

Sliding Mode Impedance Control of Flexible Base Moving Manipulators Using Singular Perturbation Method

M. Salehi, G. R. Vossoughi

Abstract — In this paper, the general problem of impedance control for a robotic manipulator with a moving flexible base is addressed. Impedance control imposes a relation between force and displacement at the contact point with the environment. The concept of impedance control of flexible base mobile manipulator is rather new and is being considered for first time using singular perturbation and new sliding mode control methods by authors. Initially slow and fast dynamics of robot are decoupled using singular perturbation method. Slow dynamics represents the dynamics of the manipulator with rigid base. Fast dynamics is the equivalent effect of the flexibility in the base. Then, using sliding mode control method, an impedance control law is derived for the slow dynamics. The asymptotic stability of the overall system is guaranteed using a combined control law comprising the impedance control law for the slow dynamics and a feedback control law for the fast dynamics.

This proposed Sliding Mode Impedance Controller (SMIC) was simulated for an advanced 10 DOF's Flexible Base Moving Manipulator (FBMM) composed of a 4 DOF's manipulator and a 6 DOF's moving base with flexibility. This controller provides desired position/force control accurately with satisfactory damped vibrations especially at the point of contact.

Index Terms— Impedance Control, Moving Base, Flexible base, Robot, Mobile Manipulator, Sliding Mode Control

I. INTRODUCTION

Robots with moving base such as macro/micro manipulators, space manipulators and URV's (underwater robotic vehicles) can be used for extending the workspace in repair and maintenance, inspection, welding, cleaning, and machining operations. Mobile manipulators have long been introduced as a way of expanding the effective workspace of robot manipulators. The assumption of base rigidity in these systems however, is often unreal and compliance of the base in most cases results in the loss of accuracy and limitations in achievable speeds. The source for base flexibility can be for example the suspension system and/or the internal structural flexibility of the base platform or joint/link

flexibility associated with a supporting manipulator/crane in a macro/micro type manipulator arrangement.

Mobile manipulators with flexible base can in general be land based, space or underwater type vehicles. In mobile manipulators, greater momentum and higher frequency vibrations produced at contact between end effector and environment provides even more impetus for dealing with such flexibilities. Achieving high performance interactive or non-interactive manoeuvres in such applications is possible only when the flexibility and base motion are both considered for in the control synthesis procedure. Simultaneous base and manipulator control in the presence of such flexibilities may be essential in many cases where the base is a floating platform. In which, the manipulator and the base motions are coupled (as in underwater ROVs or macro/micro space manipulators, Fig. 1) and the base cannot be locked in position. In land based configurations, simultaneous control of the base and the manipulator can enhance the application domain and improve the cycle time for both unconstrained and constrained manoeuvres.

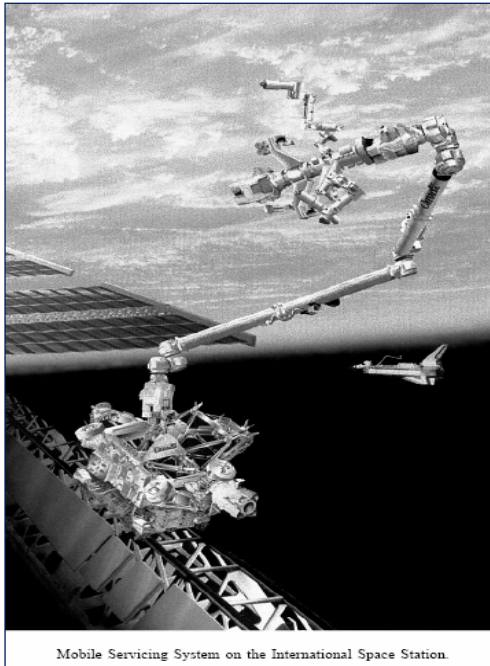
Researchers considered different control methods to improve the performance in flexible joint/link robotic systems. Modelling of flexible joint manipulators using singular perturbation method was first proposed by Khorosani and Kokotovic in 1985 [1]. Spong used perturbation method for dynamic modelling and control of manipulator with joint flexibility [2]. Singular perturbation is a unique systematic and mathematical tool for dealing with such flexibilities. This technique allows one to extract the slow and fast dynamics and formulate a separate control strategy for each subsystem. Thikhonov theorem [3] provides stability guarantees for the combined system. Among other methods, Lew introduced a simple robust control strategy for internal damping of mechanical vibrations for a manipulator with compliant (non-mobile) base [4].

