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Simplified Modeling of Thin-Walled Tubes with Octagonal Cross Section – Axial Crushing

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Abstract— **This article investigates the collapse characteristics of thin-walled tubes with octagonal cross sections during axial crushing. The tubes' axial crushing resistance (the relationship between crushing force and axial deformation) is described with a series of mathematical equations, which are derived applying global energy equilibrium theory. The derived axial crushing resistance is then used for developing simplified finite element models for the thin-walled octagonal tubes. The simplified finite element model is composed of beam elements and nonlinear springs, where the nonlinear springs were defined using the derived axial resistance and would be used to simulate the buckling behavior of the simplified model during crash analyses. The developed simplified models are used for crash analyses, and the results are compared to those from corresponding detailed models as well as from the published literatures. Relatively good agreement is achieved through these comparisons, and it shows that the simplified models can save much more computer resources and modeling labor compared to the detailed models. Explicit code LS-DYNA is used for all the modeling and simulation presented in this article**.

Index Terms— ***Thin-walled tube, octagonal cross section, nonlinear spring, simplified model***

INTRODUCTION

Thin-walled structures have been widely used in automotive industry and other engineering industries for the purpose of increasing energy absorption efficiency, weight reduction, and manufacturability. The crash behaviors of the thin-walled structures during axial collapse and their axial resistances have aroused a lot of interesting. Previous researchers have thoroughly investigated the crash behaviors of different types of the thin-walled structures and developed a series of mathematical equations to describe or predict the resistances. T. Wierzbicki and W. Abramowicz [1, 2] applied different modes of deriving the mathematical equations that accurately predict the axial resistances of the rectangular, square, and hexagonal section tubes. W. Abramowicz and N. Jones [3] also studied the crushing behavior of the circular thin-walled tubes when subjected to an axial impact during the crash.

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However, besides above thin-walled structures, the thin-walled octagonal tubes are also an important crashworthy structure due to its better characteristics in energy absorption and crushing [4]. Therefore it deserves special considerations and appropriate finite element models have to be developed to predict their crash behavior.

A. Mamalis, D. Manolakos and other co-researchers [5] simulated the crash behavior of steel thin-walled tubes subjected to axial loading, which has octagonal cross section. In this article, based on a developed method for predicting crush behavior of multicorner thin-walled tubes subjected to axial compression [2], a general equation is derived to estimate the axial resistance of the thin-walled octagonal tubes during the axial crushing. The derived axial resistance then can correctly predict the relationship between the axial load and the axial deformation of the octagonal tubes.

One of important applications of the derived thin-walled tubes' axial resistance is simplified modeling. Simplified modeling is an important modeling technique which has been extensively applied in early design stage for product evaluation, crashworthiness analyses, and computer simulation. Simplified computer model is a finite element model that composed of beam and spring elements. Compared to detailed model, it requires less modeling work and consumes less computer resources during the modeling and simulation. In developing simplified thin-walled tube models, the derived axial resistance is used to define the nonlinear spring elements, which simulate the buckling of the models. Y-C Liu and M. Day [6], H-S Kim [7], and P. Drazetic [8] have created qualified simplified models for the thin-walled tubes as well as an entire vehicle assembly and summarized general simplified modeling methodologies for such tube members. In this paper, the derived octagonal tube's axial resistance is applied to develop the simplified octagonal tube models. The developed simplified model is then used for crash analyses and the results are compared to those from the detailed model and from the published literature [5] for validation. Relatively good agreement is achieved through these comparisons and both the derived axial resistance and the developed simplified model are validated. The explicit FE code LS-DYNA is used for modeling and simulating the axial compression of the steel thin-walled octagonal tubes [9].

Axial Resistance

The method developed by W. Abramowicz and T. Wierzbicki [2] is used to predict the axial resistance of the thin-walled octagonal tubes. Based on [2], the axial crushing resistance for a multicorner thin-walled tube equals to the appropriate crushing force per one element times the number of corner elements, n. The mean crushing force per one element is

$$\frac{P_m}{M_0} = 3(A_1 A_2 A_3)^{1/3} (C/t)^{1/3} \frac{2H}{\delta_{ef}} \quad (1)$$

where

$$M_0 = (\sigma_0 t^2)/4 \quad (2)$$

In above equations, P_m is the mean crushing force; M_0 is fully plastic moment per unit length of section wall; A_1 through A_3 are coefficients and A_1 = 4.44, A_2 = π and A_3 = 2.30, respectively [2]; C is edge length of octagonal cross section; t is wall thickness; 2H is length of one plastic fold generated when the tube buckled (see figure1), δ_{ef} is called "effective crushing distance" and $\delta_{ef}/2H$ was measured as 0.73 from previous experimental observations [2]; σ_0 is the material's energy equivalent flow stress which can be approximated as: $\sigma_0 = 0.92\sigma_u$

