DOA Estimation for Wideband Signals Based on Arbitrary Group Delay

Xinggan Zhang, Yechao Bai and Wei Zhang

Abstract—**Direction-of-arrival (DOA) estimation is an important algorithm in array processing. Most traditional DOA estimation methods focus on narrow-band sources. A new wideband DOA estimation approach based on arbitrary group delay is proposed in this paper. In the proposed algorithm, echo gain at each direction is calculated based on digital group delay compensation. Digital group delay can realize time delay precisely, and has no restriction on delay step. The proposed method is able to operate arbitrary waveform and is suitable for linear frequency modulation signals, nonlinear frequency modulation signals, and even carrier free signals. Simulations with Gaussian function are carried out to show the effectiveness of the proposed algorithm.**

Index Terms—**wideband, DOA, estimation, group delay.**

I. Introduction

DOA estimation is an important algorithm in array signal processing systems [1], [2]. Various estimation methods such as Maximum Likelihood (ML), Multiple Signal Classification (MUSIC) [3], and Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) [4] were proposed in recent two decades. However, most of these methods are based on narrow band models. Several extensions to the wideband sources are also proposed [5], for example, the Coherent Signal-Subspace Processing (CSSP) [6], Wideband Cyclic MUSIC [7] and Wideband ESPRIT [8]. A frequency domain algorithm based on ML criterion called AML method has been proposed for source localization and DOA estimation [9], [10]. A method proposed in [11] performs beam-space processing using time-domain frequency-invariant beamformers (TDFIB). In this paper, we propose a new wideband DOA estimation approach based on arbitrary group delay. This method is able to operate arbitrary waveform and is suitable for linear frequency modulation signals, nonlinear frequency modulation signals, and even carrier free signals.

II. Method of Digital Group Delay

When the object direction is far from the normal direction, the delay between array elements is so large that signals need to be aligned to get a better distance resolution and a higher power gain. The time delay compensation is usually operated in two steps: the integral sampling period part and fractional sampling period part. The integral part can be simply carried out by register shifting. The subsample time delay is compensated by interpolation commonly, and the compensation effect lies on the interpolation function. Dense sampling, in which sampling frequency is much higher than the Nyquist frequency is also used in some situation to achieve a higher accuracy. Unfortunately, this method brings too much data redundancy and raises a higher demand to the sampling device. Especially, when the bandwidth of ultra-wideband signals is very high, for example 1GHz, it is hard to sample with frequency several times higher, and the expense is also too huge.

Suppose that the wideband signal is $s(t)$, and the delayed signal is $s(t\text{-}\tau)$. According to the time shifting property of continuous time Fourier transform, we can get:

$$s(t-\tau)=\mathcal{F}^{-1}\left\{\mathcal{F}\left\{s(t)\right\}e^{-j\omega\tau}\right\} \tag{1}$$

If the sampling frequency is so high that the spectral aliasing can be ignored, the digital signal delay can be implemented by

$$s(nT_s-\tau)=IDFT\left\{DFT\left\{s(nT_s)\right\}e^{-j\frac{2\pi k\tau}{NT_s}}\right\} \tag{2}$$

where T_s is the sampling period, and $s(nT_s)$ is the digital signal. The time delay τ in (2) need not to be integral multiple of the sampling period. N is the length of the operated signal, consisting of original signal and INT(τ/f_s) padded zeroes, where INT denotes rounding operation. Zeros are padded at the tail of the original signal to avoid cyclic shifting of the original signal.

Gaussian modulated sine signal is used to demonstrate the method of digital group delay. The analytical expression of the signal is

$$s(t)=e^{-\pi(\frac{t-T_0}{T_g})^2}\cos\left[2\pi f(t-T_0)\right] \tag{3}$$

where T_g, T_0 and f are the time width parameter of Gaussian window, the time center and the frequency of sine signal, respectively. T_g and f were chosen to be 50 and 0.1. T_0 was chosen to be 0 before signal is delayed. The results of simulation of digital group delay with MATLAB are shown is Fig. 1 to Fig. 2. In simulations, the signal delayed for 3.7 samples by the method of digital group delay is compared with the theoretical waveform obtained by setting T_0 to 3.7 in (3).

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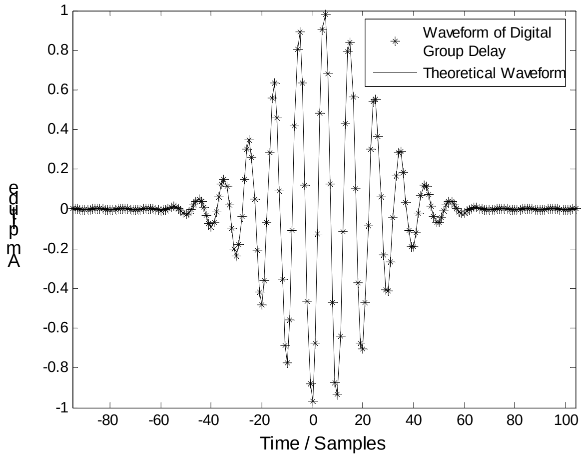

Fig. 1 Comparison between waveform of digital group delay and theoretical waveform

