Performance Improvement for PID Controller Design with Guaranteed Stability Margin for Certain Classes of MIMO Systems

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Abstract— **Our previous results indicate that there is freedom to choose some parameters in the closed-loop stabilization with guaranteed stability margin using Proportional+Integral+Derivative (PID) controllers. In this paper, such parameters are used to improve the overall performance for a class of MIMO systems. The procedure is demonstrated through an MIMO example. Sufficient conditions for the existence of PID-controllers which stabilize two classes of unstable MIMO systems with guaranteed stability margin are also derived.**

Keywords– Stabilization and tracking, PID control, performance improvement, stability margin.

I. INTRODUCTION

Due to the simplicity of Proportional+Integral+Derivative (PID) controllers and its zero asymptotic tracking error of step-input references, they are typically preferred in applications. Although the simplicity is particularly desirable because of easy implementation and from a tuning point-of-view, it also presents a major restriction. Only certain classes of plants can be controlled by using PID-controllers.

Rigorous PID synthesis methods based on modern control theory are explored recently in e.g., [9], [6], [10], [8], [5]. Sufficient conditions for PID stabilizability of multi-input multi-output (MIMO) plants were given in [5] and several plant classes that admit PID-controllers were identified.

The systematic controller design method given in [5] allows freedom in several of the design parameters. Although these parameters may be chosen appropriately to achieve various performance goals, these issues were not explored.

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Recently, sufficient conditions for some classes of MIMO systems are derived to stabilize given systems with a specified stability margin in e.g. [1], [2], [3]. Just like [5], the associated systematic design procedures also have some freedom to choose several design parameters. Although the optimal choice of those parameters for single-input single-output systems has been explored, the corresponding research for MIMO systems has not been initiated.

The goal of this paper is to first investigate how to use those parameters to improve the overall performance through a class of MIMO systems studied in [1] and [2]. Sufficient conditions for the existence of PID-controllers which stabilize two class of unstable MIMO systems with guaranteed stability margin are then derived along the same line as in [3].

The paper is organized as follows: Section II gives the problem statement of the PID controller design with guaranteed stability margin. Section III demonstrates how to use the freedom to choose parameters in a previously developed systematic procedure to improve the overall performance, through an MIMO example. Section IV presents sufficient conditions for stabilizing two classes of unstable systems by using PID controllers with guaranteed stability margin. Section V gives a short discussion, concluding remarks and some future directions.

II. PROBLEM STATEMENT

In this paper, we will use the following notations. Let $\mathbb{C}$, $\mathbb{R}$, $\mathbb{R}_+$ denote complex, real, positive real numbers. The extended closed right-half complex plane is $\mathcal{U} = \{s \in \mathbb{C} \mid \mathcal{R}e(s) \geq 0\} \cup \{\infty\}$; $\mathbf{R_p}$ denotes real proper rational functions of s; $\mathbf{S} \subset \mathbf{R_p}$ is the stable subset with no poles in $\mathcal{U}$; $\mathcal{M}(\mathbf{S})$ is the set of matrices with entries in $\mathbf{S}$; I_n is the $n \times n$ identity matrix. The H_∞-norm of $M(s) \in \mathcal{M}(\mathbf{S})$ is $\|M\| := \sup_{s \in \partial\mathcal{U}} \bar{\sigma}(M(s))$, where $\bar{\sigma}$ is the maximum singular value and $\partial\mathcal{U}$ is the boundary of $\mathcal{U}$. We drop (s) in transfer-matrices such as $G(s)$ wherever this causes no confusion. We use coprime factorizations over $\mathbf{S}$; i.e., for $G \in \mathbf{R_p}^{n_y \times n_u}$, $G = Y^{-1}X$ denotes a left-coprime-factorization (LCF), where $X, Y \in \mathcal{M}(\mathbf{S})$, $\det Y(\infty) \neq 0$.

