb's friction coefficient is $\xi = Jn\mu$

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results obtained in the Coulomb's parameter estimation with a noise in the measure of the motor position with zero mean and 10^{-3} standard deviation as done in the previous simulations. In Fig.6(a) the estimation with numerical estimations of the states is depicted. Note that is imposible to identify any value in such a figure. On the other hand, Fig.6(b) depicts an accurate estimation of the parameter estimated by using the algebraic method. The error of this last estimation is shown in Fig.6(c).

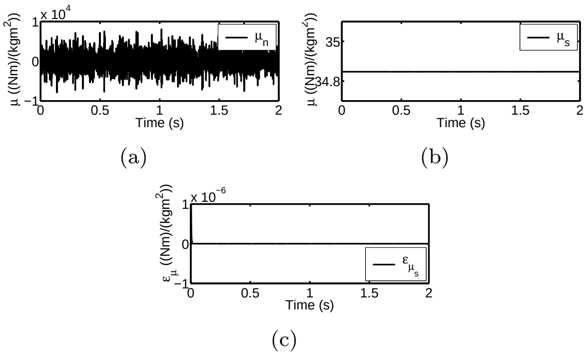


Figure 6: Coulomb estimation. (a) numerical (c) algebraic (d) algebraic error

3.2 Control application

DC motors is a topic of interest since is used as actuator in an extensive variety of robotics systems and one of the most used control methods is that based on proportional derivative *PD* controller. [10] is an example of this. The inconvenient of this sort of control is the computation of the derivative action which always introduce noises in the control voltage input due to the on-line estimation of the input derivative to the controller. Sometimes filters are required to smooth that signal. In this subsection a control application for DC motors is proposed based on the on-line algebraic estimation of the motor velocity. A PD controller is proposed, $C_{pd}(s) = k_p + k_v s$, whose gains $\{k_p, k_v\}$ can be designed by locating all the poles in closed loop of the complete system (See Fig.7) in the same location of the negative real axis. The stability condition

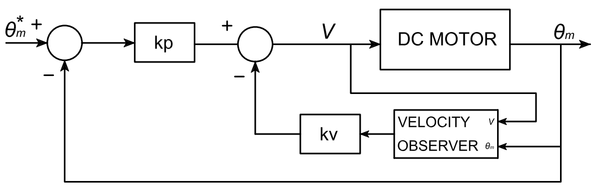


Figure 7: Closed loop PD controller with algebraic observer implementation

on the closed loop expression $(1 + G_{m_0}(s)C_{pd}(s))$ leads to the following characteristic polynomial,

$$s^2 + (kvA + B)s + k_p A = 0 \qquad (15)$$

We can equate the corresponding coefficients of the closed loop characteristic polynomial (15) with those of a desired second order Hurwitz polynomial. Thus, we can choose to place all the closed loop poles at some real value using the following desired polynomial expression,

$$p(s) = (s + a)^2 = s^2 + 2as + a^2 \qquad (16)$$

where the parameter a, strictly positive, represents the common location of all the closed loop poles. Identifying the corresponding terms of the equations (15) and (17) the parameters k_p and k_v may be uniquely obtained by computing the following equations,

$$k_p = \frac{a^2}{A},\ k_v = \frac{2a - B}{A} \qquad (17)$$

With the previous estimation of the Coulomb's friction torque, Γ_c, a compensation term is introduced in the system in order to eliminate the effect of this perturbation [4]. The compensation term is included in the control input voltage to the motor, and this is of the form:

$$\tilde{\Gamma} = \frac{\hat{\Gamma}_c}{k}(-sign(\dot{\theta}_m)) = \frac{\mu \cdot J \cdot n}{k}(-sign(\dot{\theta}_m)) \qquad (18)$$

when $\dot{\theta}_m \neq 0$. When $\dot{\theta}_m \approx 0$, the compensation term is included as done in the previous equation (18) but changing the function $sign(\dot{\theta}_m)$ by $sign(V)$. Fig.8 depicts the trajectory tracking of the motor with the PD controller with numerical computation of the motor velocity θ_{tn} and with the algebraic computation θ_t. The two signals properly track the reference trajectory θ_t^* with good performance. In Fig.9(a) the tracking error of the mo-