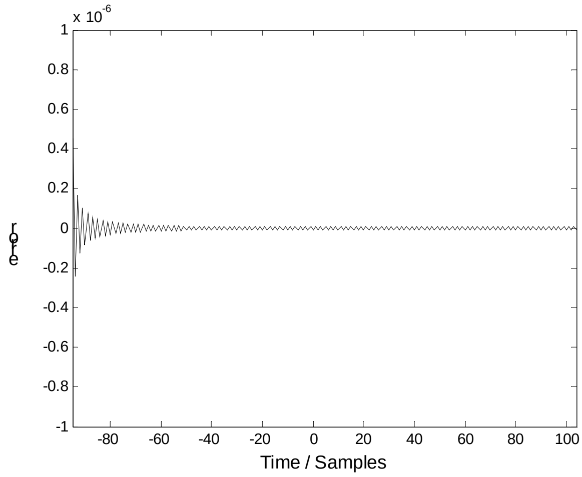

Fig. 2 Error between waveform of digital group delay and theoretical waveform

It can be seen that digital group delay can realize time delay precisely for approximate band limit signals from Fig. 1 to Fig. 2. In addition, digital group delay operates the integral sampling period part and fractional sampling period part together, and has no restriction on delay step. Furthermore, this method has no requirement for the signal waveform, and is suitable for linear frequency modulation signals, nonlinear frequency modulation signals, and even carrier free signals.

III. DOA Estimation Based on Digital Group Delay

A simple time delay manifests itself as the slope of an additive linear phase in the phase spectrum, which is an additional group delay

$$\tau_g(f) = -\frac{1}{2\pi}\frac{d\varphi}{df} = -\tau \tag{4}$$

τ_g is a constant.

Consider an equally spaced linear array composed of N_e identical elements. The distance between consecutive sensors is d. The relative delay of the ith element is

$$\tau_i = \frac{id\sin\alpha}{c} \tag{5}$$

where α is the beam direction, and c is the velocity of electromagnetic wave.

The target echoes of array elements are aligned according to each direction. At the direction α, signals are aligned completely after group delay compensation, and the gain after match filter reaches the maximum. In contrast, at other directions, echo signals can not be totally aligned, and the gain decreases.

DOA estimation based on arbitrary group delay compensates the echo signals according to different beam directions, and find the target direction by locating the maximum of the match filter gain. Considering that the searching directions are discrete and the accurate is limited, we fit a convex parabola around the peak and obtain the required direction by calculating the symmetry axis of the parabola.

IV. Simulation

Gaussian function [10] (GGP, generalized Gaussian pulse) was employed in the simulation (using MATLAB) to demonstrate the effectiveness of the proposed algorithm. One of forms based on experimentation with UWB impulse radiators and sensors is:

$$f(t) = \frac{E_0}{1-\alpha}\left\{ e^{-4\pi\left(\frac{t-t_0}{\Delta T}\right)^2} - \alpha e^{-4\pi\alpha^2\left(\frac{t-t_0}{\Delta T}\right)^2} \right\} \tag{6}$$

where E_0 is peak amplitude at time $t=t_0$, α is scale parameter. In the simulation, $E_0=1$, $\Delta T=2\mu s$ and $\alpha=10$. In addition, element amount N_e, distance between consecutive sensors d and the sampling frequency were chosen to be 100, 0.4m and 100MHz, respectively.

The root mean square error (RMSE) in the DOA estimation versus signal to noise ratio (SNR) is shown in Table I. The target is fixed at direction 45.5°. The simulation for each SNR was performed 1000 times.

Table I The RMSE in the DOA estimation versus SNR

SNR (dB)	-10	0	10	20	30
RMSE (°)	1.4457	0.4584	0.1627	0.0579	0.0187

From Table I, it is easily seen that the proposed algorithm has a very small RMSE, and the proposed algorithm also performs well at low SNR.

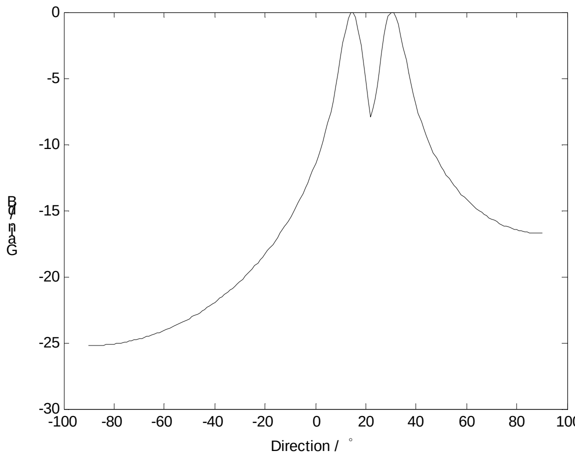

Fig. 3 DOA estimation for two targets at 15° and 30°,

SNR=20dB

Fig. 3 shows the resolution performance of the proposed algorithm. In the simulation, E_0, ΔT, α and the sampling frequency were chosen to be 1, 2μs, 200 and 1GHz, respectively. The echo contains two targets located at 15° and 30°, and SNR is 20dB.

V. Conclusion

The digital group delay can realize time delay precisely for approximate band limit signals. In this paper, we apply the digital group delay method to DOA estimation. The simulations show the effectiveness of the proposed algorithm. The proposed algorithm has no requirement for the signal waveform, and is suitable for wideband and ultra-wideband radars, sonar systems, and microphone arrays.

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