Consider the linear time-invariant (LTI) MIMO unity-feedback system $Sys(G, C)$ shown in Fig. 1, where $G \in \mathbf{R_p}^{m \times m}$ is the plant's transfer-function and $C \in \mathbf{R_p}^{m \times m}$ is the controller's transfer-function. Assume that $Sys(G, C)$ is well-posed, G and C have no unstable hidden-modes, and $G \in \mathbf{R_p}^{m \times m}$ is full (normal) rank. We consider the realizable form of proper PID-controllers given by (1), where K_p, K_i, $K_d \in \mathbb{R}^{m \times m}$ are the proportional, integral, derivative constants, respectively, and $\tau \in \mathbb{R}_+$ [4]:

$$C_{pid} = K_p + \frac{K_i}{s} + \frac{K_d\, s}{\tau s + 1}\ . \qquad (1)$$

For implementation, a (typically fast) pole is added to the derivative term so that C_{pid} in (1) is proper. When $K_i = 0$, we have a PD-controller.

Definition 2.1: a) $Sys(G, C)$ is said to be stable iff the transfer-function from (r, v) to (y, w) is stable. *b)* C is said to stabilize G iff C is proper and $Sys(G, C)$ is stable. $\triangle$

The problem addressed here is the same as that in [1]: Suppose that $h \in \mathbb{R}_+$ is a given constant. Can we find a PID-controller C_{pid} that stabilizes the system $Sys(G, C_{pid})$ with a guaranteed stability margin, i.e., with real parts of the closed-loop poles of the system $Sys(G, C_{pid})$ less or equal to $-h$? It is clear that this goal is not achievable for some plants. Furthermore, even when it is achievable, it may be possible to place the closed-loop poles to the left of a shifted-axis that goes through $-h$ only for certain $h \in \mathbb{R}_+$.

To deal with such a problem, we first perform a simple transformation from s-space to $\hat{s}$-space as follows.

$$\hat{s} := s + h, \ \text{ or } \ s =: \hat{s} - h \qquad (2)$$

Define

$$\hat{G}(\hat{s}) \ := \ G(\hat{s} - h)\ ; \qquad (3)$$

Similarly, define $\hat{C}_{pid}$ as

$$\hat{C}_{pid}(\hat{s}) := K_p + \frac{K_i}{\hat{s} - h} + \frac{K_d\,(\hat{s} - h)}{\tau(\hat{s} - h) + 1}\,. \qquad (4)$$

The original problem is then reduced to ask if $Sys(\hat{G}, \hat{C})$ can be stabilized by $\hat{C}_{pid}(\hat{s})$.

III. PERFORMANCE CONSIDERATION

As mentioned earlier, certain classes of MIMO systems admit PID-controller with guaranteed stability margin under proper sufficient conditions. Furthermore, the sufficient conditions lead to systematic design procedures. Also there are some freedom to choose these parameters in the design procedures. We shall use the first class of MIMO systems we considered in [1] to demonstrate how to use these parameters to improve the overall performance.

Consider the same class of MIMO problems as in [1] denoted by $\mathcal{G}_h$ which can be described as follows. If $G \in \mathcal{G}_h \subset \mathbf{S}^{m \times m}$, then the given plant G has no pole with real parts in $[-h, 0]$. Assume that $G(s)$ has no transmission-zeros (or blocking-zeros) at $s = 0$, i.e., $G(0)$ is invertible (note that this condition is necessary for existence of PID-controllers with nonzero integral-constant K_i [5]). The plant G may have transmission-zeros (or blocking-zeros) elsewhere in $\mathcal{U}$ but not at $s = 0$.

Let $\mathcal{S}_h(G)$ denote the set of all PID-controllers that stabilize $G \in \mathcal{G}_h$, with real parts of the closed-loop poles of the system $Sys(G, C_{pid})$ less or equal to $-h$; i.e.,

$$\mathcal{S}_h(G) := \{\, C_{pid} \,|\, \hat{C}_{pid} \text{ stabilizes } \hat{G}(\hat{s}) \,\}\,. \qquad (5)$$

We then have the following proposition from [1].

