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Computational Modeling of Damage and Self-Repair Processes of Engineering Materials

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Abstract— **Continuum damage mechanics is extended to cover the self-repair process as well as the damage process. The repair variable and its evolution equation are newly introduced to consider the self-repair process. The evolution equation of the repair variable is proposed, based on Dyson's equation of creep cavity growth. The validity of the proposed modeling is illustrated through the simulations for the self-repair processes of the creep-damaged steel by sintering and the fatigue-damaged polystyrene by annealing.**

***Index Terms*— Computational Modeling, Damage Mechanics, Self-Repair, Steel, Polystyrene**

I. Introduction

The researches on self-repair materials with self-repairing functions as in living things are being activated for the purpose of increasing safety, reliability and economy of materials and structures. The reunion of molecular chains in high-polymer materials, the surface repair in corrosion-resistant steels, the creep void repair in heat-resistant steels, the surface crack recovery in ceramic materials and the adhesive dispersion in concrete/composite materials have been studied in recent years [1].

The analytical method based on continuum damage mechanics [2] is a powerful tool as a modeling and simulation technique for the damage and fracture behaviors of these materials. It is expected that simulations of the self-repair processes of materials and structures as mentioned above will be made possible by extending the concept of damage mechanics to the modeling of the self-repair processes which are the inverse processes to the damage and fracture behaviors. The study conducted by Barbero et al. [3] for fiber-reinforced high-polymer composite materials and the research by Toi and Hirose [4] for steels at high temperature are the pioneering works trying to extend damage mechanics to the self-repair materials.

In the present study, the concept of damage mechanics is formally extended to include the self-repair processes as well as the damage and fracture behaviors. The repair variable and the repair evolution equation describing the self-repair processes are introduced, which correspond to the damage variable and the damage evolution equation for the damage and fracture behaviors respectively. The effects of damage and repair on the constitutive equation are independently taken into account in the formulation of Barbero et al. [3], however, the summation of both may affect it, depending on the microscopic damage and repair mechanisms of the materials to be considered. The extension in the latter case is mainly discussed in the present study.

The self-repair processes for two kinds of engineering materials are simulated as examples for the application of the above-mentioned concept. The process of self-repair by sintering under compressive loading is calculated for the boiler plate steel 1.3Mn-0.5Mo-0.5Ni (abbreviated as SBV2) subjected to creep damage under tensile loading. The process of self-repair by annealing is also analyzed for the polystyrene (abbreviated as PS) subjected to fatigue damage. The validity of the present modeling is illustrated by comparing the obtained results with the corresponding experimental results [5], [6].

The extension of damage mechanics to the self-repair process is described in Section 2. Section 3 presents the constitutive equation system for damage evolution and self-repair. The repair evolution equation based on the constrained void evolution model in solids given by Dyson [7] is proposed. In Sections 4 and 5, the self-repair simulations for the boiler plate steel and the polystyrene are carried out and the results are compared with the test results. Section 6 contains concluding remarks.

II. Extension of Damage Mechanics to Self-Repair Process

The damage variable in continuum damage mechanics is defined as the ratio of the damaged area (void area) due to microvoids and microcracks to the total area, which is the sum of the damaged area and the undamaged area, on the cross-section in the representative volume element. This ratio is expressed by the scalar damage variable independent on directions in the isotropic damage theory, which is denoted by D_F to distinguish it from the real damage D explained later. D_F, which is a monotonically increasing function, has the value of zero when no damage exists and has the value of one when complete damage occurs. Then the following conditions must be satisfied:

$$\dot{D}_F \geq 0 \tag{1a}$$

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$$0 \le D_F \le 1 \tag{1b}$$