As a space application, Finzi studied dynamic modeling and control strategies of mobile manipulator in space [5]. Hootsmans and Dubowsky addressed the joint motion control strategy of a macro/micro manipulator on a large mobile manipulator for improving the structural vibrations [6]. Torres and Dubowsky proposed a simple damping algorithm for errors associated with an elastically mounted space manipulator [7]. Mavroidis and Dubowsky proposed Inferred End-Point Control for long reach manipulator with base vibration [8]. These investigations are experimental and address error compensation of base vibrations without any stability and accuracy analysis.

This research was proposed and supported in Sharif University of Technology and Center of Excellence in Design, Robotics and Automation.

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Mobile Servicing System on the International Space Station.

Fig. 1. Macro/Micro Mobile Servicing Manipulator on the International Space Station

The pioneering work in stiffness /impedance control is by Salisbury and Hogen [9,10]. Kazerooni presented a frequency domain interpretation and design method, and proposed an implementation more suitable for use with industrial robots [11]. The problem of impedance control and dynamic stability of mobile manipulators (without flexibility) has been addressed by Inoue [12]. The concept of virtual / generalized impedance was proposed by Lao and Donath to avoid obstacles by redundant manipulators [13]. Modeling and impedance control of a two- manipulator system handling a flexible beam was addressed by Yan and Lin [14]. Multiple impedance control of cooperative manipulator in space was proposed by Mossavian, Papadaouplos and Poulakakis as an approach for handling large cargo in space [15]. To reduce contact forces in a mobile manipulators, simple damping-based posture control has been proposed by Kang and his colleagues [16]. Flexibility hasn't been considered in any of the above investigations.

Position/force control of flexible joint robots using singular perturbation method has been proposed by Hu [17]. Roy and Whitcomb used adaptive coefficients for force control law and he achieved better response using this control law [18]. A research group at DLR Aerospace Research Centre have studied impedance control of light link manipulators with fixed base and joint flexibility. They proposed a new approach based on decoupled dynamics of torque and position errors [19,23]. Subudhi addressed dynamic modeling and control of manipulators with combined joints and links flexibilities using singular perturbation method [20]. Impedance control of rigid mobile manipulator was studied by Tan and his colleagues [21] and experimental results were presented with a mobile PUMA 560. Hang proposed a fuzzy control law for impedance control and was able to achieve a better response when impedance parameters were selected based on fuzzy rule base [22]. Vossoughi and Karimzadeh addressed the

general impedance control of a flexible link manipulator using singular perturbation method and they presented simulation results of impedance control for a 2 DOF manipulator with fixed base [24].

Vibration of flexible base is according to situations of the Perturbation Theorem, because base flexibility of FBMM for all applications including suspension system, tyre or structural flexibility is less than 0.001 totally ($K \geq 1000 N / m$). Therefore, proposed SMIC using singular perturbation method can be used for all applications of FBMM.

II. DYNAMIC MODEL

Consider following general dynamics of flexible moving mobile manipulator (FBMM) by (1);

$$\tau = M(X)\ddot{X} + C(X, \dot{X})\dot{X} + K(X)X + G(X) + N(u_r, \dot{u}_r)$$

$$X = [y, x_1, x_2, \dots, \theta_1, \theta_2, \dots]^T$$

$$\tau = [0, F_{x1}, F_{x2}, \dots, \tau_1, \tau_2, \dots]^T \quad (1)$$

Where, y is base flexibility vector, x_1, x_2, \dots are base DOF's, $\theta_1, \theta_2, \dots$ are angular movements of manipulator links, F_{x1}, F_{x2}, \dots are applied force to base and τ_1, τ_2, \dots are applied torques to links. M, C, K, G, N represent inertia matrix, damping and centrifugal and Coriolis terms matrix, stiffness matrix, gravity matrix and matrix of road input to base or so on.