Substitutes all the values into (1) and multiplies it by eight for the octagonal section, the axial resistance of the steel thin-walled octagonal tubes can be approximated by the function:

$$\frac{P_m}{M_0} = 97.77(C/t)^{0.4} \quad (3)$$

Substitutes (2) to (3), the mean crushing force can be obtained from the given octagonal tube's dimensions and material properties:

$$P_m = 24.44\sigma_0 t^{1.6} C^{0.4} \quad (4)$$

To find the length of plastic fold 2H, minimizing equation (1) with respect to H by using $\partial P_m / \partial H = 0$ yields

$$H = 0.97\sqrt[3]{tC^2} \quad (5)$$

The calculated P_m is the axial resistance of the octagonal tubes and will be used to determine the nonlinear spring element for developing the simplified model together with the calculated H.

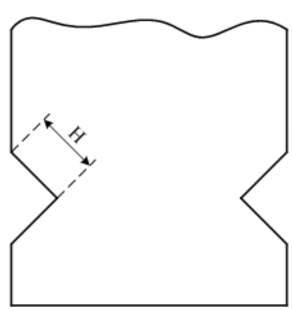

Figure1. Half plastic folding wave, H

Development of Simplified Model

Both detailed and simplified thin-walled octagonal tube models are created using LS-DYNA. The detailed model is composed of 4-node Belytschko-Tsay shell elements while the simplified model is generated based on the detailed model following Liu and Day's method [6], which only has beam elements and nonlinear springs. The derived axial resistance is applied for defining the nonlinear spring elements. Crash analyses are performed on both models to validate the developed simplified model as well as the presented axial resistance of the thin-walled octagonal tube. Relatively good correlations are achieved through comparing the analysis results from the simplified model to those from the detailed model and from the published literature [5].

Detailed Model

The detailed model for the thin-walled octagonal tube is shown in figure2, which is then used for the crash analysis. The detailed model is created using the full integration shell element: 4-node Belytschko-Tsay shell element with 5 integration points through the thickness. During the crash analysis, the beam model is fully constrained at one end. An initial velocity 1m/s is applied on the other end to make the model move along the Z direction. Table1 lists all the related conditions and properties of the thin-walled straight beam. In the crashworthiness analysis, all the geometries and material properties of this model and the impact conditions are the same as those from the published literature [5], therefore the obtained analysis results are comparable. After the analysis, the dynamic results are compared to those from the simplified model, which will be presented in the latter section.

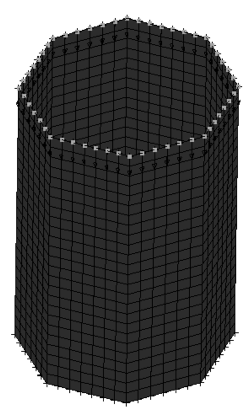

Figure2. Detailed thin-walled octagonal tube model

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Table1. Properties and impact conditions

Material Properties	
Young's modulus	207GPa
Density	7830kg/m^3
Yield Stress	250MPa
Ultimate Stress	448MPa
Hardening modulus	630MPa
Poisson's ratio	0.3
Geometries	
Total length	127mm
Cross section	Edge 31.8mm
Wall thickness	1.52mm
Impact conditions	
Added mass	280kg
Initial velocity	1m/s
Crash time	10sec

Simplified Model

Before creating the simplified model, the characteristic of the nonlinear spring elements is first defined. Substituting the material properties from table1 into equations (4) and (5), the length of the plastic folding wave is calculated as 2H = 22.39mm, and the mean crushing force is P_m = 78.5KN. The calculated axial resistant then is used to define the nonlinear spring; figure3 plots its F – δ relationship. From figure3, the developed nonlinear spring begins to deform when the crushing force reaches 78.5KN. After its deformation reaches 22.39mm, the spring fails and stops deforming.

