The Most Accurate Estimation For The Total Arc Length Of The Astroids On The Positive Cartesian

Necat Tasdelen

Abstract

This work is targeting to find the perimeter of an ellipse. It is not only for the ellipse, but for the total arc length of all the astroids on the positive Cartesian. No algebraic calculi were used but only graphical solutions. It is based on the overlapping of the real graphs with the model graphs. That is a simulation. *The real graphs are known* [5]. In this concern, the equation:

(x/a)^r+(y/b)^r=1 is considered
(r=2) case of an ellipse is studied
The overall error % =**0.000004753119960** is reached.

The actual record for (error %) being =**0.00145** [1], this is a shocking result.

Why such estimations when exact values exist? Exact values do not exist, except for (r=2)
Why estimation? It is necessary for the people's understanding. Kepler, Euler, Ramanujan were academicians but also they were close to the people's understanding of science.

The history of this work starts in year 1956. In 1959, I gave my one-line estimation formula **(a^s+b^s=L^s)** to my math-Prof.Weiyrich who refused it [2], due to his wrong comment. In 2000 this formula was registered in a tricky style to the name of Roger Maertens [3]. Since then the formula was attempted for correction by researchers who never succeeded to comment it, due to their insufficient knowledge about the prove [4] of the formula. Think about this illegal registration when it is said to belong to Hölder. Also search for a prove document similar to [4] for Hölder. Here you will discover my comments on my formula.

Keywords: shocking-reasoning, accurate-estimation, arc-length

Introduction

This work is about a new reasoning to reach to the most accurate approximation for the perimeter of an ellipse. It is valid for the **total** arc length, on the positive Cartesian, for all the astroids expressed by the equation:

necattasdelen@ttmail.com
TURKEY

(x/a)^r+(y/b)^r=1 (1)

Case	r=2	is an ellipse
Case	r=1	is a line segment
Case	r=2/3	is an astroid
Case	r=9783.01	is also an astroid.

Astroid is the general name for (1)

(a,b)	are the semi-axes lengths
(r)	is the power of the astroid.

The relation to reach to this shocking accuracy is expressed by the one-line formula:

(a^s+b^s=L^s) (2)

Today, there are some people who still insist that this formula has been sent from the Sky [3]. No. It has a reaching reasoning, an algebraic solution [4], dated from 1956-1959 and resumed as follows:

Consider the astroid family
(x/a)^r+(y/b)^r=1 (3)
When we have a relation
f(a,b)=0 (4)
we may speak of an envelope for this family.
The envelope is freely chosen in the form

(x/A)^t+(y/B)^t=1 (5)
where t=t(x)

(5) is not an astroid, except (t=Constant)

Say (B/A=E)
Say (r*t/(t-r))=s (6)

Then, the relation (5) is written as:

(A^t-x^t*(1-N))^(s/r)*A^t=a^s*N^(s/r)+(b/E)^s (7)

where

N=(1+(dt/dx)*(x/t^2)*(t*ln(x/A)+(y/x)^t*(A/B)^t*t*ln(y/B)))(8)

Suppose we think about the envelope of (3), when it has **constant total arc length** on the positive Cartesian. When an arc length is at the research target, the relation (5) is solvable with approximation methods as follows:

ISBN: 978-988-98671-0-2

Consider the classic segment expression
dL^2=dx^2+dy^2 (9)

Here, for all approximation, (d) will means (delta), then we write

dL=(1/n)*(a^2+b^2*((n^r-(i+1)^r)^(1/r)-(n^r-i^r)^(1/r))^2)^(1/2) (10)

where

n= the total segments quantity in the positive Cartesian
i= intermediary segments quantity, finally (i=n).

The relation (4) is symmetric when we treat **total** arc lengths:
-(a and b) may change their position, their places, in the evaluation of the **total** arc length.
The relation (4) is written with the same parameters as long as (b/a=TAN) is constant.
-the "**lieu**" of the touching point of the astroid with the envelope is a line
Then we write:

(dt/dx=dt/dTAN*dTAN/dx=0) for a given set of (a,b). That is (b/a=TAN=Constant) .And we get

N=1
A^s=((a*E)^s+b^s)*E^s (7 solved)

When (A=B), say (A=B=K) and (K/a=L1) we write

a^s+b^s=K^s and with (b/a=TAN)
1+TAN^s=L1^s (11)
Where
L1=unit *total* arc length on the positive Cartesian.