Proposition 3.1: (*A sufficient condition):*

Let $h \in \mathbb{R}_+$ and $G \in \mathcal{G}_h$ be given. If for some $\hat{K}_p \in \mathbb{R}^{m \times m}$, $\hat{K}_d \in \mathbb{R}^{m \times m}$ and $\tau < 1/h$, the given $h \in \mathbb{R}_+$ satisfies

$$h < \frac{1}{2}\gamma(h, \hat{K}_p, \hat{K}_d), \qquad (6)$$

where $\gamma = \gamma(h, \hat{K}_p, \hat{K}_d)$ is defined as

$$\gamma(h, \hat{K}_p, \hat{K}_d)$$

$$:= ||\hat{G}(\hat{s})(\hat{K}_p + \frac{\hat{K}_d(\hat{s}-h)}{\tau(\hat{s}-h)+1}) + \frac{\hat{G}(\hat{s})G(0)^{-1} - I}{\hat{s}-h}||^{-1}, \quad (7)$$

then there exists a PID-controller C_{pid} of the form in (1) that stabilizes $G \in \mathcal{G}_h$, with real parts of the closed-loop poles of the system $Sys(G, C_{pid})$ less or equal to $-h$. Furthermore, a PID-controller $C_{pid} \in \mathcal{S}_h(G)$ is given by

$$C_{pid} = (\alpha+h)\hat{K}_p + \frac{(\alpha+h)G(0)^{-1}}{s} + \frac{(\alpha+h)\hat{K}_d\ s}{\tau s+1}\ , \quad (8)$$

where $\hat{K}_p, \hat{K}_d \in \mathbb{R}^{m\times m}$ are chosen under condition (6), $\tau < 1/h$, and $\alpha \in \mathbb{R}_+$ satisfies

$$h < \alpha < \gamma(h, \hat{K}_p, \hat{K}_d) - h\ . \quad (9)$$

△

As mentioned in [1], the above sufficient condition can be used to synthesize a PID controller systematically as follows: Given $h \in \mathbb{R}_+$ and $G \in \mathcal{G}_h$, define

$$\beta \triangleq max\{x | p = x + jy,\ \ where\ p\ is\ a\ pole\ of\ G(s)\}; \quad (10)$$

then $-h > \beta$. Choose any $\hat{K}_p$ and $\hat{K}_d$ and compute $\gamma(h, \hat{K}_p, \hat{K}_d)$ given by (7). If $\gamma(h, \hat{K}_p, \hat{K}_d) > 2h$ as in condition (6), then it is possible to find $\alpha \in \mathbb{R}_+$ satisfying (9). The PID-controller $C_{pid} \in \mathcal{S}_h(G)$ is then given by (8). If (6) is not satisfied, the process can be repeated for a smaller h value. Despite the systematic procedure, we can see there are plenty of freedom in choosing parameters. We will illustrate how to use these parameters to improve the overall performance through the same MIMO example as the example 3.3 in [1].

Example 3.1: Consider the quadruple-tank apparatus in [7] which consists of four interconnected water tanks and two pumps. The output variables are the water levels of the two lower tanks, and they are controlled by the currents that are manipulating two pumps. The transfer-matrix of the linearized model at some operating point is given by

$$G = \begin{bmatrix} \frac{3.7b_1}{62s+1} & \frac{3.7(1-b_2)}{(23s+1)(62s+1)} \\ \frac{4.7(1-b_1)}{(30s+1)(90s+1)} & \frac{4.7b_2}{90s+1} \end{bmatrix} \in \mathbf{S}^{2\times 2}. \quad (11)$$

One of the two transmission-zeros of the linearized system dynamics can be moved between the positive and negative real-axis by changing a valve. The adjustable transmission-zeros depends on parameters b_1 and b_2 (the proportions of water flow into the tanks adjusted by two valves). For the values of b_1, b_2 chosen as $b_1 = 0.43$ and $b_2 = 0.34$, the plant G has transmission-zeros at $z_1 = 0.0229 > 0$ and $z_2 = -0.0997$.

By (10), $\beta = -1/90 = -0.0111$. Suppose that $h = 0.004$, and $\tau = 0.05$. The design in [1] has chosen the following parameters.

$$\hat{K}_p = \begin{bmatrix} -22.61 & 37.61 \\ 72.14 & -43.96 \end{bmatrix}, \quad (12)$$

$$\hat{K}_d = \begin{bmatrix} 5.28 & 6.21 \\ 6.53 & 7.84 \end{bmatrix}. \quad (13)$$

The quantity γ can be computed as $\gamma = 0.0099 > 2h = 0.008$. By choosing $\alpha = 0.5\gamma$, the maximum of the real-parts of the closed poles is -0.0059, which is then less than $-h = -0.004$ and fulfills the requirement.