Motion equations are decoupled as below:

$$\begin{bmatrix} 0 \\ \tau_\Theta \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \ddot{\Theta} \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} \dot{y} \\ \dot{\Theta} \end{bmatrix} + \begin{bmatrix} Ky \\ 0 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} + \begin{bmatrix} -Ku_r - c\dot{u}_r \\ 0 \end{bmatrix}$$

$$\Theta = [x_1, x_2, \dots, \theta_1, \theta_2, \dots]^T \quad (2)$$

Where, u_r is road input or corresponding variable as input to the base. Now, new parameters μ (base flexibility), Q and ζ (quasi-steady state) will be defined as following relations.

$$\mu = 1 / K(X) \quad \text{as } \mu, K \text{ are Scaler}$$

$$\mu = K(X)^{-1} \quad \text{as } \mu, K \text{ are Matrix}$$

$$Q = y - u_r = \mu h^{-1} \zeta \quad (3)$$

K is stiffness coefficient or matrix and M^{-1} is inverse matrix of M . This matrix is considered for decoupling the slow and fast dynamics. h is a scaling factor.

$$M^{-1} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \quad (4)$$

Decoupling relations are as below:

$$\begin{aligned} \ddot{\Theta} &= H_{22}\tau_\Theta - H_{21}[C_{11}(\mu h^{-1}\dot{\zeta} + \dot{u}_r) + C_{12}\dot{\Theta}] - H_{22} \cdot \\ &[C_{21}(\mu h^{-1}\dot{\zeta} + \dot{u}_r) + C_{22}\dot{\Theta}] - H_{21}h^{-1}\zeta + H_{21}c\dot{u}_r \\ &- H_{21}G_1 - H_{22}G_2 \end{aligned} \quad (5)$$

$$\begin{aligned} \mu h^{-1} \ddot{\zeta} &= H_{12} \tau_{\Theta} - H_{11} [C_{11} (\mu h^{-1} \dot{\zeta} + \dot{u}_r) + C_{12} \dot{\Theta}] - \\ &H_{12} [C_{21} (\mu h^{-1} \dot{\zeta} + \dot{u}_r) + C_{22} \dot{\Theta}] - H_{11} h^{-1} \zeta + H_{11} \dot{c} \dot{u}_r \\ &- H_{11} G_1 - H_{12} G_2 - \ddot{u}_r \end{aligned} \quad (6)$$

III. SLOW DYNAMICS

Slow dynamics is equivalent dynamics with rigid base. In this case, flexibility coefficient of base is considered infinite parameter and in result, μ is zero. $\mu = 0$ is substituted in relation (6). We defined a new equivalent static parameter $\bar{\zeta}$ in this case. Also, $\bar{\tau}_{\Theta}$ is equivalent torque vector for slow dynamics.

$$\begin{aligned} \mu = 0 \Rightarrow \\ \bar{\zeta} &= h H_{11}^{-1} [H_{12} \bar{\tau}_{\Theta} - H_{11} (C_{11} \dot{u}_r + C_{12} \dot{\Theta}) - H_{12} \cdot \\ &(C_{21} \dot{u}_r + C_{22} \dot{\Theta}) + H_{11} \dot{c} \dot{u}_r - H_{11} G_1 - H_{12} G_2 - \ddot{u}_r] \end{aligned} \quad (7)$$

Finally, slow dynamics of FBMM is specified with substituting relation (7) into relation (5) :

$$\begin{aligned} \bar{\tau}_{\Theta} &= \hat{M} \ddot{\Theta} + \hat{C} \dot{\Theta} + \hat{G} \\ \hat{M}, \hat{C}, \hat{G} &\text{ are corresponding matrixes for slow dynamics.} \end{aligned} \quad (8)$$