Here, the power (s) is to be commented. The expression (6) indicates that (s) is a variable, that each ellipse has its own (s).But, for a coarse estimation, we may agree that (s) is a constant
Then we write,

R^s+R^s=(R*L1)^s

and knowing (L1=Pi/2) for a circle, the orthogonal case of the ellipse, we find

s=ln(2)/ln(L1)=1.5349853566138…

For a line segment (r=1) we will find
s1=ln(2)/ln(L1)=2 knowing (L1=2^(1/2))
For a classic astroid (r=2/3) we will find
s2/3=ln(2)/ln(L1)=1.709511… knowing (L1=1.5)

The fine estimation needs that (6) be commented correctly. In this concern, we evaluate (L1) with (n) segments and find the graph of (s).This is a *real graph* [5].

Use, at least (n=5 000 000 000) segments for an accurate (L1).There are no integral calculi.
This reasoning is general for the total interval (0<r<infinity).

Figure (1) shows the coarse graph (stage0) for (dL1=L1Estimated-L1Real) with s=1.5349853……
Figure (2) shows the error % graph at stage(0)

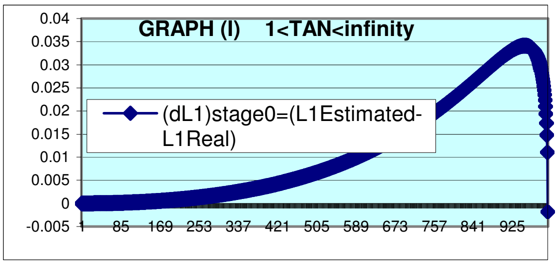

Fig.1 shows the coarse dL1 graph at (stage 0)

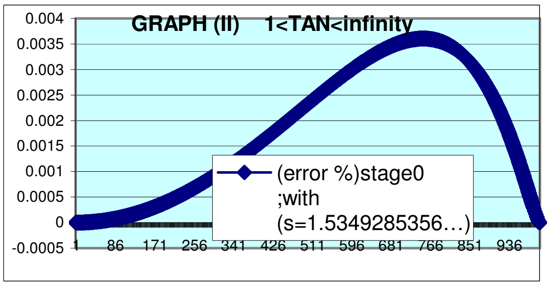

Fig.2 shows the error % graph at (stage 0)

Max. (dL1)stage0 =0.034155353209052
at TAN=b/a=25.45169958
Max.(error %)stage0 =0.003605936813090
at TAN=b/a=5.006784983

Figure (3) shows (sReal) graph for the ellipse. The Real graphs are **known** [5]

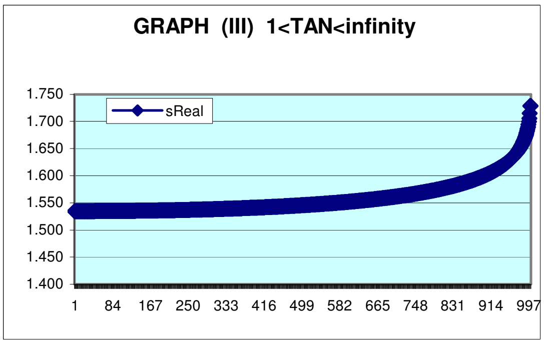

Fig.3 shows (sReal) graph for the ELLIPSE

This graph looks like an astroid, seated on a polynomial curve. So, we write an astroidal math-model with which we try to overlap the real graph (s)

Here are the math-model (sMod) and its parameters. Parameters are known [5]. Only (p) is estimated. This is a personal estimation. Figure (4) shows the overlapping, with (p=2.98)

sMod=d1+b1*(1-((x-c1)/a1)^p)^(1/p)+(F+m1*x)

Table 1 shows the parameters and their values for (stage1)

	AUTOMAT 1
parameters	**values**
a1	1000
(sm-sM)=b1	-0.193966223722475
c1	0
d1	0.000000000000000
p	***2.980000000000000***
sM=F	1.728894759383850
m1	0.000000000000000
sm	1.534928535661380