On the other hand, when the existing damage is repaired in some way and the damaged area decreases, the decreased damage area is called the repaired area and the ratio of the repaired area to the total area is called the repair variable, which is denoted by D_R here. D_R, which is a monotonically decreasing function, is zero when no repair exists and its absolute value has the upper limit of D_F when there is repair. Therefore the following conditions must be satisfied:

$$\dot{D}_R \le 0 \tag{2a}$$

$$0 \le \left|D_R\right| \le D_F \tag{2b}$$

As the microscopic mechanisms for damage and repair are generally different with each other, the evolutions for damage and repair are expressed by the different evolution equations as follows:

$$\dot{D}_F = F(s_{Fi}), \quad i = 1 \sim n_F \tag{3}$$

$$\dot{D}_R = R(s_{Ri}), \quad i = 1 \sim n_R \tag{4}$$

where s_{Fi}, the number of which is n_F, and s_{Ri}, the number of which is n_R, are the state variables contained in the damage and the repair evolution equation, respectively.

As the damage and the repair evolution as explained above are simply increase and decrease of the area of damage opening, the real damage variable at every time D and its time rate $\dot{D}$ can be evaluated by the summation of both variables as follows:

$$D = \alpha D_F + \beta D_R \tag{5a}$$

and

$$\dot{D} = \alpha \dot{D}_F + \beta \dot{D}_R \tag{5b}$$

where α and β are the material constants to consider the interactive effects in the case when the damage and the repair proceed simultaneously. There are no interactive effects when $\alpha = 1$ and $\beta = 1$ in the identified material constants based on the experimental results. Some interactive effects can be considered to exist, when $\alpha \ne 1$ or $\beta \ne 1$. The influence of the damage and the repair variable on the material constitutive equations can be taken into account by replacing the conventional stress σ with the effective stress defined by using the real damage variable D which is the sum of both variables according to the strain-equivalence hypothesis [2]. The resulting constitutive equation is expressed by

$$\varepsilon = C(\sigma/(1-D), \cdots\cdots) \tag{6}$$

The damage and the repair of the heat-resistant steel and the polystyrene discussed in the present study can be considered to be examples of the above-proposed modeling, because they are the phenomena of evolution and shrinkage of the microvoids.

In the adhesive diffusion in concrete and composite materials, the pre-inserted adhesive capsule is fractured by the crack propagation and the adhesives fill up the cracks. In such a case, the influence of the repair variable D_R on the constitutive equation must be independently considered, as the different material such as adhesives repair the damage openings. The resulting constitutive equation is expressed by

$$\varepsilon = C(\sigma/(1-D), D_R, \cdots\cdots) \tag{7}$$

It is considered that the above-mentioned extension, which makes possible the applications of continuum damage mechanics to the self-repair processes as well as the damage and fracture processes, offers simulation models useful for the prediction for the damage and the repair of self-repair materials.

III. Evolution Equations for Damage and Self-Repair Processes

A. Constitutive Equation

The following constitutive equation, in which the strain-equivalence hypothesis [2] is applied to the well-known Hooke's law, is used:

$$\overline{\sigma}_{ij} = \frac{\sigma_{ij}}{1-D} = D^e_{ijkl}\,\varepsilon^e_{kl} = D^e_{ijkl}\left(\varepsilon_{kl} - \varepsilon^i_{kl}\right) \tag{8}$$

where the following notations are used: $\overline{\sigma}_{ij}$, the effective stress; σ_{ij}, the nominal stress; D^e_{ijkl}, the stress-strain matrix of an elastic solid; ε^e_{kl}, the elastic strain; ε_{kl}, the total strain; ε^i_{kl}, the inelastic (creep for SBV2 and viscoplastic for PS) strain; D, the real damage variable defined by Eq. (5a).