IV. FAST DYNAMICS

Fast dynamics is equivalent dynamic for FBMM with flexibility. Perturbation parameter, ε and new state variables are considered as following relations.

$$\begin{cases} \mu \text{ is scalar} \Rightarrow \varepsilon = \sqrt{\mu} \\ \mu \text{ is matrix} \Rightarrow \mu = \varepsilon \varepsilon^T = L_{\mu} D_{\mu} L_{\mu}^T; \\ D_{\mu} \text{ diagonal} \Rightarrow \varepsilon = L_{\mu} \sqrt{D_{\mu}} \end{cases}$$

$$Z_1 = h^{-1} \zeta, Z_2 = \varepsilon h^{-1} \dot{\zeta} \quad (9)$$

If μ is matrix, we can use Cholesky decomposition for calculating ε . Dynamic equations of slow and fast subsystems can be rewritten as following forms using singular perturbation method:

$$\begin{cases} \dot{X}_1 = X_2 \\ \dot{X}_2 = H_{22} \tau_{\Theta} - H_{21} [C_{11} (\varepsilon Z_2 + \dot{u}_r) + C_{12} X_2] - \\ H_{22} [C_{21} (\varepsilon Z_2 + \dot{u}_r) + C_{22} X_2] - H_{21} Z_1 + \\ + H_{21} \dot{c} \dot{u}_r - H_{21} G_1 - H_{22} G_2 \end{cases} \quad (10)$$

$$\begin{cases} \varepsilon \dot{Z}_1 = Z_2 \\ \varepsilon \dot{Z}_2 = H_{12} \tau_{\Theta} - H_{11} [C_{11} (\varepsilon Z_2 + \dot{u}_r) + C_{12} X_2] - \\ H_{12} [C_{21} (\varepsilon Z_2 + \dot{u}_r) + C_{22} X_2] - H_{11} Z_1 + H_{11} \dot{c} \dot{u}_r \\ - H_{11} G_1 - H_{12} G_2 - \ddot{u}_r \end{cases} \quad (11)$$

Let $s = t / \varepsilon$ be the fast timescale, $\eta_1 = Z_1 - h^{-1} \bar{\zeta}$ and $\eta_2 = Z_2$ be the standard fast state variables. Now, we have

final form of above relation. We will have following relation:

$$\begin{cases} \frac{d\eta_1}{ds} = \eta_2 \\ \frac{d\eta_2}{ds} = H_{12} \tau_f - H_{11} \eta_1 \end{cases} \quad \tau_f = \tau_{\Theta} - \bar{\tau}_{\Theta} \quad (12)$$

This is a linear state space system for fast dynamics. Unforced system is stable because H_{11} is a positive definite matrix. τ_f can be considered as control input for fast dynamics.

V. SLIDING MODE IMPEDANCE CONTROL (SMIC)

General dynamics of FBMM were decoupled using singular perturbation method. Singular perturbation method is the most important method for decoupling general systems including small perturbation parameters. Slow and fast dynamics will be controlled and then combined control law is proposed. We propose new impedance control method for slow dynamics using sliding method control law. Also, feedback torque control law is considered for asymptotic stability guarantee of fast dynamics.

Impedance control is a dynamic relation between position and force. Impedance relation indicates desired impedance by matrixes M_m, B_m, K_m, K_f :

$$\begin{aligned} M_m \ddot{e} + B_m \dot{e} + K_m e &= -K_f e_f \\ e &= x(t) - x_d(t) \end{aligned} \quad (13)$$

$$e_f = F(t) - F_d(t)$$

M_m, B_m, K_m, K_f are impedance positive definite matrixes and x_d, F_d are desired position and force vectors. Now, we consider combined sliding surface as following form:

$$s_c = \dot{e} + F_1 e + F_2 Z_c \quad (14)$$

So, following relation indicates compensating dynamics for combined sliding surface related to Z_c :

$$\dot{Z}_c = A Z_c + K_1 e + K_2 \dot{e} + K_3 e_f \quad (15)$$

K_1, K_2, K_3 are compensating positive matrixes. A is semi-negative definite matrix .