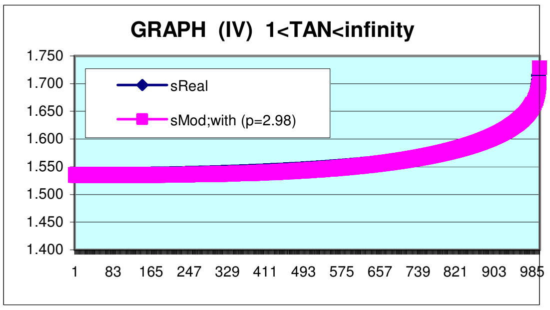

Fig .4 shows the overlapping of the graphs

(x) is the angular absis of (TAN=b/a), so that:
when (TAN=1,x=0) and when (TAN=infinite, x=1)
Practically, we use x=1000
When TAN=b/a=infinite. For this:
divide the angle (90o-45o) in 1000 linear (d alpha) and write:

(Angle alpha=Radian alpha*180/Pi) and
[x=(angle apha-45o)/(45o/1000)]

For unit evaluation of L1,it is evident that (a=1;b=TAN) and (1<TAN<infinity)

Graph (IV) shows a very nice overlapping,with (p=2.98) but we have to control the error %.

The error % is defined as:

Error %=(L1Estimated-L1Real)/L1Estimated

Figure (5) shows the error % graph, at this first stage of the approximation, with (p=2.98).

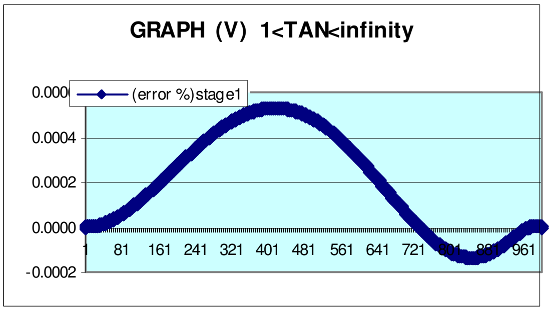

Fig.5 shows the error % graph at (stage1)

We have (max.error %) **=0.000529950426610**.
Better than the actual world record of (max.error %) **=0.00145**

With another value of (p),we will get another graph .We can diminish the error % where we want it diminished. Or equalize the max-min error %.
Elasticity in reasoning !

dL1=(L1Estimated-L1Real) is much more important. See Fig. (VI).

Max. dL1 (stage1) =0.001353949406581

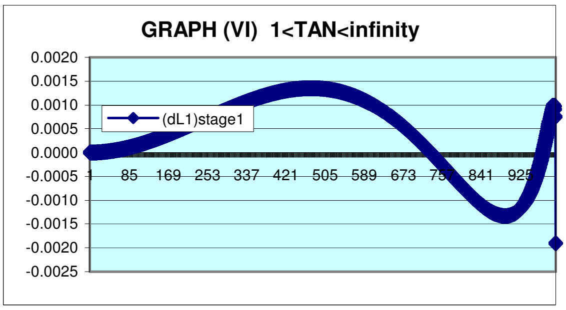

Fig.6 shows dL1 graph at (stage1)

Knowing (error % and dL1) at any point (at any given TAN) we can evaluate (L1Real).
(L1Real) will not change according our chose of the parameter (p).

Example: at the max.error point, where (x=415), we have
dL1 =0.001294322464019 and
error % =0.00052995042661. Then,

L1Estimated=(dL1/error %) =2.44234630076549
will vary according (p)
L1Real=(L1Estimated-dL1)=2.4410519783147
will stay constant

ISBN: 978-988-98671-0-2

When considering the graphs (I to VI) we think that (p) should be a variable to have (dL1=0)

So, we get the graph (VII) for (pReal) which will make (dL1)stage1=0 **at every point.**

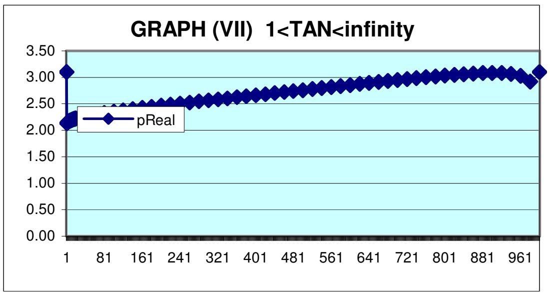

Fig.7 shows the (pReal) graph

This graph looks like an astroid seated on a polynomial curve. We write a (pMod) in order to overlap the (pReal).Here are the math-models and its parameters. Figure (8) shows the overlapping

pMod=d2+b2*(1-((x-c2)/a2)^q)^(1/q)+(G+m2*x)

sMod=d1+b1*(1-((x-c1)/a1)^**pMod**)^(1/**pMod**)+(F+m1*x)