The step response of such a feedback system $T(s)$ is shown in Fig. 2 by solid lines. To see how the choice of α will affect the overall performance, let us choose α equals to its extreme values. In Fig. 2, the dashed lines correspond to $\alpha = h + 10^{-6}$ and the dotted lines for $\alpha = \gamma - h - 10^{-6}$. It is consistent with the intuition that higher gain in PID-controller causes higher overshoot in this example. For the rest of illustration, we simply choose α equals to its lower bound and optimize on $\hat{K}_p$ and $\hat{K}_d$.

From the observation in Fig. 2, let choose the model transfer function $T_{m12}(s)$ and $T_{m21}(s)$ to be zero and both $T_{m11}(s)$ and $T_{m22}(s)$ to be the same as the prototype second order model plant, with $\zeta = 0.8$ and $\omega_n = 0.01$; i.e.,

$$t_m = \frac{\omega_n^2}{s^2 + 2\zeta\omega s + \omega_n^2} \quad (14)$$

The step response for $T_m(s)$ is shown in Fig. 3 as the dashed lines. The original design is shown as the dotted lines.

We want the actual closed-loop step response $s_o(t)$ to be as close as possible to the step response $s_m(t)$ using the model plant T_m. Let us consider the cost function

$$error = \frac{1}{1000}\sum_{i=1}^{2}\sum_{j=1}^{2}\int_0^{1000}(s_{oij}(t) - s_{mij}(t))^2 dt, \quad (15)$$

where $s_o(t)$ denotes the step response for a choice of $(\hat{K}_p,\ \hat{K}_d)$. The goal is to minimize *error* by choosing the best $(\hat{K}_p,\ \hat{K}_d)$. The step response of the optimal design corresponds to this model plant is shown in Fig. 3 by the solid lines. We can see that it is closer to the given model plant than the original design and is a reasonable choice.

To see how the optimal design changes according to the model plant, let now keep $\zeta = 0.8$ and change $\omega_n = 0.02$ for a faster response. The step response of the new model plant is shown in Fig. 4 by the dashed lines, and that of the original design by the dotted lines. The optimal step response is also shown by the solid lines for the new model plant. We can see the solution is reasonable and has a much faster response than before. $\triangle$

IV. PID CONTROLLER SYNTHESIS

In this Section, we will develop the sufficient conditions of synthesizing PID controllers for two classes of unstable MIMO systems. We will use the two-step design method by first obtaining the Proportional+Derivative (PD) controller, and then adding the Integral (I) portion to form the overall PID controller as in [5].

Proposition 4.1: (*Systems with a small RHP zero):* Let $G(s)$ have no pole at $s = 0$. It has one zero at $s = z_1$, where $z_1 \geq 0$. Let $h \geq 0$. Choose any $k_d \geq 0$ and $0 < \tau_1 < \frac{1}{h}$. Define

$$\Phi_1(\hat{s}) = \hat{x}_1 \hat{G}^{-1}(\hat{s}) \hat{Y}(h)^{-1} / [1 + \frac{k_d(\hat{s}-h)}{\tau_1(\hat{s}-h)+1}] - I, \quad (16)$$

$$\text{where } \hat{x}_1 = (\hat{s}-h) - z_1, \quad (17)$$

$$\hat{y}_1 = a_1(\hat{s}-h)+1, \text{ with } a_1 > 0, \quad (18)$$

$$\hat{G} = \hat{Y}^{-1}\hat{X} = (\frac{\hat{x}_1}{\hat{y}_1}\hat{G}^{-1})^{-1}(\frac{\hat{x}_1}{\hat{y}_1}I). \quad (19)$$

Denote

$$Y(0) := \hat{Y}(h) = -z_1 G^{-1}(0) \quad (20)$$

If $0 \leq z_1 < ||\Phi_1(\hat{s})/(\hat{s}-h)||^{-1}$, then for any α satisfying

$$0 < \alpha < ||\frac{\Phi_1(\hat{s})}{(\hat{s}-h)}||^{-1} - z_1, \quad (21)$$

the PD controller $C_1(s)$ below stabilizes $G(s)$:

$$C_1(s) = \frac{1}{(z_1+\alpha)}[1 + \frac{k_d(s)}{\tau_1 s + 1}]Y(0). \quad (22)$$