B. Damage Evolution Equation

The following equation given by Lemaitre [2] is used as the damage evolution equation (3):

$$\dot{D}_F = \left(-\frac{Y}{S_1}\right)^{S_2} \dot{\varepsilon}^i_{eq} \tag{9}$$

where $\dot{\varepsilon}^i_{eq}$ is the equivalent inelastic strain rate. The following conditions for damage evolution are assumed:

$$\dot{D}_F = 0 \quad when \quad \varepsilon^i_{eq} < \varepsilon_{pd} \tag{10a}$$

$$\dot{D}_F > 0 \quad when \quad \varepsilon^i_{eq} \ge \varepsilon_{pd} \tag{10b}$$

$$0 \le D_F \le D_{cr} \tag{10c}$$

When the accumulated equivalent inelastic strain ε^i_{eq} exceeds the critical inelastic strain for damage initiation, the inelastic damage evolves. When the damage variable D_F reaches the critical damage D_{cr}, the mesocracking occurs in the material. In Eq. (9), S_1 and S_2 are the damage strength material parameters. Y is the strain energy density release rate expressed by the following equation:

$$-Y = \frac{\sigma^2_{eq}}{2E(1-D)^2} R_v \tag{11}$$

where

$$R_v = \left\{ \frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2 \right\} \quad (12)$$

In Eqs. (11) and (12), the following notations are used: E, Young's modulus; ν, Poisson's ratio; R_v, the triaxiality function; σ_H, the hydrostatic pressure; σ_{eq}, the equivalent stress.

The damage strength material parameters S_1, S_2 contained in the damage evolution equation (9) and the parameters ε_{pd}, D_{cr} contained in the damage evolution conditions (10) are the material constants depending upon the sorts of materials and the types of damage. As the inelastic damage discussed in the present study is a low-speed deformation damage, none of these material constants depends on the strain rate. Therefore,

$$S_1 = S_{10} \quad (13a)$$

$$S_2 = S_{20} \quad (13b)$$

$$\varepsilon_{pd} = \varepsilon_{pd0} \quad (13c)$$

$$D_{cr} = D_{cr0} \quad (13d)$$

where the following notations are used: S_{10} and S_{20}, the static damage strength material parameters; ε_{pd0}, the static critical strain for damage initiation; D_{cr0}, the static critical damage for mesocracking.

C. Repair Evolution Equation

The repair evolution equation is derived in the following. Kyono et al. [5] applied the evolution model of creep voids given by Dyson [7] to the creep void sintering and calculated the sintering velocity using the following equation in which the tensile strain rate in the void evolution model is replaced with the compressive strain rate:

$$\frac{dr}{dt} = -\frac{\dot{\varepsilon}\lambda^2 d}{16r^2} \quad (14)$$

where the following notations are used: r, the radius of creep voids; $\dot{\varepsilon}$, the compressive creep strain rate; λ, the distance between creep voids; d, the size of crystal grains.

The following equation can be derived from Eq. (14):

$$\frac{dr^2}{dt} = -\frac{\dot{\varepsilon}\lambda^2 d}{8r} \quad (15)$$

As the left-hand side of Eq. (15) and the denominator of the right-hand side are proportional to the reduction rate of void areas and the square root of void areas respectively, the damage repair equation has the following form:

$$\dot{D}_R \propto -D^{-1/2}\dot{\varepsilon}^i_{eq} \quad (16a)$$

Its generalized form is given by the following equation:

$$\dot{D}_R \propto -D^n\dot{\varepsilon}^i_{eq} \quad (16b)$$

The following equations for the evolution of real damage can be obtained, combining Eqs. (9) and (16b) with Eq. (5b). No repair occurs in the inelastic damage process under a tensile stress, then

$$\dot{D} = \dot{D}_F = \left(-\frac{Y}{S_1}\right)^{S_2} \dot{\varepsilon}^i_{eq} \quad (17)$$

It can be considered that the compressive damage and the repair due to sintering or annealing take place simultaneously in the inelastic damage/repair process under a compressive stress. Therefore, the following equation can be obtained:

$$\dot{D} = \dot{D}_F + \dot{D}_R = K_1\left(-\frac{Y}{S_1}\right)^{S_2} \dot{\varepsilon}^i_{eq} - K_2 D^n \dot{\varepsilon}^i_{eq} \quad (18)$$

where K_1 and K_2, which are the material constants for compressive damage and repair, correspond to α and β in Eq. (5b) respectively.