Table 2.shows the parameters and their values for (stage2)

	AUTOMAT 2
parameters	values
a2	500
b2	0.3475
c2	500
d2	0.000
q	4
G	1.95
m2	0.000935

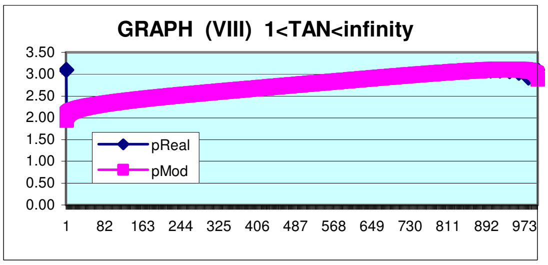

Fig .8 shows the overlapping

Knowing (a=1; b=TAN), we know the (angle alpha) and (x)
We use (pMod) values in (sMod) and we get

L1Estimated=(1+TAN^**sMod**)^(1/**sMod**)

At this stage2, the (error %)stage2 and (dL1)stage2 are shown on the graphs (IX and X)

Maximum (error %)stage2 =0.000019980316582
(dL1)stage2 =0.000592851604697
at the max-error point.
Maximum (dL1)stage2 =0.001153601015915
for the range (1<TAN<infinity)

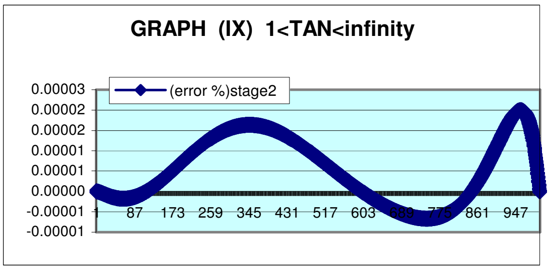

Fig.9 shows the error% at (stage2)

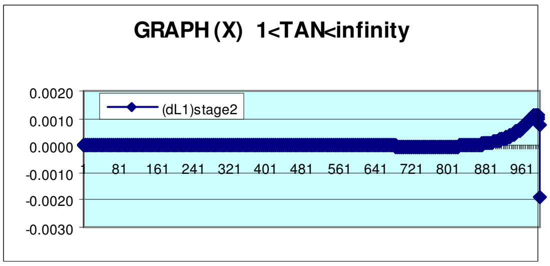

Fig.10 shows dL1 graph at (stage2)

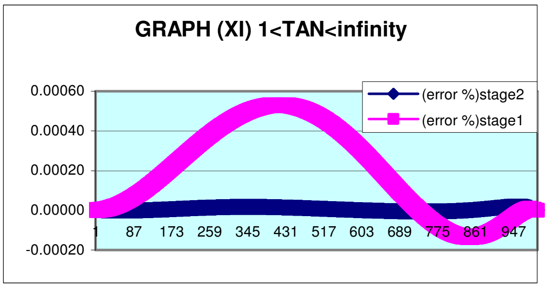

Fig.11 is a comparison of the error graphs

Error % (stage2) is better than the error % (stage1)
This means: the parameters were estimated correctly

When this accuracy level is not sufficient, we may go one step ahead.
In this concern,the parameters (a2,b2,c2,d2,q,G,m2) of (pMod) should be variables for a finest evaluation. We get their (Real graphs) [5] when we write:

(dL1)stage2=0 for (b2)
(dL1)stage3=0 for (m2)
(dL1)stage4=0 for (G)

We write (math-Models) for these parameters. They all look astroidal, seated on a polynomial curve. Respective math-Models and the parameters are as follow:

b2Mod=d3+b3*(1-((x-c3)/a3)^r)^(1/r)+(H+m3*x^v3+n3*x^w3)
pMod =d2+**b2Mod***(1-((x-c2)/a2)^q)^(1/q)+(G+m2*x)
sMod =d1+b1*(1-((x-c1)/a1)^**pMod**)^(1/**pMod**)+F+m1*x)

Table 3 shows the parameters and their values for (stage3)

