Dynamic Phasor Modeling and EMT Simulation of USSC

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Abstract – **This paper deals with the modeling of unified series-shunt compensator (USSC) which include an analysis of a simple test system based on dynamic phasor model. Its aim is to investigate the overall efficiency of USSC for power quality (PQ) analysis and results will be compared with EMTP like simulation. The dynamic phasor model is implemented in Matlab/Simulink toolbox where as the EMT model simulation of the USSC uses the PSCAD/EMTDC software. Credible solutions to the PQ problems on the distribution network have been analyzed using dynamic phasor model and EMT model simulation techniques. Simulation results of the USSC dynamic phasor model including the system makes a perfect agreement with the detailed time-domain EMTP like PSCAD/EMTDC simulation. It is found that the dynamic behavior of USSC phasor model have very good potential application in analyzing overall PQ issues, faster in speed and higher accuracy as compared with PSCAD/EMTDC simulation.**

Index Terms-- **Dynamic phasor model, unified series-shunt compensator, power quality, switching, control system**

I. INTRODUCTION

DEVELOPMENT of power electronic devices such as flexible ac transmission systems (FACTS) and custom power devices play an important role in emerging deregulated power system with versatile new control capabilities [1]. FACTS and custom power devices such as unified power-flow controller (UPFC), synchronous static compensator (STATCOM) dynamic voltage restorer (DVR), solid-state transfer switch, solid-state fault current limiter are developed for improving voltage regulation, steady-state and dynamic stability, reliability and power quality of the system [2, 3]. These individual devices are useful for compensating particular types of power quality problems. So as to solve the above problems, a new kind of USSC was developed [4] to mitigate a wider range of power-quality problems. The USSC is almost similar to the UPFC except that the UPFC is used in transmission systems whilst the USSC is used in distribution systems. Furthermore, UPFC inverters are in shunt-series connected whereas the USSC are in series-shunt connected. One important point is note that not much work has been carried out in the development of a USSC.

Modeling and simulation of power electronic devices in a system are generally developed using electromagnetic transient program (EMTP) simulation and quasi-steady-state (QSS) approximations [5, 6]. The EMTP concept is universally accepted for simulation of complex power system containing non-linearities, power electronic components and their controllers. However, due to limitation of computer storage and computational time, the implementation of a large power system in an EMTP is difficult, time critical in simulation with extremely small step-length and therefore, not suitable for transient stability study [7]. The QSS approximations models, although, commonly used in electro-mechanical transient simulation, the model is not adequate enough to catch the dynamic behavior of the switching [8]. Moreover, the time domain simulations of QSS is a significant computational burden and offers little insight into problem sensitive to design quantities and no basis for design for protection scheme [9]. Hence the QSS is impractical for simulation of power electronic device such as UPFC or USSC in large-scale power system.

Power electronic elements of USSC controller are difficult to model accurately due to their switching behavior [10]. As the EMTP and QSS approximations are not adequate enough to model the dynamic behavior of the switching, a simplified model of USSC with sufficient engineering accuracy is therefore required. The simplified USSC model must be fast and accurate modeling, simulation and be used in control design for improve overall PQ and dynamic stability analysis.

Dynamic phasor technique has been developed using generalized averaging procedure to study the effects of power electronic devices such as FACTS controller, custom power devices and HVDC transmission system [11]. Dynamic phasor theory has a great potential and its offer a number of advantages over conventional methods. Firstly, dynamic phasor can be used to compute the fast electromagnetic transient with larger step size, so that it makes simulation potentially faster than conventional time domain EMTP like simulation. Secondly, it has wider band width in the frequency domain than that of traditional QSS approximation. Thirdly, by keeping the dominant components in Fourier coefficient series, it can catch significant impact on switching of the power electronic devices.

In this paper, a generalized averaging technique is applied to obtain the dynamic phasor model of USSC including a simple test system for overall PQ analysis. Dynamic phasor model of USSC switching function and its control system are also elaborately described in this paper. A comparison of dynamic phasor model and PSCAD/EMTDC simulation is studied to see the close agreement between the two techniques. The dynamic phasor model has potential

This work was supported under grant: UKM-GUP-NBT-08-03-123.