Proof:

Choose the RCF for $\hat{C}_1$ as

$$\hat{C}_1 = \hat{N}_c \hat{D}_c^{-1} = (\hat{C}_1)(I)^{-1}. \quad (23)$$

Let

$$\hat{M}_1 = \hat{X}\hat{C}_1 + \hat{Y} = (\frac{\hat{x}_1}{\hat{y}_1}I + \hat{Y}\hat{C}_1^{-1})\hat{C}_1 \quad (24)$$

$$\hat{M}_1 = (\frac{\hat{x}_1}{\hat{y}_\alpha}I + \frac{\hat{y}_1}{\hat{y}_\alpha}\hat{Y}\hat{C}_1^{-1})\frac{\hat{y}_\alpha}{\hat{y}_1}\hat{C}_1, \quad (25)$$

where

$$\hat{y}_\alpha = (\hat{s}-h) + \alpha \quad (26)$$

$$\hat{M}_1 = [I + \frac{(z_1+\alpha)}{\hat{y}_\alpha}(\frac{\hat{y}_1}{(z_1+\alpha)}\hat{Y}\hat{C}_1^{-1} - I)]\frac{\hat{y}_\alpha}{\hat{y}_1}\hat{C}_1 \quad (27)$$

$$\hat{C}_1(\hat{s}) = \frac{1}{(z_1+\alpha)}\hat{Y}(h)[1 + \frac{k_d(\hat{s}-h)}{\tau_1(\hat{s}-h)+1}], \quad (28)$$

$$\hat{M}_1 = [I + \frac{(z_1+\alpha)(\hat{s}-h)}{(\hat{s}-h)+\alpha}\frac{\Phi_1(\hat{s})}{(\hat{s}-h)}]\frac{\hat{y}_\alpha}{\hat{y}_1}\hat{C}_1 \quad (29)$$

Note that $\Phi_1(h) = 0$, we have $\frac{\Phi_1(\hat{s})}{(\hat{s}-h)} \in \mathcal{M}(\mathbf{S})$. With the choice of α, $\hat{M}_1$ is unimodular by the small-gain theorem [11]. Thus, $\hat{C}_1(\hat{s})$ stabilizes $\hat{G}(\hat{s})$. As the result, $C_1(s)$ stabilizes $G(s)$. $\triangle$

Proposition 4.2: (*Systems with two small RHP zeros):* Let $G(s)$ have no pole at $s = 0$. It has two zeros at $s = z_1$ and $s = z_2$, where $z_1 \geq 0$, $z_2 \geq z_1$ and $z_j \in R$. Choose any $k_2 > 0$. Define

$$\Phi_2(\hat{s}) := \frac{\hat{x}}{k_2(\hat{s}-h)+1}\hat{G}^{-1}(s)\hat{Y}(h)^{-1} - I, \quad (30)$$

where

$$\hat{y} = (a_1(\hat{s}-h)+1)(a_2(\hat{s}-h)+1),$$

$$\hat{x} = ((\hat{s}-h)-z_1)((\hat{s}-h)-z_2), \quad (31)$$

$$\hat{G} = \hat{Y}^{-1}\hat{X} = (\frac{\hat{x}}{\hat{y}}\hat{G}^{-1})^{-1}(\frac{\hat{x}}{\hat{y}}I). \quad (32)$$

Denote

$$Y(0) := \hat{Y}(h) = z_1 z_2 \hat{G}^{-1}(0) \quad (33)$$

If $2(z_1 + z_2) < ||\Phi_2(\hat{s})/(\hat{s} - h)||^{-1}$, then for any $\alpha > 0$ and $\beta > 0$ satisfying

$$\alpha + \beta < ||\Phi_2(\hat{s})/(\hat{s} - h)||^{-1} - 2(z_1 + z_2), \quad (34)$$

and

$$\eta/\rho < 1/h, \quad (35)$$

where

$$\eta = \alpha + \beta + 2z_1 + 2z_2, \text{ and } \rho = \alpha\beta + \beta z_1 + \alpha z_2, \quad (36)$$

the PD controller $C_2(s)$ below stabilizes $G(s)$:

$$C_2(s) = [\frac{k_2 s + 1}{\eta s + \rho}]Y(0). \quad (37)$$

Proof:

