# Tracking Control of Robots Using Decentralized Robust Pid Control For Friction And Uncertainty Compensation

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ABSTRACT Dexterous and skilled motions in robot manipulators require reliable and robust joint controllers for achieving accurate joint motion tracking despite uncertainties in the robot dynamics, external disturbances, friction and unknown pay load. For the compensation of these effects a Decentralized Robust H∞ PID controller is applied to tracking problem of robotic manipulator, which guarantees arbitrary disturbance attenuation.

Index Terms -- robot, robust , friction ,PID, uncertainty

### I. INTRODUCTION

Mechanical Robot manipulators are subjected to uncertainties due to mass of the robot links, pay load changes, friction and external disturbances. For specialized applications Robot manipulators require reliable and robust joint controllers for achieving accurate joint motion tracking despite uncertainties. This need prompted considerable research efforts in the area of robot motion control.

The present paper is motivated by the fact that despite extensive research on robust motion control and adaptive motion control of robots, it is hard to find industrial robot manipulators that use these controllers. In fact most industrial robots employ simple independent PD or PID joint controllers [3]. The reason for such wide spread use of these controllers appears to be their simplicity and satisfactory performance for set point control. However this PID controller does not provide good performance in tracking applications such as painting, spraying and path following where joint angle q (t) must be close to its desired value during the entire trajectory. At lower speed the frictional disturbances affect the control performance and at higher speed the dynamic disturbances affect it. Hence control performance in such applications cannot be guaranteed in the presence of these disturbances with PID controller, as well as their stability also can not be guaranteed.

Therefore a natural alternative for the conventional PID controller is to provide the controller with robustness, whose design procedure for robust and accurate trajectory tracking does not require the knowledge of the robot dynamics. In this paper a robust  $H_{\infty}$  optimal linear PID tracking motion controller, which is easily tunable, guaranteeing a desired performance under the dynamic disturbances, friction disturbances etc. is applied for tracking controller is robust in the sense of  $L_2$  gain ( $\Upsilon$ ) attenuation from external disturbances. External disturbances in the form of static and dynamic friction models, pseudo randomly time varying noise and a parametric uncertainty such as change in the payload is applied to the

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Robotic system. Robustness of the controller (i.e. there is no increase in the tracking error) is observed against these disturbances. A comparative study of the performance of this robust PID controller with the conventional controller is made for the tracking control problem of robotic manipulator by taking different types of friction, disturbances and modeling uncertainty into account

The rest of paper is organized as follows- Section II presents robot dynamics with both static and friction models. Robust  $H\infty$  PID controller and conventional controller and their implementation for tracking control problem are given in section III. Simulation of tracking control problem of Robot Manipulators with frictions acting on it along with parametric variations and bounded disturbances is dealt in section 1V. Finally section V gives the conclusion.

#### **II. SYSTEM MODEL**

#### 2.1 Robot Dynamics

The dynamics of revolute joint type of robot can be described by following nonlinear differential equation [9]

$$T = M (q) \dot{q} + N (q, \dot{q}) - \dots (1)$$
  
where  $N(q,\dot{q}) = V(q,\dot{q})\dot{q} + G(q) + F_d(\dot{q}) + F_s(\dot{q}) + T_d$ 

With  $q \in R^n$  as the join position variables  $\in R^n$ , T as vector of input torques, M(q) is the inertia matrix which is symmetric and positive definite,  $V(q, \dot{q})$  is the coriolis and centripetal matrix, G (q) includes the gravitational forces,  $F_d(\dot{q})$  represents dynamic friction forces and  $F_s(\dot{q})$  are static friction forces acting independently in each joints and  $T_d$  is vector of unknown but bounded disturbances.

#### 2.2 Friction model.

The classical model for friction involves incorporating coulomb and viscous friction models. The following static model of friction is considered in this paper: [4]

$$F = F_v \dot{q} + F_c \operatorname{sgn}(\dot{q}) \quad -----(2)$$

Where Fv is a diagonal matrix with diagonal elements as static friction constants, similarly Fc is a diagonal matrix representing coulomb friction constants,  $\dot{q}$  is joint angle velocity for each link of the manipulator

As opposed to classical static friction model, dynamic friction models attempt to incorporate a variety of other friction characteristics such as stiction, zero slip displacement, stribeck effect etc. Dynamic friction models also tend to capture effectively the changing friction characteristics that are caused

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primarily due to wear and aging. The dynamic friction model known as Dhal model [4] in the sliding regime is given by Eq(3)

Where  $\sigma_0$ =stiffness coefficient,

*w*=external disturbances accounts for inadequacies of friction modeling

General friction model is given as a superposition of viscous friction and coulomb friction i.e. dynamic friction

 $F = \sigma_1 \dot{q} + F_d$ 

Where  $\sigma_1$  is viscous friction coefficient.