	AUTOMAT 3
parameters	values
a3	500
b3	0.330000
c3	500
d3	0
r	5
H	0.650000
m3(+)	0.000038
v3	1
n3 (-)	0
w3	1

m2Mod=d4+b4*(1-((x-c4)/a4)^t^(1/t)+(J+m4*x^v4+n4*x^w4)
pMod =d2+**b2Mod***(1-((x-c2)/a2)^q)^(1/q)+(G+**m2Mod***x)
sMod=d1+b1*(1-((x-c1)/a1)^**pMod**)^(1/**pMod**)+(F+m1*x)

Table 4.shows the parameters and their values for (stage4)

	AUTOMAT 4
parameters	values
a4	500
b4	0.0009381
c4	500
d4	0
t	10
J	0.000001
m4(+)	0.0000000700
v4	1
n4(-)	-0.000000065
w4	1

GMod=d5+b5*(1-((x-c5)/a5)^u)^(1/u)+(K+m5*x^v5+n5*x^w5)
pMo=d2+**b2Mod***(1-((x-c2)/a2)^q)^(1/q)+(**GMod+m2Mod***x)
sMod=d1+b1*(1-((x-c1)/a1)^**pMod**)^(1/**pMod**)+(F+m1*x)

Table 5 shows the parameters and their values for (stage 5)

	AUTOMAT 5
parameters	values
a5	500
b5	0.098750000
c5	500
d5	0
u	10
K	1.851440000000000
m5	0.000000010000000
v5	1.2
n5	0
w5	1

Here are their graphs

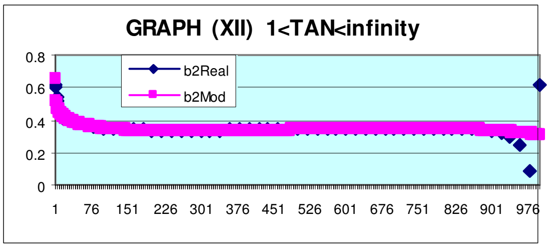

Fig.12 shows the overlapping

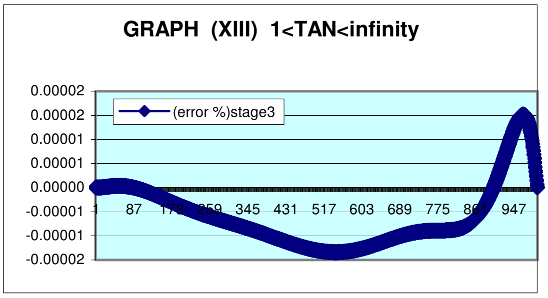

Fig.13 shows the error % graph at (stage3)

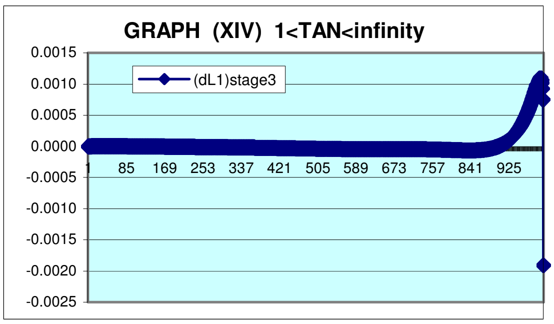

Fig.14 shows the dL1 graph at (stage3)