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advantages in enhancing the power system stability as well as overall PQ solution in term of faster speed and higher accuracy.

II. Basic Concept of Phasor Dynamics

The concept of phasor dynamic is based on generalized averaging theory in the form of time-varying Fourier coefficient [11]. A complex periodic (time-domain) waveform $x(\tau)$ can be represented on the interval $\tau \in (t-T, t)$ using a Fourier series of the form

$$x(\tau) = \sum_{k=-\infty}^{\infty} X_k(t)\, e^{jk\omega_s \tau} \qquad (1)$$

where $\omega_s = 2\pi/T$ and $X_k(t)$ is the complex time-varying Fourier coefficient, which refers as dynamic phasors. The k[th] coefficient of dynamic phasor at time t is determined by the following averaging operation,

$$X_k(t) = \frac{1}{T}\int_{t-T}^{t} X(\tau)\, e^{-jk\omega_s \tau} d\tau = \langle x \rangle_k (t) \qquad (2)$$

where $<x>_k(t)$ is the complex quantity, which denotes the averaging operation. Dynamic phasor method is the function of frequency decomposition that focused on the dynamics of the Fourier coefficient. There are two key properties of the phasors and they are described as follows,

A. k-phasor Differential Properties

A key factor for dynamic phasor development is that the derivative of k[th] Fourier coefficient is given by the following expression,

$$\frac{dX_k}{dt}(t) = \left\langle \frac{dx}{dt} \right\rangle_k (t) - jk\omega_s X_k(t) \qquad (3)$$

This formula is easily verified using (1) and (2), and integration by parts.

B. Phasor Properties of a Product

The k[th] phasor of a product of two time-domain variables, $x(\tau)$ and $y(\tau)$ equals a discrete time convolution that can be obtained by the following expression,

$$\langle x\, y \rangle_k = \sum_{i=-\infty}^{\infty} \langle x \rangle_{k-i} \langle y \rangle_i \qquad (4)$$

III. Dynamic Phasor Modeling of USSC

The USSC is a combination of series and shunt voltage source inverters used for overall power quality mitigation in distribution system as shown in Fig. 1. The basic configurations of USSC consists of two voltage source inverter (VSI) connected in series and shunt with the line through a set of series and shunt injecting transformer. The dc terminals of two VSI are connected together and their common dc voltage is supported by a capacitor bank [4]. VSI-E realized the injection of $\dot{V}_{pq}$ in series with the distribution line by controlling magnitude and phase angle which in turn control current and power flow to the system. On the other hand, VSI-B is used to inject current I_{pq} to regulate the constant voltage V_R and V_d. The inverters output voltage V_E and V_B are obtained in term of inverter modulation index (m_E and m_B) of sinusoidal pulse width modulation (SPWM) controller and the measured dc link capacitor voltage V_d.

$$V_E = m_E V_d \qquad (5)$$
$$V_B = m_B V_d$$

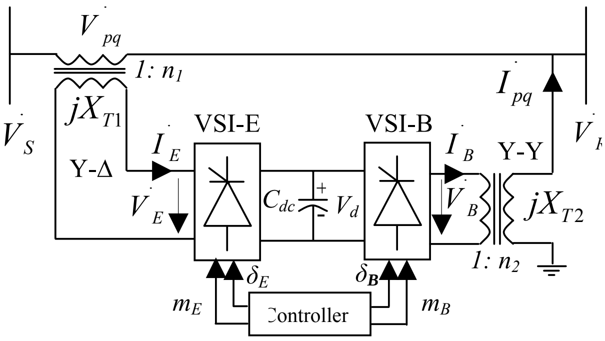

Fig. 1 Basic configuration of USSC

The phase angle of inverter output voltages are θ_E and θ_B, respectively, whose are controlled by the firing angle δ_E and δ_B. The relations among them are as follows,

$$\theta_E = \theta_S - \delta_E \qquad (6)$$
$$\theta_B = \theta_S - \delta_B$$

A. Modeling of USSC including Switching Function

Let us consider the equivalent circuit of phase 'a' as the reference phase and its valve switching simulated by the ideal switch-state function S_{Ea} and S'_{Ea} ($S_{Ea} + S'_{Ea}=1$) as shown in Fig. 2. The small resistance r_s is considered as an equivalent power loss. In VSI-E, the v_{Ea} and i_{Ea} are related at the ac system neutral point as follows,

$$v_{Ea} = v_{EaH} + v_{Hn} \qquad (7)$$
$$= [(i_{Ea} r_s + v_{dc}).\, S_{Ea} + (i_{Ea} r_s).\, S'_{Ea}] + v_{Hn}$$