It must be considered $s = \dot{s} = 0$ for reaching to desired sliding mode:

$$\dot{Z}_c = -F_2^{-1} (\ddot{e} + F_1 \dot{e}) \quad (16)$$

$$Z_c = -F_2^{-1} (\dot{e} + F_1 e)$$

We will have following relation by substituting equation (16) into equation (15);

$$\begin{aligned} \ddot{e} + (F_1 - F_2 A F_2^{-1} + F_2 K_2) \dot{e} + (F_2 K_1 - F_2 A F_2^{-1} F_1) e &= \\ = -F_2 K_3 e_f \end{aligned} \quad (17)$$

K_1, K_2, K_3 are specified by comparison between two relations (17) and (13) as desired impedance relations;

$$\begin{aligned} K_1 &= F_2^{-1} M_m^{-1} K_m + A F_2^{-1} F_1 \\ K_2 &= F_2^{-1} M_m^{-1} B_m - F_2^{-1} F_1 + A F_2^{-1} \\ K_3 &= F_2^{-1} M_m^{-1} K_f \end{aligned} \quad (18)$$

Sliding mode law was defined as following relation:

$$\dot{s}_c = -F(s_c) = -k \cdot \text{sat}(s_c) - \alpha \cdot s_c - \beta \int_0^t s_c dt \quad (19)$$

Where α, β, k are positive definite and diagonal matrixes. Function sat is as below:

$$\text{sat}(s_c) = \begin{cases} \text{sign}(s_c) & |s_c / \phi| > 1 \\ s_c / \phi & |s_c / \phi| \leq 1 \end{cases} \quad (20)$$

This function causes to prevent the chattering specification of real state variables. Chattering coefficient, ϕ , is a positive definite vector for reducing the rate of sliding surface variable.

VI. IMPEDANCE CONTROL OF SLOW DYNAMICS

Now, we propose new sliding mode impedance control for slow dynamics. We specify tracking error and then we will achieve desired acceleration vector of links and base.

$$\begin{aligned} e &= x - x_d \\ \Rightarrow \dot{e} &= J(\Theta)\dot{\Theta} - \dot{x}_d \\ \Rightarrow \ddot{e} &= \dot{J}\dot{\Theta} + J\ddot{\Theta} - \ddot{x}_d \end{aligned} \quad (21)$$

Using sliding mode control law, desired acceleration vector is given by following relation:

$$\begin{aligned} \dot{s} &= -F(s) = -k \cdot \text{sat}(s) - \alpha \cdot s = \ddot{e} + F_1 \dot{e} + F_2 \ddot{e} \\ \Rightarrow \dot{J}\dot{\Theta} + J\ddot{\Theta} - \ddot{x}_d + F_1(J(\Theta)\dot{\Theta} - \dot{x}_d) + F_2 \ddot{e} &= -F(s) \\ \Rightarrow \ddot{\Theta} &= -J^{-1}(Ls + \dot{J}\dot{\Theta}) \end{aligned} \quad (22)$$

Where,

$$\begin{aligned} Ls &= F_2 A Z + F_2 K_1 e + (F_1 + F_2 K_2) \dot{e} + F_2 K_3 e_f \\ -\ddot{x}_d &+ F(s) \end{aligned} \quad (23)$$

J is corresponding Jacobian Matrix of FBMM slow dynamics and $\dot{J} = \partial J / \partial t$. J^{-1} is pseudo-inverse Jacobian matrix for redundant manipulator and is considered by following relation.

$$J^{-1} = J^T (J J^T + k_m E)^{-1} \quad (24)$$

J^T is matrix transpose of J and k_m and E are identity scale factor and matrix for redundancy management of FBMM.