# III. CONTROL DESIGN FOR TRACKING CONTROL OF

# MECHANICAL MANIPULATORS WITH FRICTION

In the motion control of mechanical manipulator the primary concern is the tracking of the coordinates q to some desired trajectory qd (t), the error is defined as

$$\hat{M}(\ddot{q}^{d} + k_{v}\dot{e} + k_{p}e) + \hat{V}(\dot{q}^{d} + k_{v}e + k_{p}\int e) + \hat{G} - u = T \quad --(5)$$

where ( M, C, G ) is the estimated or nominal dynamic parameters of the exact (M,C,G). The disturbance W is formulated as below

If u is taken as zero in eq. (5) then this control law is known as the conventional computed torque law controller, which is globally stable when uncertainties and friction are not present (i.e.  $T_d=0,F_d=0,Fs=0$ )[3]

The following nonlinear  $H^{\infty}$  disturbance attenuating control u with the closed loop stability [2] can be applied to the reference error output passive system

$$u = -k (\dot{e} + k_{v} e + k_{p} \int e)$$
 -----(7)

Where  $k=1/\gamma^2$  and  $\gamma$  is  $L_2$  gain. For a given  $\gamma >0$  and for a set of  $k_p$  and  $k_v$  so that the inequality  $kv^2 > 2 k_{p \text{ is}}$  satisfied, the control u does not depend upon any dynamic parameters of the robotic system although  $H_{\infty}$  optimal control proposed by Acho et.al. does [6]. This is its major advantage, as due to this property it can be applied generally without any knowledge of the system dynamics, only it has to be Euler Lagrange system.

For decentralized  $H_{\infty}$  optimal control, if the nominal dynamics is given of the form  $(\hat{M}, \hat{C}, \hat{G}) = (\text{diag}\{\hat{m}_i\}, 0, 0)$ , and the reference error feedback

gain matrix k is set to diag  $(k_i)$ , then a decentralized control is obtained by combining (5) and (7) as

$$\tau_{i} = \hat{m}_{i} \quad \ddot{q}_{i}^{d} + (k_{v} \quad \hat{m}_{i} + k_{i}) \quad \dot{e}_{i} + (k_{p} \hat{m}_{i} + k_{v} k_{i}) e_{i} + k_{p} k_{i} \int e_{i}$$

$$i=1,.n$$

Eq.8 is a kind of decentralized PID control, but the control still has a property of H $\infty$  disturbance attenuation. For simplicity and decentralized structure  $\hat{M} = I$  is chosen.

#### IV. SIMULATION

The problem of tracking control of a planar robotic arm having two degrees of freedom, having both the joints as revolute is taken up in this simulation study. The dynamics of the manipulator satisfies Eq. (1) and is affected by various friction models as given in section 2.2, bounded disturbances and model uncertainties

The dynamic equation matrices for two-link manipulator [7] are of following form

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$$M(q) = \begin{bmatrix} 8.77 + 1.02 \cos(q2) & .76 + .51 \cos(q2) \\ .76 + .51 \cos(q2) & .62 \end{bmatrix}$$
$$\begin{bmatrix} -0.51 \sin(q2)\dot{q}_2 & -0.51 \sin(q2)(\dot{q}_1 + \dot{q}_2) \\ .62 \end{bmatrix}$$

$$V(q, \dot{q}) = \begin{bmatrix} -0.51\sin(q_2)q_2 & -0.51\sin(q_2)(q_1 + q_2) \\ 0.51\sin(q_2)\dot{q}_1 & 0 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} 74.48\sin(q1) + 6.174\sin(q1+q2) \\ 6.174\sin(q1+q2) \end{bmatrix}$$

Manipulator is commanded to track the following desired trajectory shown in figs (1 & 2) given by Eq.(9)

$$q^{d}_{1}=0.3 \sin (0.7t - \Pi/2)+0.3 \sin(0.1t - \Pi/2)+0.7$$
  
 $q^{d}_{2}=0.5 \sin (0.9t - \Pi/2)+0.5 \sin (0.1t - \Pi/2)+1.1$ 

Friction parameters chosen are as following.