IV. Results for Boiler Plate Steel (SBV2)

A. Tensile Creep Damage

The creep damage constitutive equation [4] has been identified by using the material test results for SBV2 cylindrical specimens which are the creep test results under the tensile stresses of 118[MPa] and 157[MPa] [5]. The identified time-histories for the tensile creep strain as well as the test results are shown in Fig. 1. The real damage variable D in the calculations of the present section is evaluated by Eq. (17) which contains no repair processes. The symbol × on the identified curves in the figure indicates the fracture point ($D = D_{cr}$).

It can be seen from the results of Fig. 1 that the creep damage model in the present study does not represent clearly the reduction of the creep strain rate in the primary creep, but express well the stationary creep progress in the secondary creep and the creep rate increase and the creep fracture in the tertiary creep. The identified results agree well with the two creep test results under different stresses. It has been confirmed that the identified creep damage constitutive model can represent the creep damage process in good accuracy.

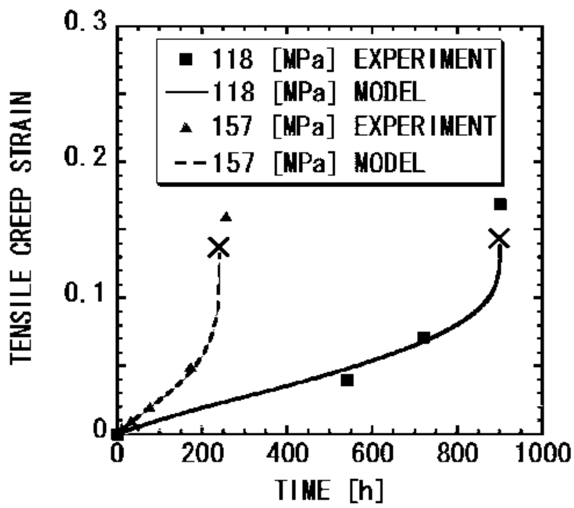


Fig. 1 Tensile creep curves for SBV2

B. Self-Repair by Sintering

The uniaxial, tensile creep test of 540[hours] was conducted for SBV2 at the constant temperature of 550[℃] under the tensile stress of 118[MPa] [5]. It was followed by the repair of creep damage by sintering under five levels of uniaxial, compressive stresses 59, 88, 116, 147, 177[MPa] [5]. Figures 2 and 3 show the results of the simulations for these five experimental results [5] by the identified creep damage constitutive model. The real damage variable D in this calculation has been evaluated by Eq. (18) with $n = -1/2$ considering the repair process.

Figure 2 shows the time-history of the compressive creep strain during the loading of the compressive stress. As seen from the figure, the present results represent well the transient material softening phenomena after the stress reversal and totally correspond well with the experimental results, although the correspondence depends on the compressive stress levels.

Figure 3 shows the relation between the repair rate of damage and the compressive creep strain. The repair rate of damage on the vertical axis, which is the recovery rate for the density of the material decreased by the tensile creep in the experiment, corresponds to the reduction rate of the damage variable D in the analysis. The horizontal axis indicates the compressive creep strain. As seen from Fig. 3, the time rate of repair decreases with the progress of the repair in the experimental results, however, this phenomena has not appear in the creep damage constitutive modeling proposed in the present study. On the contrary, the time rate of repair has increased with the relative increase of the influence of the second term in Eq. (18). Although there remains a room for improvement in the present modeling, the repair evolution equation is physically valid to some extent as the dependence of the repair rate on the compressive stress levels at the initial stage has been successfully simulated.