Control torque/force vector has been given as following relation by substituting equation (22) into equation (8);

$$\bar{\tau}_{\Theta} = \tau_{slow} = -\hat{M} J^{-1} Ls + (\hat{C} - \hat{M} J^{-1} \dot{J}) \bar{\Theta} + \hat{G} \quad (25)$$

VII. CONTROL LAW OF FAST DYNAMICS

Now we have two reduced order subsystems in (8) and (12). Torque control law is used for stability guarantee and vibration damping. It is considered feedback control law as following form for vibration damping control of link torques and base control forces:

$$\begin{aligned} \tau_f &= \tau_{fast} = -K_p \eta \\ \eta &= [\eta_1, \eta_2]^T \end{aligned} \quad (26)$$

We can consider nonlinear observer for indicating the vector η by measurable state variables, y and \dot{y} using strain gauge sensors and accelerometer.

VIII. COMBINED CONTROL METHOD, SMIC

Combined control method is considered as following relation using singular perturbation theorem.

$$\tau = \tau_{slow} + \tau_{fast} \quad (27)$$

This relation provides desired impedance and vibration damping and stability guarantee of FBMM. According to Tikhonov's Theorem, real state variables converge to the slow/fast state variables with the order of ε as following relations:

$$\begin{aligned} X_1 &= \bar{X}_1 + O(\varepsilon), X_2 = \bar{X}_2 + O(\varepsilon) \\ Z_1 &= h^{-1} \bar{z} + \eta_1 + O(\varepsilon), Z_2 = \eta_2 + O(\varepsilon) \end{aligned} \quad (28)$$

As a result of singular perturbation method and Tikhonov's Theorem, if slow and fast dynamics are stable, stability of combined dynamics will be proved.

IX. SMIC FOR A FBMM MODEL

Complete and advanced model of FBMM is considered with application of welding, cleaning or so on. FBMM model is 10 DOF's Flexible Base Mobile Manipulator (FBMM) composed of a 4 DOF's manipulator and 6 DOF's moving base with tyre flexibility or springs of suspension system. Non-holonomic constraint was considered for manipulator's base. Steering is possible by torque control of rear wheels. Model has been shown in Fig. 2. Also, $[\theta_l \ \theta_r \ \psi_b]$ are dependent state variables. Where, state variables including fast and slow variables are;

Slow DOF's:

$$[x_b \ y_b \ \psi_b \ \theta_1 \ \theta_2 \ \theta_3 \ \theta_4]$$

$$\text{or} \quad [\theta_l \ \theta_r \ \theta_1 \ \theta_2 \ \theta_3 \ \theta_4]$$

$$\text{Fast DOF's: } [z_b \ \theta_b \ \varphi_b] \quad (29)$$

A) Motion Equations of 10 DOF's FBMM

Motion equations of this model were derived using two dynamic methods, Lagrange Method and Direct Path Method (DPM). They were compared, simulated and confirmed. We provided two general packages; FBMMLAG and FBMMDPM using MAPLE software for driving motion equations and exporting to the SIMULINK, MATLAB for dynamic and control simulation.

B) FBMM specifications

$$m_b = 20 \text{ kg}, a = 1 \text{ m}, b = 0.5 \text{ m}$$

$$m_1 = 5 \text{ kg}, m_2 = 5 \text{ kg}, m_3 = 3 \text{ kg}, m_4 = 2 \text{ kg}$$

$$\begin{aligned}
 L_1 &= 2 \text{ m}, L_2 = 2 \text{ m}, L_3 = 1.5 \text{ m}, L_4 = 1 \text{ m} \\
 I_{b_x} &= 1.67 \text{ kg.m}^2, I_{b_y} = 6.67 \text{ kg.m}^2, I_{b_z} = 8.3 \text{ kg.m}^2 \\
 I_{L_1} &= 1.67 \text{ kg.m}^2, I_{L_2} = 1.67 \text{ kg.m}^2, I_{L_3} = 0.56 \text{ kg.m}^2, \\
 I_{L_4} &= 0.17 \text{ kg.m}^2
 \end{aligned}
 \tag{30}$$