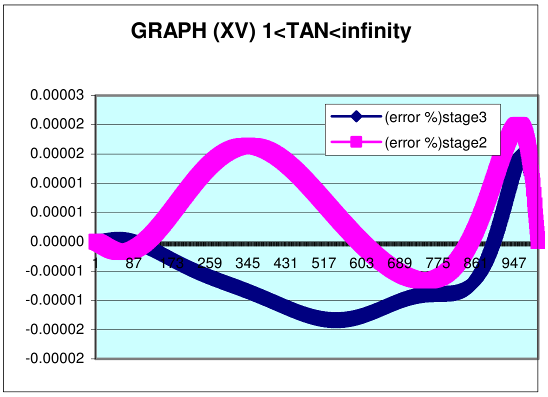

Fig.15 shows a comparison of error% graphs

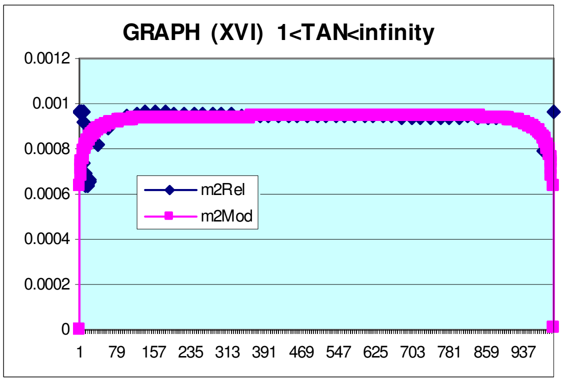

Fig.16 shows the overlapping

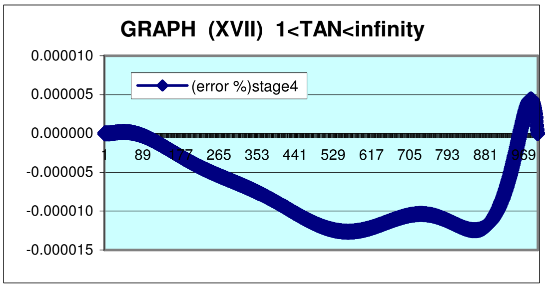

Fig.17 shows the error % graph at (stage4)

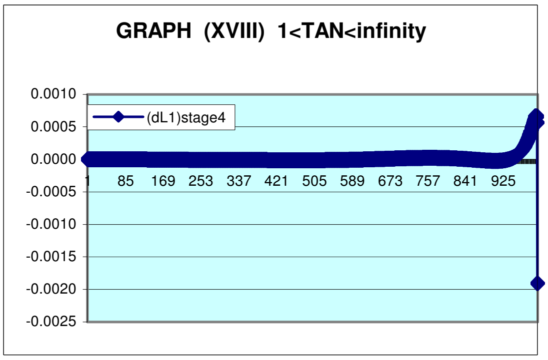

Fig.18 shows dL1 graph at (stage4)

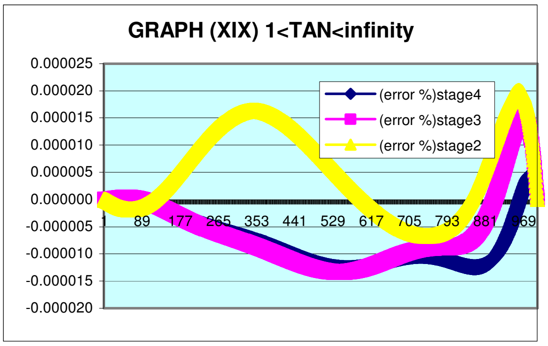

Fig.19 is a comparison of error % graphs

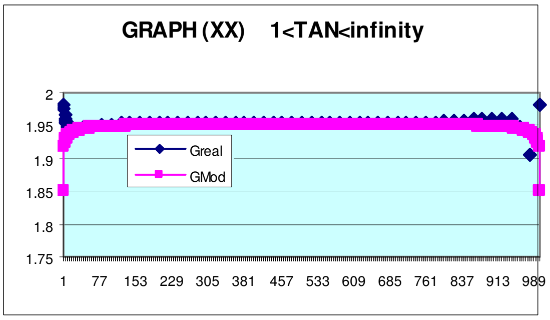

Fig.20 shows the overlapping

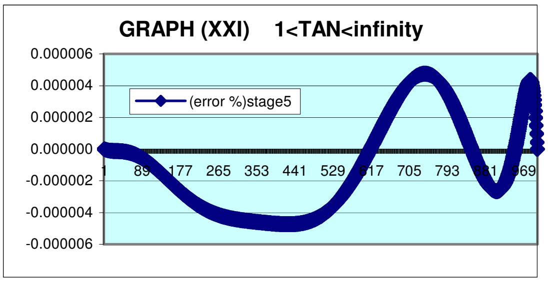

Fig .21 shows the error % graph at (stage5)

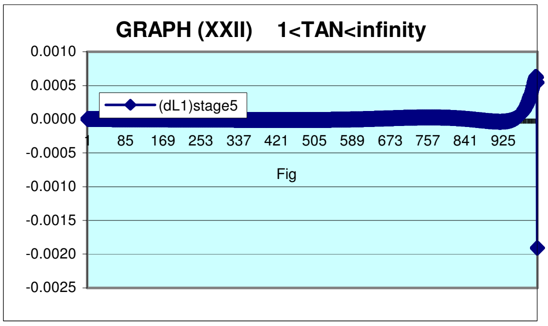

Fig.22 shows dL1 graph at (stage5)