For a balance ac system, it is easy to drive

$$v_{Hn} = -\frac{1}{3} v_{dc} \sum_{j=a,b,c} S_{Ej} \qquad (8)$$

Substituting (8) into (7), we have,

$$v_{Ea} = i_{Ea} r_s + v_{dc}.\, S_{Ea} - \frac{1}{3} v_{dc} \sum_{j=a,b,c} S_{Ej} \qquad (9)$$

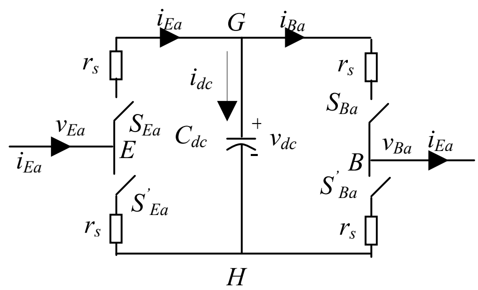

Fig. 2 Equivalent circuit of STATCOM

The dc capacitor dynamics can be described as

$$c_{dc}\frac{dV_{dc}}{dt}=i_{dc}\;=i_{Edc}-i_{Bdc}\qquad(10)$$
$$=\sum_{j=a,b,c}(i_{Ej}\,S_{Ej}-i_{Bj}S_{Bj})$$

According to (9) and (10), keeping fundamental frequency component (*k=1*) and dc component (*k=0*), based on phasor product properties, the dynamic phasor of v_{Ea} and v_{dc} are,

$$\langle V_{Ea}\rangle_1=\langle I_{Ea}\rangle_1 r_s+\langle V_{dc}\rangle_0.\langle S_{Ea}\rangle_1-\frac{1}{3}\langle V_{dc}\rangle_0\sum_{j=a,b,c}\langle S_{Ej}\rangle_1\qquad(11)$$

$$\frac{d\langle V_{dc}\rangle_0}{dt}=\frac{1}{C_{dc}}\sum_{j=a,b,c}(\langle I_{Ej}\rangle_1\langle S_{Ej}\rangle_{-1}-\langle I_{Bj}\rangle_1\langle S_{Bj}\rangle_{-1})\qquad(12)$$

For shunt inverter VSI-B, an equation similar to (11) can also be developed.

The switch-state function S_{Ea} and $S^{'}_{Ea}$ of inverter VSI-E are discrete and periodic function of time, which is determined by SPWM control. The fundamental wave component and dc component of the switching function S_{Ea} can be represent as [10],

$$d_{Ej}=\frac{m_E}{2}\cos(\omega t-\delta_E-\sigma_j)+\frac{1}{2}\qquad(13)$$

Where $j = a, b, c$ and $\sigma_a=0$, $\sigma_b=2/3\pi$, $\sigma_c=4/3\pi$

For phase ‘a’, the dynamic phasors of the dc and fundamental wave components are,

$$\langle d_{Ea}\rangle_0=\frac{1}{2},\ \langle d_{Ea}\rangle_1=\frac{m_E}{4}e^{j\theta_E}=\langle d_{Ea}\rangle_{-1}=\frac{m_E}{4}e^{-j\theta_E}\qquad(14)$$