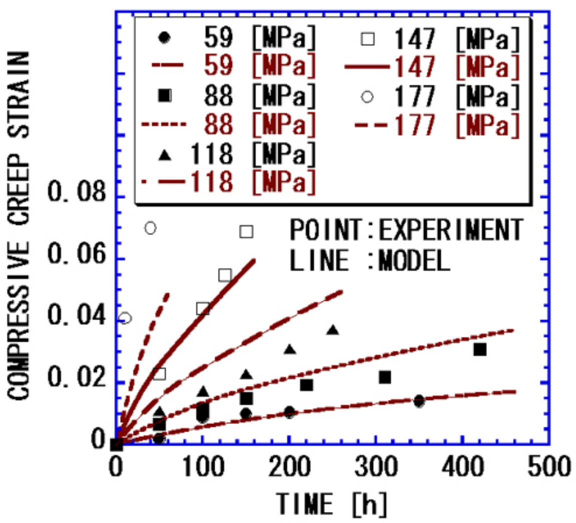


Fig. 2 Compressive creep curves for SBV2

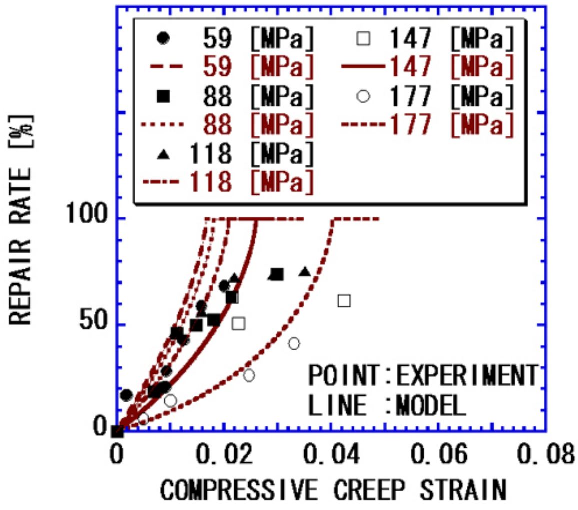


Fig. 3 Repair rate versus compressive creep strain for SBV2

V. RESULTS FOR POLYSTYRENE (PS)

A. Tensile Fatigue Damage

The tensile fatigue damage analysis [8] has been conducted for the model of Fig. 4 under the repeated loading with the maximum stress of 6MPa and the frequency of 0.83Hz [6].

The craze and crack have developed from the notch-tip in the direction perpendicular to the load. Figure 5 shows the relation between the number of cycles and the length of craze and crack. The calculated result agrees well with the experimental result [6].

The specimens in which the crack and craze length normalized by the specimen width has reached 0.2, 0.3, 0.4 and 0.5 are identified as 20%-, 30%-, 40%- and 50%-PS model.

B. Self-Repair by Annealing

The self-repair analysis has been conducted for 20%-, 30%-, 40%- and 50%-PS model subjected to fatigue damage. The real damage variable D in this calculation has been evaluated by Eq. (18) with $\sigma_{eq} = 0$ (i.e. $Y = 0$) and $n = 1/2$ considering the repair process.

Figures 6 and 7 are the damage variable distribution and the equivalent stress distribution, respectively, for 50%-PS model. The damage except for the crack has been repaired by the repair evolution equation. It is seen that the craze has been repaired and the residual stress has been removed. The similar results have been obtained for the other three models.

C. Tensile Strength before and after Annealing

Figure 8 shows the calculated results for the tensile strength before and after annealing. The increase of the tensile strength by annealing has been confirmed. The qualitative tendency of the increase of the repair rate with the increase of the crack and craze length has been well simulated. The calculated results have corresponded well with the experimental results [6].