Choose the RCF for $\hat{C}_2$ as

$$\hat{C}_2 = \hat{N}_c \hat{D}_c^{-1} = (\hat{C}_2)(I)^{-1}. \quad (38)$$

Let

$$\hat{M}_2 = \hat{X}\hat{C}_2 + \hat{Y} = (\frac{\hat{x}}{\hat{y}}I + \hat{Y}\hat{C}_2^{-1})\hat{C}_2 \quad (39)$$

Let

$$\hat{y}_\alpha = (\hat{s} - h) + \alpha + z_1 \text{ and } \hat{y}_\beta = (\hat{s} - h) + \beta + z_2 \quad (40)$$

$$\hat{M}_2 = (\frac{\hat{x}}{\hat{y}_\alpha \hat{y}_\beta}I + \frac{\hat{y}}{\hat{y}_\alpha \hat{y}_\beta}\hat{Y}\hat{C}_2^{-1})\frac{\hat{y}_\alpha \hat{y}_\beta}{\hat{y}}\hat{C}_2 \quad (41)$$

$$\hat{M}_2 = (I - \frac{\eta(\hat{s} - h) + \rho}{\hat{y}_\alpha \hat{y}_\beta}I + \frac{\hat{y}}{\hat{y}_\alpha \hat{y}_\beta}\hat{Y}\hat{C}_2^{-1})\frac{\hat{y}_\alpha \hat{y}_\beta}{\hat{y}}\hat{C}_2 \quad (42)$$

$$\hat{M}_2 = [I + \frac{(\eta(\hat{s} - h) + \rho)(\hat{s} - h)}{\hat{y}_\alpha \hat{y}_\beta}(\frac{\Phi_2(\hat{s})}{\hat{s} - h})]\frac{\hat{y}_\alpha \hat{y}_\beta}{\hat{y}}\hat{C}_2 \quad (43)$$

Note that $\Phi_2(h) = 0$, we have $\frac{\Phi_2(\hat{s})}{(\hat{s}-h)} \in \mathcal{M}(\mathbf{S})$. With the choice of α and β, $\hat{M}_2$ is unimodular by the small-gain theorem [11]. Thus, $\hat{C}_2(\hat{s})$ stabilizes $\hat{G}(\hat{s})$ and $C_2(s)$ stabilizes $G(s)$. △

To get the PID-controller, we use the two-step design procedure used in [5] and [3]. Let $C_{pd}(s)$ be a PD-controller stabilizing $G(s)$ with guaranteed stability margin specified by h, which can be either $C_1(s)$ or $C_2(s)$ in previous propositions. Equivalently, $\hat{C}_{pd}(\hat{s})$ is a PD-controller stabilizing $\hat{G}(\hat{s})$, and

$$\hat{H}_{pd}(\hat{s}) := \hat{G}(I + \hat{C}_{pd}\hat{G})^{-1} \in \mathcal{M}(\mathbf{S}). \quad (44)$$

Finding an I-controller for $\hat{H}_{pd}(\hat{s})$ is the special case of Proposition 3.1 by letting $\hat{K}_p = 0$ and $\hat{K}_d = 0$ as given in [3]. Once $\hat{C}_I(\hat{s})$ is found, the controller $\hat{C}_{pid}(\hat{s}) = \hat{C}_{pd}(\hat{s}) + \hat{C}_I(\hat{s})$ is a PID-controller stabilizing $\hat{G}(\hat{s})$ [11].

Proposition 4.3: (*PID-controller):*
Let $G(s)$ have no pole at $s = 0$. Define

$$\gamma(h) := ||\frac{\hat{H}_{pd}(\hat{s})H_{pd}(0)^{-1} - I}{\hat{s} - h}||^{-1}. \quad (45)$$

For any given PD-controller $C_{pd}(s)$ stabilizing $G(s)$ with guaranteed stability margin specified by h (≥ 0), if we can find an α such that

$$h < \alpha < \gamma(h) - h, \quad (46)$$

the PID-controller

$$C_{pid}(s) := C_{pd}(s) + C_I(s), \quad (47)$$

where

$$C_I(s) = \frac{(\alpha + h)H_{pd}(0)^{-1}}{s}, \quad (48)$$

stabilizes $G(s)$ with guaranteed stability margin specified by h. △

V. CONCLUSIONS

For stable plants whose poles have negative real-parts less than a pre-specified $-h$, we illustrated how to use the freedom in choosing parameters to improve the overall performance of feedback systems using PID-controllers with a guaranteed stability margin. The optimization procedure depends on the sufficient conditions for the existence of PID controllers stabilizing such a class of MIMO systems. Preliminary numerical results are presented through an MIMO example. We also derive the sufficient conditions for the existence of PID-controllers which stabilize two classes of unstable MIMO systems with guaranteed stability margin.