Substituting dynamic phasor d_{Ej} in (14) for S_{Ej} in (11) and separating real and imaginary parts, we obtain the overall dynamic phasor model of USSC are as,

$$\langle V_{Ea}\rangle_1^r=\langle I_{Ea}\rangle_1^r.\,r_s+\langle V_{dc}\rangle_0.\frac{m_E}{4}\cos\theta_E$$
$$\langle V_{Ea}\rangle_1^i=\langle I_{Ea}\rangle_1^i.\,r_s+\langle V_{dc}\rangle_0.\frac{m_E}{4}\sin\theta_E\qquad(15)$$
$$\langle V_{Ba}\rangle_1^r=-\langle I_{Ba}\rangle_1^r.\,r_s+\langle V_{dc}\rangle_0.\frac{m_B}{4}\cos\theta_B$$
$$\langle V_{Ba}\rangle_1^i=-\langle I_{Ba}\rangle_1^i.\,r_s+\langle V_{dc}\rangle_0.\frac{m_B}{4}\sin\theta_B$$

$$\frac{d\langle V_{dc}\rangle_0}{dt}=\frac{m_E}{C_{dc}}\frac{3}{2}[\langle I_{Ea}\rangle_1^r.\cos\theta_E+\langle I_{Ea}\rangle_1^i.\sin\theta_E]\qquad(16)$$
$$-\frac{m_B}{C_{dc}}\frac{3}{2}[\langle I_{Ba}\rangle_1^r.\cos\theta_B+\langle I_{Ba}\rangle_1^i.\sin\theta_B]$$

Generalized averaging theory of ac and dc components of USSC series and shunt inverter dynamic phasor (15) and (16) are interfaced with ac network can be represented as,

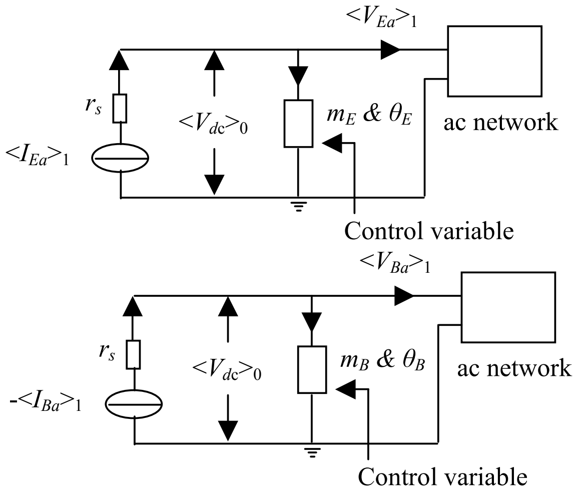

Fig. 3 USSC dynamic phasor equivalent circuit interfacing with ac network

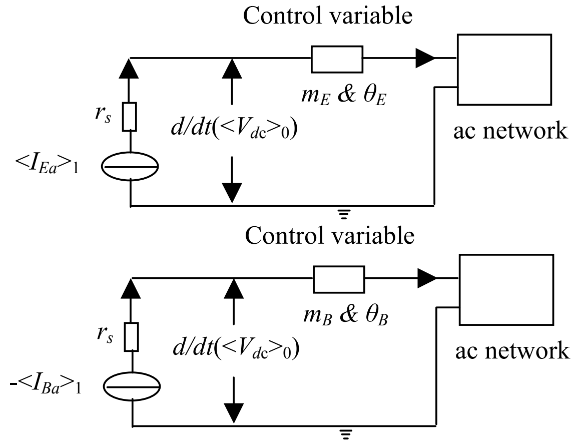

Fig. 4 USSC dc circuit dynamic phasor equivalent interfacing with ac network

where, $<Vdc>_0$ is the state variable; m_E, δ_E, m_B, and δ_B are the control variable, acquired from the control system of the USSC.

Thus, at steady-state, the dynamic phasor model of (15) and (16) interfaced with the ac network equations can measure the phasor dynamics of USSC and system parameters.

B. Modeling of USSC control

The series and shunt VSI control strategies of USSC adopted in this paper are shown in Fig. 5 and Fig. 6, respectively. The constant ac terminal voltage is achieved by controlling m_E and δ_E of the SPWM controller in VSI-E as shown in Fig. 5. The USSC output series compensation voltage $\dot{V}_{pq}$ is decomposed as $\dot{V}_p$ and $\dot{V}_q$, these voltages have significant effect on controlling real and reactive power flow of the system. The PI controlled PLL generated synchronizing signal of the system voltage θ_S, which can be used efficiently to calculate the USSC output voltage phase angle θ_E. The constant load current is achieved by controlling

m_B and δ_B of the SPWM controller in VSI-B as shown in Fig. 6. If the phase angle θ_S of V_S and USSC control output m_E, δ_E, m_B and δ_B are

known, the dynamic phasor model of USSC with an interface to the ac system can be used to represent transient stability analysis.