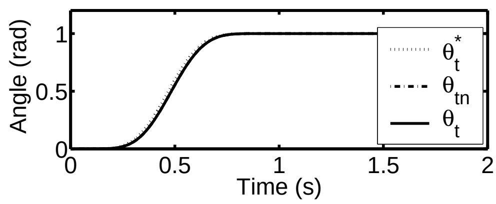
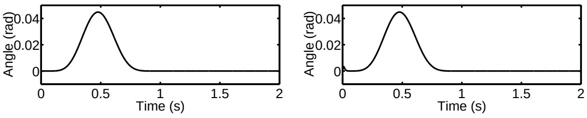


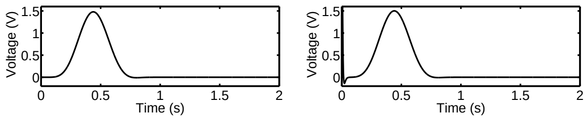
Figure 8: Trajectory tracking of the closed loop system. θ_t^* Command trajectory. θ_{tn} system response with numerical PD. θ_t system response with algebraic PD.

tor position when numerical PD is used is presented. By comparing such error with that of the Fig.9(b), in which the tracking error of the motor position with algebraic PD is depicted, we can observe that the two signals are the same in phase and magnitude. And it can be observe the same characteristic in Fig.10(a) and Fig.10(b) where the control input voltages to the DC motor are depicted. This accuracy tracking of both control schemes is due to the gains of the controllers which force the system to track the command trajectory by minimizing the error in the feedback. However, in real life we always find noises and errors which corrupt the measuring data, in this case, the encoder is not an infinite precisely measure system,



(a) (b)

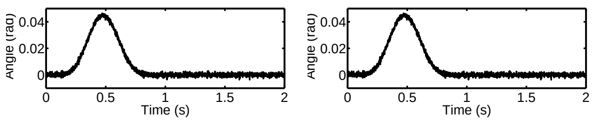
Figure 9: Tracking errors of the closed loop systems. (a) Tracking error with numerical PD implementation. (b) Tracking error with algebraic PD implementation.



(a) (b)

Figure 10: control input voltages to the DC motor. (a) With numerical PD implementation. (b)With algebraic PD implementation.

therefore, noises are include the control system due to the limited precision of the apparatus. We consider a noise corrupting the data with zero mean and 10^{-3} standard deviation as considered in the previous simulations. The trajectory tracking of the motor position with numerical PD θ_{tn} and with algebraic PD θ_t with noisy measure is similar to those of the Fig.8. Both trajectories follow properly the reference θ_t^*. Fig.11(a) and Fig.11(b) depict the tracking errors of the previous signals respectively. Note the noise is introducing an aleatory component in the error. Although the two errors has the same amplitude the control input voltage to the DC motor of the numerical PD has not a smooth shape, on the contrary, such a voltage would saturate the amplifier which has the limits in ± 10 (V) (see Fig.12(a)). In contrast, the control input voltage, when algebraic PD is used, has a smoother profile with much less control effort, therefore, such a signal would never saturate the amplifier. As a consequence, the amplifier would not suffer overheating.

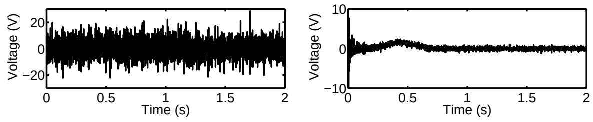


(a) (b)

Figure 11: Tracking errors of the closed loop systems with noise in the measure. (a) Tracking error with numerical PD. (b) Tracking error with algebraic PD.

4 Conclusions

A fast, non-asymptotic algebraic state estimation method has been successfully applied to estimates the Coulomb ´s friction coefficient. Performance studies show that the



(a) (b)

Figure 12: control input voltages to the DC motor in the system with noise. (a) With numerical PD. (b)With algebraic PD.

algebraic method provides satisfactory estimates even in the presence of significant noise levels. In addition, the state feedback controller is designed. Closed-loop simulation runs show that the state estimates calculated by algebraic method can be used efficiently.

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