C) Initial conditions

$$\begin{aligned}
 [x_b \ y_b \ z_b \ \theta_b \ \phi_b \ \psi_b \ \theta_1 \ \theta_2 \ \theta_3 \ \theta_4]_{(t=0)} &= \\
 = \left[0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \frac{\pi}{3} \ \frac{\pi}{9} \ \frac{\pi}{9} \right]
 \end{aligned}
 \tag{31}$$

D) Equivalent Stiffness of suspension system or tyres

$$\begin{aligned}
 K(z_b) &= K_1 z_b + K_2 \\
 K_1 &= 1000 \text{ N/m}^2, K_2 = 10000 \text{ N/m}
 \end{aligned}
 \tag{32}$$

E) Desired Position and Force

Desired path is the motion of FBMM in direction x and y. Then End Effector of manipulator will contact the wall with stiffness $K=1000 \text{ N/m}$ in direction y at the specified point. Desired End Effector trajectory on the wall is a linear path as welding or cleaning process. Also, principle angles (orientation) of End Effector (Fourth link) on the wall, $\psi_b + \theta_1$ and $\theta_2 + \theta_3 + \theta_4$, must be constant and remained 85 and 0 degree as uniform application. Desired y position and force on the wall were selected as $y_d = 4.5 \text{ m}$ and $F_{dy} = 2.5 \text{ N}$. In this simulation, it is assumed that tangential and friction forces on the wall's surface are negligible.

$$\begin{aligned}
 \left[\theta_{1end} \ \theta_{2end} \ x_{end} \ y_{end} \ z_{end} \right] &= \\
 = \left[\psi_b + \theta_1 \ \theta_2 + \theta_3 + \theta_4 \ x_e \ y_e \ z_e \right] \\
 \left[\theta_{1end} \ \theta_{2end} \ x_{end} \ y_{end} \ z_{end} \right]_{des.} &= \\
 = \left[85^\circ \ 0 \ 0.1x_{time} \ 4.5m \ -0.25x_{time} \right] \\
 F_d y &= 2.5N
 \end{aligned}
 \tag{33}$$

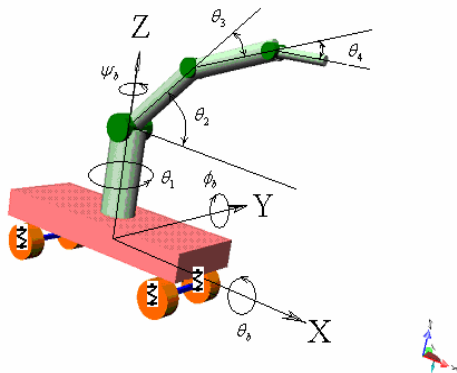


Fig. 2. Advanced FBMM model DOF's:

$$[x_b \ y_b \ z_b \ \theta_b \ \phi_b \ \psi_b \ \theta_1 \ \theta_2 \ \theta_3 \ \theta_4]$$

X. SIMULATION RESULTS

Simulation results are shown for sample 10 DOF's FBMM under SMIC. New proposed impedance control (SMIC) is used for reaching to specified and desired impedance (position, orientation and force). Fig. 3 shows position & principle angles of End Effector and Contact force. Base flexibility of FBMM for all applications including suspension system, tyre or structural flexibility is less than 0.001 totally ($K \geq 1000 \text{ N/m}$). Therefore, proposed SMIC using singular perturbation method can be used for all applications of FBMM.

Fig.3(a) shows that Y motion of End Effector was provided at 4.5 meter on the wall after contact. Principle angles of End Effector are shown in Fig. 3. Principle angles of End Effector ($\psi_b + \theta_1, \theta_2 + \theta_3 + \theta_4$) on the wall are fixed on desired value 85 and 0 degree by SMIC. They were shown in Fig. 3(b,c) and were provided before contact moment at 2.88 sec. This figure shows that all desired position and force are provided by SMIC. Finally, Fig. 4 shows that three dimensional motion of End Effector before and after contact. So, SMIC provided all desired position, orientation and force (impedance) for a real advanced FBMM model for industrial processes. Results show that SMIC provides desired path and contact force accurately as defined impedance parameters.