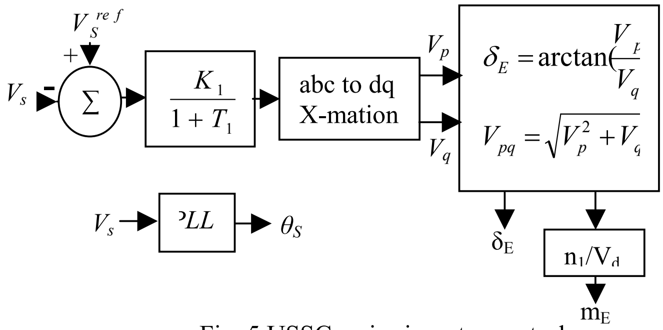

Fig. 5 USSC series inverter control

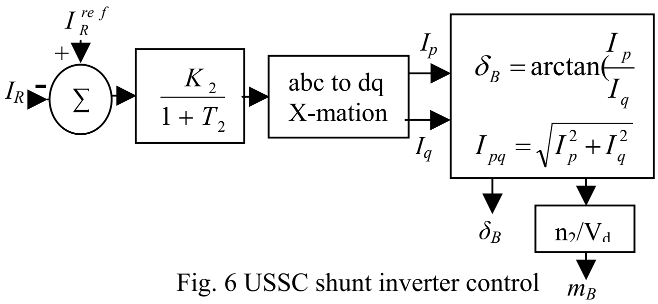

Fig. 6 USSC shunt inverter control

IV. Results and Discussion

A simplified test distribution system has been implemented to perform PQ analysis using USSC, in order to verify dynamic phasor model and later comparing it to the EMT model simulation. The dynamic phasor model and EMT model simulation of the USSC including the system is implemented in Matlab/Simulink toolbox and PSCAD/EMTDC, respectively. To illustrate the results obtained with the models of the network elements, the USSC is connected in shunt to the loads and series to the 22 kV, 10 MVA systems as shown in the single line diagram in Fig. 7. The effectiveness of USSC as a unified compensator for voltage regulation, sag compensation, flicker reduction, harmonic elimination, unbalance mitigation, UPS mode function, power flow control etc. are evaluated using phasor dynamics and EMTP like PSCAD/EMTDC simulation. Table I shows the detailed load characteristics of various ratings in the test distribution system.

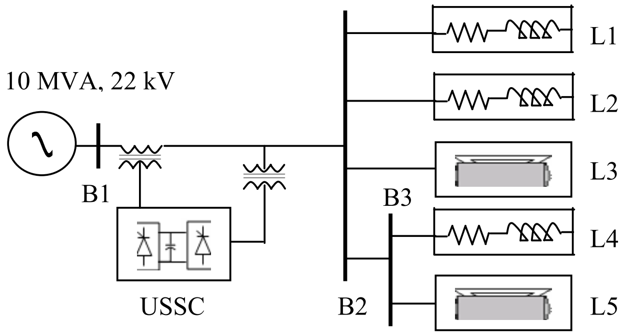

Fig. 7 USSC test distribution system

Table 1. Loads in the test distribution system

Load (L)	Type	Rating	Parameters	
			R (ohms)	L (Henry)
L1	RL	1.2 MVA, 0.90 pf	363.33	0.46
L2	RL	0.5 MVA, 0.95 pf	871.21	1.119
L3	IM	1600 H P, 50 Hz	---	---
L4	IM	1600 H P, 50 Hz	---	---
L5	RL	1.2 MVA, 0.90 pf	363.33	0.46