$$\boldsymbol{\sigma}_{0} = \begin{bmatrix} 1 & 0 \\ 0 & 1.2 \end{bmatrix}, \quad \boldsymbol{\sigma}_{1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, F_{c} = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$$

And  $T_d$ =[-10; 20] as bounded disturbance torque

# Case1) tracking performance with no friction and uncertainties

For conventional computed torque controller of Eq.(5) with u=0, gain kp and kv are tuned to 100 and 20 respectively. Robust H $\infty$  PID controller given by Eq (8), is tuned with L<sub>2</sub> gain  $\gamma$  for the same kp and kv. The optimal performance is obtained for  $\gamma$ =0.0275.

Table1---Results for case1 (Ref.Figs.3-4 and 5-6)

case1 (Without friction)	Absolute value of max. Tracking error		2 norm of Tracking error		
	In joint angle1	In joint angle2	In joint angle1	In joint angle2	
Computed torque law	0.2549	0.7927	8.3983	20.4356	
Robust PID	0.0172	0.0053	0.3241	0.1296	

# Case2) with different friction models

With the same simulation set up as above, static and dynamic friction models given in section II. and fixed disturbance torque  $T_d$  are applied to the dynamic equation of the robotic manipulator. The purpose of this study is to investigate the controller under different friction models and fixed disturbances. The tracking error with conventional computed torque controller is shown in figs (3-4)- and with Robust H $\infty$  PID in figs (5-6). Tracking error in case of conventional controller increases whereas in robust controller it remains approximately the same with the application of friction and fixed disturbances.

#### Table2---Results for case2 (Ref.Fig 3-4 & Fig 5-6)

Case2 (With friction)		Absolute value of max. Tracking error		2 norm of Tracking error		
		In joint angle1	In joint angle2	In joint	In joint	
				angle 1	angle2	
Compu ted	static	0.3263	0.8704	8.5634	21.6851	
torque	dynami c	0.3978	0.7	9.623	17.2601	
Robust pid	static	0.0170	0.0063	0.3136	0.1345	
	dynami c	0.0172	0.0053	0.3241	0.1296	

Case3) with pay load changes / parametric uncertainty along with different friction models and torque disturbances The purpose of this study is to investigate the performance of controller under payload changes and pseudo randomly generated disturbance torque. Consider the same simulation set up as above except that mass  $m_2$  is perturbed to  $m_2 + \Delta m_2$  The uncertain parameter is set to 10% of its nominal value. Along with this, pseudo randomly generated torque disturbance is also applied. Tracking error for the two joints in case of computed torque is shown in figs (7-8) and with Robust  $H \approx$  PID is shown in figs (9-10) It has been observed that robust controller gives the robust performance whereas conventional controller performance deteriorates quite much with uncertainty.

Table3Results for o	case3 (Ref. I	Figs.7-8 & 9-10)
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Case3(with parametric variation)		Absolute value of max. Tracking error		2 norm of Tracking error	
		In joint angle1	In joint angle2	In joint angle1	In joint angle2
Compu ted torque	No friction	0.2588	0.8038	8.5484	20.703
	static	0.3293	0.9006	8.6243	22.320
	dynamic	0.4048	0.7280	9.7650	17.78
Robust PID	No friction	0.0188	0.0059	0.3513	0.1428

# V. CONCLUSION

Decentralized Robust H $\infty$  PID controller is implemented for accurate trajectory tracking of mechanical robot manipulator. The controller has several important features. Its decoupled structure allows ease of implementation, fault tolerance and isolation. It guarantees robustness to different frictions, disturbances and payload changes without requiring any specific knowledge of robot dynamics or parameters, which is verified by extensive simulation studies under a variety of conditions.

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Fig 1 Desired Trajectory of joint angle1



Fig 3 tracking error in joint angle 1 with friction with computed torque



Fig 5 tracking error in joint angle 1 with friction with robust PID



Fig 2 Desired Trajectory of joint angle2



Fig 4 tracking error in joint angle 2 with friction with computed torque



Fig 6 tracking error in joint angle 2with friction with robust PID



Fig 7 tracking error in joint angle 1 with friction and parametric variation with computed torque



Fig 9 tracking error in joint angle 1 with friction and parametric variation with robust PID



Fig 8 tracking error in joint angle 2 with friction and parametric variation with computed torque



Fig10 tracking error in joint angle 2 with friction and parametric variation with robust PID