Future directions of this study will involve extension to more classes of unstable MIMO plants. In addition, optimal parameter selections for the corresponding MIMO cases will be explored.

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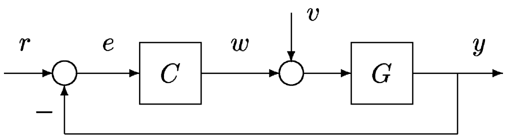

Fig. 1. Unity-Feedback System $Sys(G,C)$.

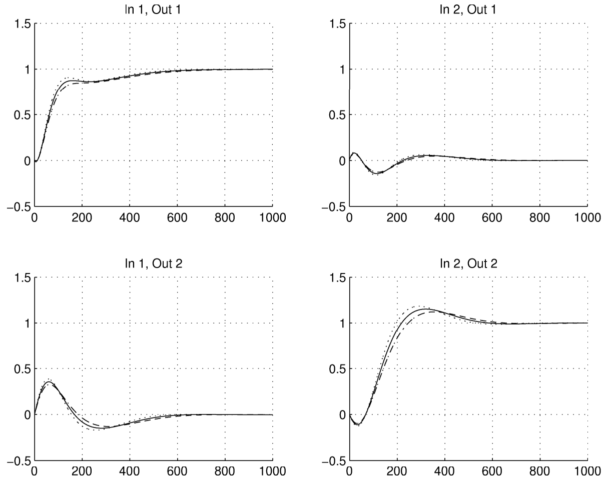

Fig. 2. Step responses for original design in Example 3.1

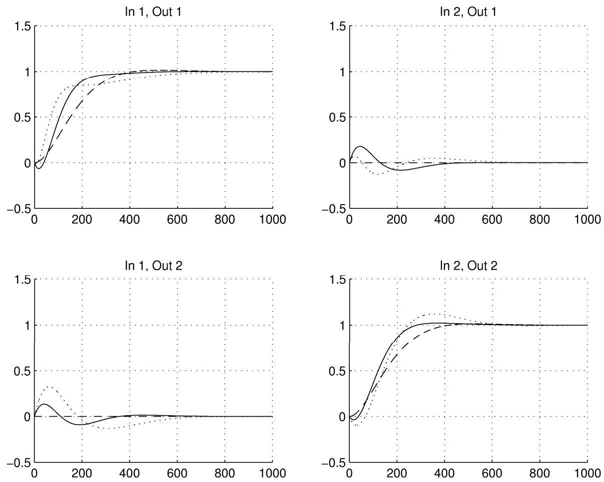

Fig. 3. Step responses for three transfer matrices in Example 3.1

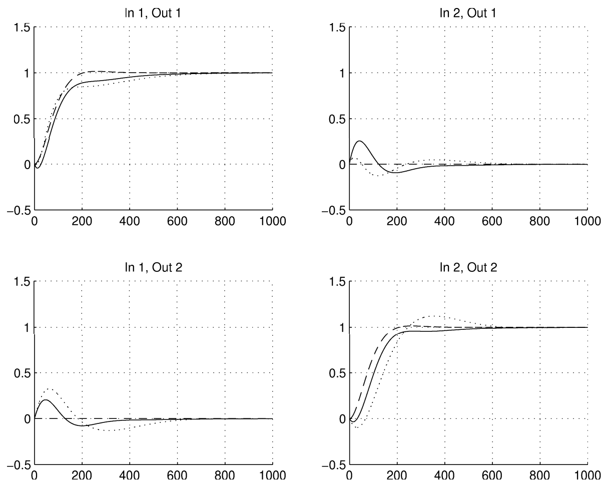

Fig. 4. Step responses for new model plant in Example 3.1