A. USSC dynamic phasor model validation

To illustrate the effectiveness and validate the USSC dynamic phasor model, PSCAD/EMTDC simulation of the test distribution system is carried out. Results are superimposed to observe the accuracy between the waveforms. Fig. 8 shows the USSC model in compensating voltage sag, in which red line represents the prediction of dynamic phasor model, while the blue line corresponds to the time-domain PSCAD/EMTDC simulation of phase 'A'. A voltage sag condition is simulated by creating a balance three-phase to ground fault at time t = 1.5 s for a duration 0.2 s as shown in Fig. 8 (a). The system with the USSC connected, load voltage manage to recover from 0.50 p.u. to its rated voltage due its voltage sag compensation capability as shown in Fig. 8 (b). Both, Fig. 8 (a) and Fig. 8 (b) shows that the dynamic phasor models of USSC load voltage in both conditions are closely hug with the PSCAD/EMTDC simulation. In addition, due to larger integration step, dynamic phasor of USSC model has faster simulation speed of 2.7 s, while PSCAD/EMTDC simulation cost 8.7 s, which is much larger than that of dynamic phasor model.

a)
b)

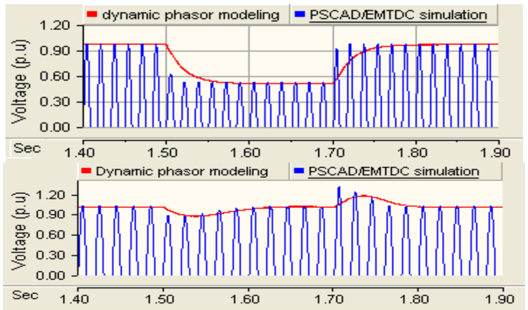

Fig. 8 Load voltage of a test system using dynamic phasor model and PSCAD/EMTDC simulation a) without USSC b) with USSC

B. USSC capability on flicker reduction

To illustrate the use of the USSC in reducing voltage flicker, simulations were carried out by first connecting a variable electric load of 5.2 MVA, 22 kV as the source of voltage flicker. The rms value of phase a voltage flicker is shown in Fig. 9 (a). It is found that voltage flicker index is 0.40 measured by FFT technique in which the value exceeds IEEE standard limit of 0.07. However, with the USSC connected, it is noted that the calculated voltage flicker index is reduced to 0.02 as shown in Fig 9 (b). These results prove that the USSC can be used to reduce voltage flicker. Results as shown in Fig. 9 (a) and Fig. 9 (b) imply that the dynamic phasor models are effective and provide very good accuracy compared to PSCAD/EMTDC simulation.

a)
b)

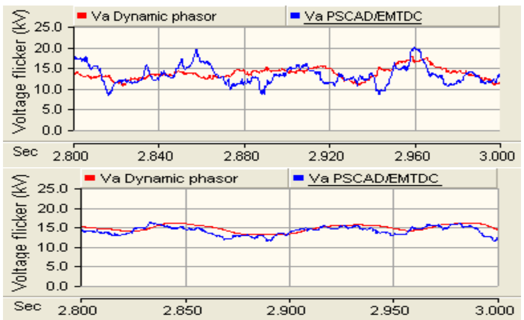

Fig. 9 RMS voltage flicker using dynamic phasor model and PSCAD/EMTDC simulation a) without USSC b) with USSC

C. USSC Harmonic reduction analysis

Keeping frequency component of $k = 1, 2, 3$ ... and dc component $k = 0$ in (9) and (10), the dynamic phasor of total harmonic distortion (THD) is measured. The test result of dynamic phasor model and PSCAD/EMTDC simulation shows that USSC can generate almost 61% of THD at fault period of 1.5 s to 2.25 s due to its high frequency switching loss as shown in Fig 10 (a). This value is still higher than the acceptable level of 5%. In order to reduce the harmonics, a passive LC filter is connected at the load of the distribution system. Fig 10 (b) shows the dynamic phasor model and PSCAD/EMTDC simulation of THD of the system including USSC with connected passive filter. It can be seen that the harmonics are suppressed and the THD of the system is reduced to 2.05% from 61%, which is far below the IEEE standard THD limit of 5%. Fig. 10 (a) and Fig. 10 (b) confirm that the dynamic phasor models make an excellent agreement in terms of consistency and accuracy with the time-domain PSCAD/EMTDC simulation as well as faster in computing the simulation. Hence, in both cases, USSC with and without filter, the rms values of dynamic phasor model are in satisfactory agreement to the PSACD/EMTDC simulation.