ISBN: 978-988-98671-0-2

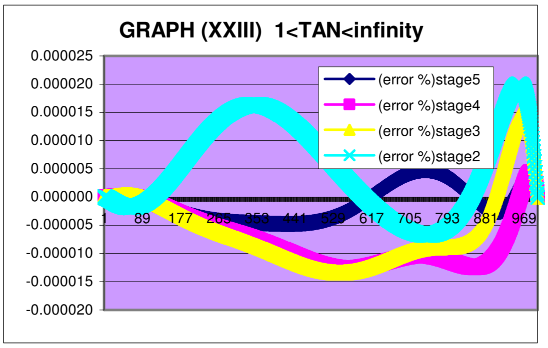

Fig.23 is a comparison of error % graphs

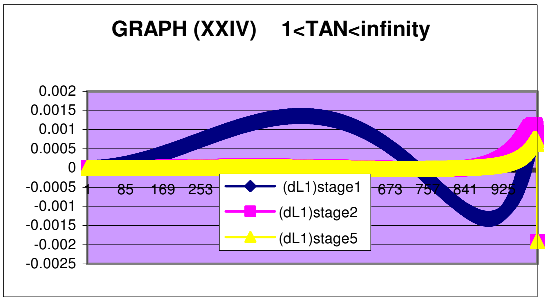

Fig.24 is a comparison of dL1 graphs

The improvements are as follows:

Max.(error %)stage3	=0.000015169457087 at TAN=39.83780595
Max.(dL1)stage3	=0.001101293826650 at TAN=181.9062712
Max.(error %)stage4	=0.000012645431103 at TAN=2.791356503
Max.(dL1)stage4	=0.000674186830850 at TAN=424.4123962
Max.(error %)stage5	=0.000004753119960 at TAN=4.828817352
Max.(dL1)stage5	=0.000637394755302 at TAN=424.4123962

The parameters which I used are not IMPERATIVE! Not MUST! Not mandatory! Better parameters may be proposed for a most accurate estimation.
When we have calculated (b2Mod, m2Mod, GMod) we use these values in

pMo =d2+**b2Mod***(1-((x-c2)/a2)^q)^(1/q)+(**Gmod+m2Mod***x)
and then,
sMod=d1+b1*(1-((x-c1)/a1)^**pMod**)^(1/**pMod**)+(F+m1*x)

will give

L1Estimated =(1+TAN^**sMod**)^(1/**sMod**)

Conclusion

All these calculations are done with simple macro programs. Ready, available on request [5]
We do not use eccentricity, but TAN, as it is general for [0<r<infinity]

We do not need to calculate and to get the sum of 2500 000 000 integral terms.

The following figure (25) is a comparison of Ramanujan's estimations and my estimations. The interval (1<TAN<10) is specially chosen, to escape the high error %.of Ramanujan when TAN goes>10.The parameters (not MUST) have been modified to reach to this comparison.

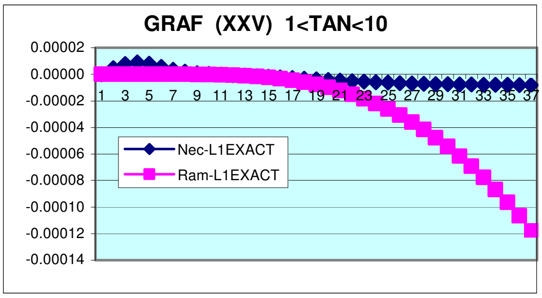

Fig.25 is a comparison of dL1(Ram,Nec) graphs

It was impossible to beat Ramanujan's estimations For TAN<3.25

I thank Mr.Paul Bourke/Australia for the helps he brought to this project. He worked really hard.
See
http://local.wasp.uwa.edu.au/~pbourke/geometry/ellipsecirc for details

Notes:

[1] Internet/Go to Google/write necat tasdelen/see Ellipse Perimeter Approximation. www.ebyte.it
[2] Refusal letter of Weiyrich. A translation. Upon request
[3] Internet/Go to Google/write necat tasdelen/see Circumference,Perimeter of an Ellipse-Numericana-Ynot
[4] Manuscripts. Now computerized.Upon request
[5] 12 e-mails, tables and macros, to be enlarged and activated by the receiver.
Total (2.100 KB).Upon request

Necat Tasdelen
necattasdelen@ttmail.com
TURKEY