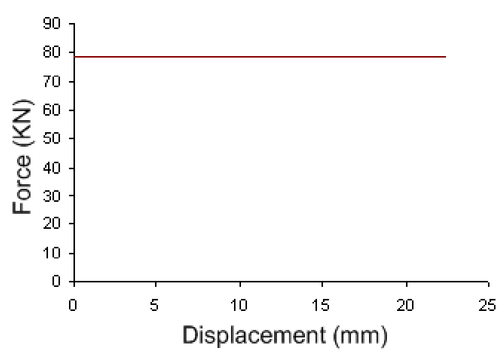

Figure3. F – δ curve of developed nonlinear spring

Since LS-DYNA doesn't include the octagonal section as its standard section type, therefore the octagonal cross section has to be developed by the users. The modeling method introduced by Len Schwer [10] is referenced here to develop the octagonal cross section for the beam elements. According to Schwer's method, in LS-DYNA cards, *SECTION_BEAM and *INTEGRATION_BEAM are modified to define the octagonal cross section. The Hughes-Liu beam with cross section integration is still used to define the beam elements and 24 integration points are defined along the octagonal cross section (figure4). For each integration point, its normalized s and t coordinates (-1 ≤ s ≤ 1, -1 ≤ t ≤ 1) and weighting factor are calculated and input to fully define the cross section. From figure4, the s or t coordinates equal to the integration point's coordinates (s_i, t_i) divided by the maximum coordinate of the cross section (S and T), and the weighting factor equals the area associated with the integration point divided by actual cross sectional area, which is 1/24.

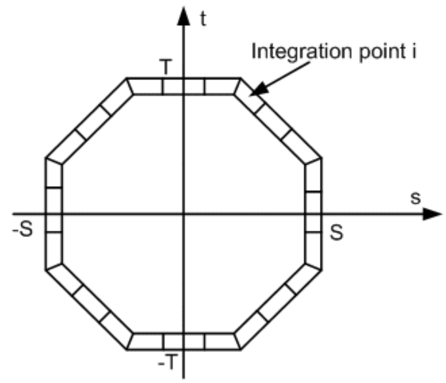

Figure4. Definition of integration points for octagonal cross section

The simplified finite element model (figure5) for the thin-walled octagonal tube is then created applying Liu and Day's method [6]. In the developed simplified model, the Hughes-Liu beam elements are used for building a pure beam-element model and the developed cross sectional information is assigned. Afterwards, the entire beam is divided into several equal segments whose length equals the length of one plastic fold, 22.39mm. Developed nonlinear spring elements connect these segments together, with the resistance displayed in figure3. Besides, appropriate boundary conditions are applied in order to ensure that the simplified model only deforms along its length.

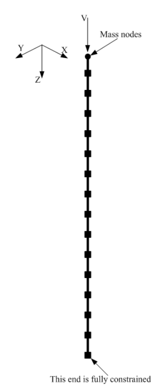

Figure5. Simplified thin-walled octagonal tube model

Analyses and Discussion

Same crash analyses are performed on both detailed and simplified models. The results come from both models are compared and shown in figure5 and table2.

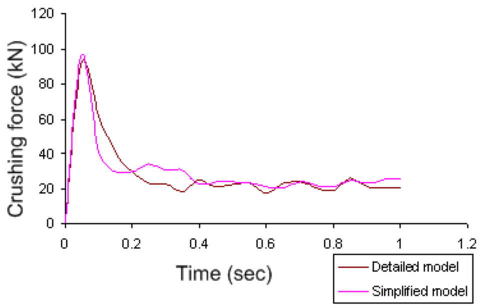

Figure5. Comparisons of crash results –thin-walled octagonal tube model, (a) deformed configurations, (b) crushing forces

Table2. Comparisons of crash results from detailed and simplified octagonal section tube models.

	Detailed model	Simplified model	Difference (%)
Peak crushing force (kN)	91.8	96.3	4.9
Mean crushing force (kN)	28.5	29.6	3.9
Elements	844	108	

From the results of the comparisons, it is verified that the developed simplified octagonal tube model is qualified for replacing the detailed model to be used for crash analyses. Figure5 verifies that during the analyses, the detailed and simplified models showed similar axial shortening and underwent similar crushing force history. Table2 shows that the numerical results yielded from the simplified model correlated to those from the detailed model very well. The errors between the peak and mean crushing force are below 10%, and the simplified model contains only about 1/8 of the number of elements of the detailed model. Also, in comparing the obtained force values to the published literature (from [5] the peak force is 92.9kN), it is seen that the detailed model created here yields same results as the model presented in [5], the tiny difference (about 1%) may be caused by the different techniques applied for creating the base plate and the impact interface.

Conclusions

This paper derives the axial resistance of the thin-walled octagonal tubes which are subjected to axial crushing. The derived resistance is used to develop the simplified thin-walled octagonal tube model. Through a series of numerical analyses and comparisons, the efficiency and accuracy of the developed simplified models are verified and the octagonal tube's axial resistance derived here is proved to be correct in predicting the collapse behavior of the thin-walled octagonal tubes during the axial crushing. The explicit code LS-DYNA provides lot conveniences in both modeling and simulation.

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