XI. CONCLUSION

The demand of Flexible Base Mobile Manipulator (FBMM) has risen in recent years and the applications are many and varied. This research proposed new Combined Sliding Mode Impedance Control (SMIC) for FBMM using singular perturbation method. FBMM applications include robotic manipulator mounted on the mobile vehicle in space, under water or on the land. These applications include position/force control requirements so on welding, cleaning, machine tooling, construction, finishing and inspection. Meanwhile, assumption of rigid base is unreal for all kind of FBMM. Impact value of mobile manipulators depends to both base motion and links masses which cause greater vibration on the flexible base at the contact point.

As new concept, combined control of slow and fast dynamics (SMIC) is proposed for impedance control using new application of sliding mode control law and singular perturbation method. SMIC guarantees asymptotic stability of FBMM. Of course, this new method of impedance control can be used as general impedance control method for every kind of FBMM.

10 DOF's FBMM model was considered including of a 4 DOF's manipulator and a 6 DOF's moving base with flexibility. SMIC provides desired path, orientation and contact force between End Effector and environment. It guaranties stability of slow and fast dynamics. Also it causes to damp any high frequency and domain vibration at the contact point completely. Contact force was damped at the contact point rapidly.

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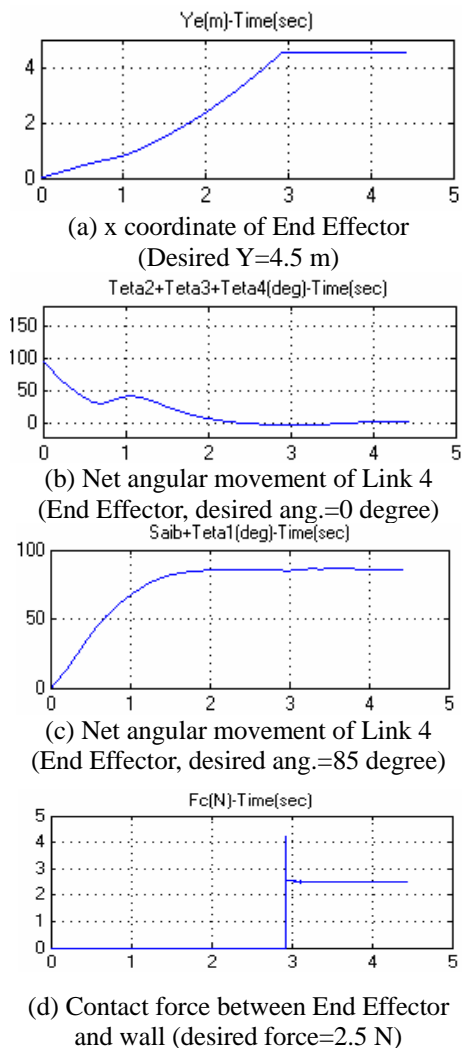


Fig. 3. Position/principle angles of End Effector and contact force

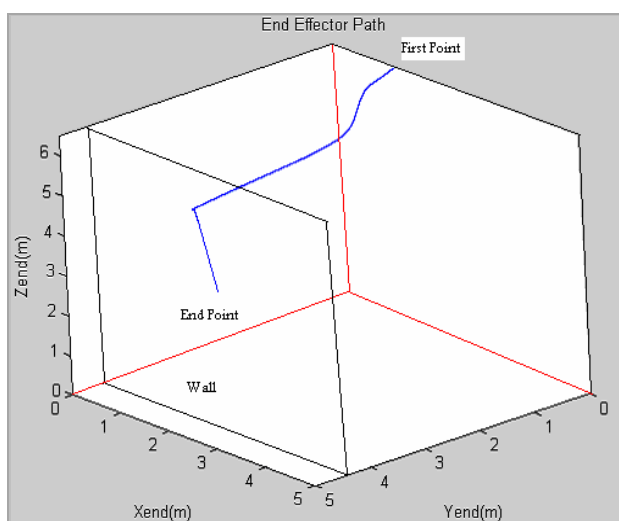


Fig. 4. 3D motion of End Effector

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