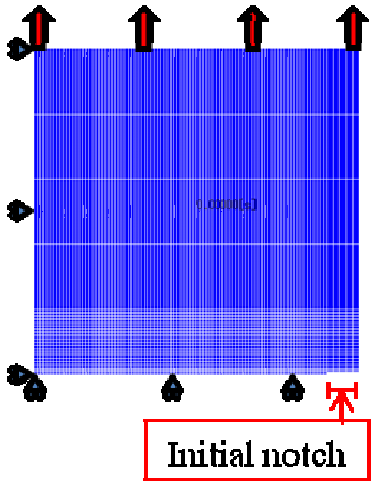


Fig. 4 Finite element mesh for notched PS

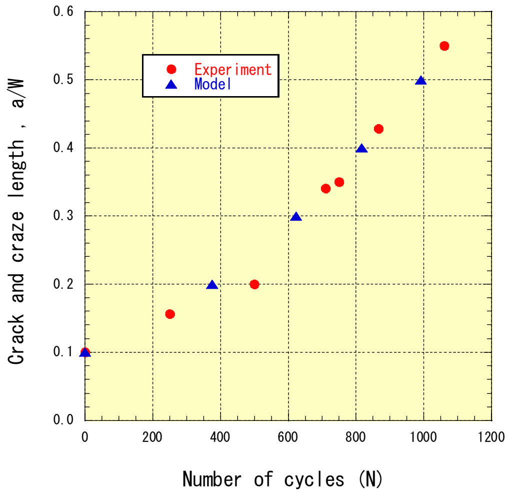
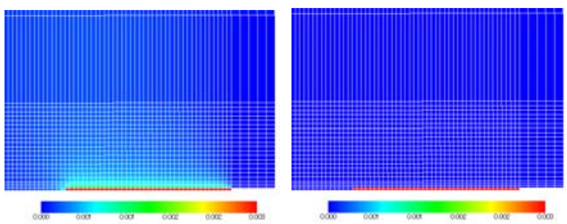
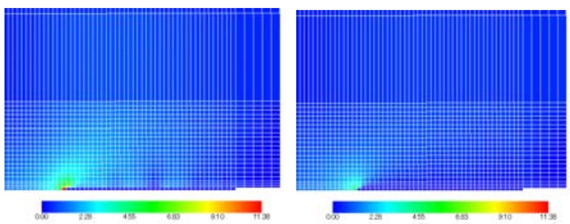


Fig. 5 Crack and craze length for PS



(a)before annealing (b)after annealing

Fig. 6 Distribution of damage for fatigued PS



(a)before annealing (b)after annealing

Fig. 7 Distribution of equivalent stress for fatigued PS

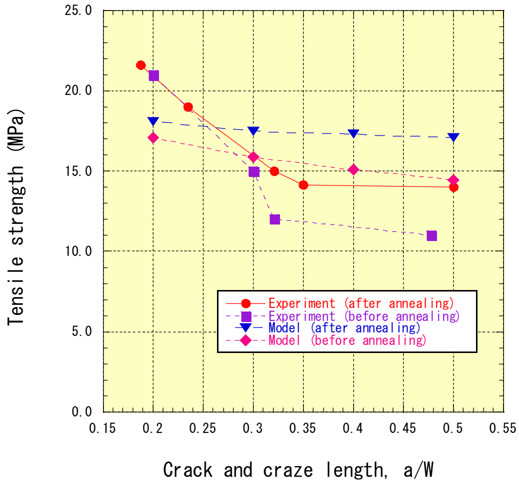


Fig. 8 Change in tensile strength of fatigued PS by annealing

VI. Conclusion

The concept of continuum damage mechanics has been extended so as to contain the self-repair process as well as the damage fracture behavior in the present study. The repair variable and the repair evolution equation have been introduced to describe the self-repair process, corresponding to the damage variable and the damage evolution equation. The detailed formulation has been conducted for the case when both influence additionally on the constitutive equations, depending upon the microscopic damage and the repair mechanism of the material.

The self-repair processes have been simulated for two kinds of engineering materials, the creep-damaged steel and the fatigue-damaged polymer. The validity of the present computational modeling has been illustrated by comparing the calculated results with the corresponding test results. It is considered that the extension of continuum damage mechanics to the self-repair process as proposed in the present study can also be applied effectively to brittle materials such as ceramics and concrete.

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