a)
b)

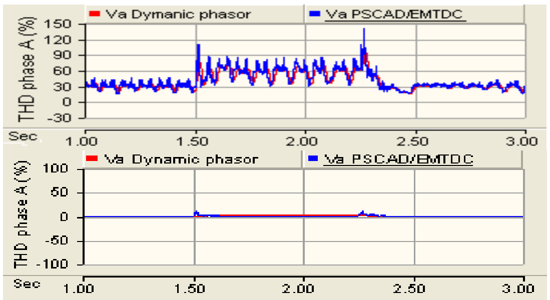

Fig. 10 THD of a USSC system using dynamic phasor model and PSCAD/EMTDC simulation a) without filter b) with filter

D. USSC unbalance voltage mitigation analysis

To investigate how USSC with system connected mitigates PQ problem, we have created an unbalance condition involving of unbalance voltage. This is done by applying two single phase to ground fault on the phase 'A' and 'C' of the RL load, L4 at time t = 0.5 s for duration of 0.1 s. Fig. 11 (a) shows the dynamic phasor model and PSCAD/EMTDC simulation of unbalance voltage without USSC connected to the system. The percentage of imbalance is found to be about 27.5 %, which is considered as severe when compared to the IEEE standard unbalance of 2%. By introducing the USSC application, however, dynamic phasor model and PSCAD/EMTDC simulation for the unbalance load voltage profile has been improved. It can be seen that the phase 'a' and 'c' voltages are increased to its approximate rated value. Thus the system load voltages are more balanced as shown in Fig. 11 (b).

a)
b)

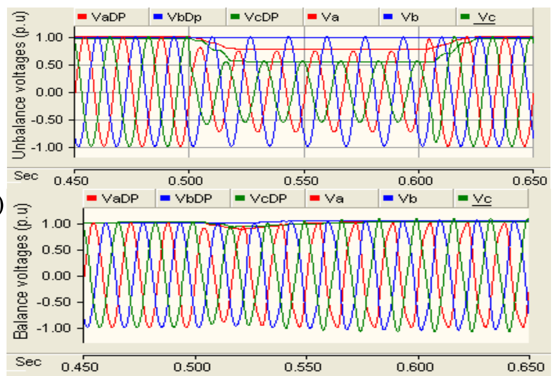

Fig. 11 Load voltages using dynamic phasor model and PSCAD/EMTDC simulation a) unbalance without USSC b) balance with USSC

ISBN:978-988-17012-6-8

The percentage of unbalance is about 1.9 %, in which it is within the limit of IEEE standard of unbalance. Fig. 11 (b) also shows that during the fault period a slight displacement between the phases has been noted due to the switching control of the USSC.

In both cases, dynamic phasor model excellently match the time-domain PSCAD/EMRDC simulation. It was also observed that the dynamic phasor model is typically 2 or 3 time faster than PSCAD/EMTDC simulation. Additionally, with the sudden load unbalance voltage of PSCAD/EMTDC simulation, the dynamic phasor model are still able to provide satisfactory agreement of tracking between the waveforms.

E. USSC function as UPS

To illustrate the use of USSC as an uninterruptible power supply (UPS) mode, an outage condition is created by applying a balance three phase to ground fault on the RL load L1 at time t = 1.5 s for a fault duration of 0.2 s. Fig. 12 (a) shows that the phase A voltage at load L1 drops to zero due to the 3-phase fault, where fault impedance X/R ratio equal to 1. In Fig. 12 (b), the sag down voltage is compensated with USSC connected by increasing load voltage zero to its rated value. The load voltage returns to almost its rated value as a result of the voltage sag compensation capability of the USSC. It also can be seen that the minimum and maximum load voltages at the starting and ending fault period of dynamic phasor model and PSCAD/EMTDC simulation, respectively are still within the IEEE standard limit. In both cases, dynamic phasor model and the time-domain EMT model simulation waveform match excellently. For the case of sudden voltage sag down, both techniques are still able to provide satisfactory results in which the waveforms are very much in agreement.

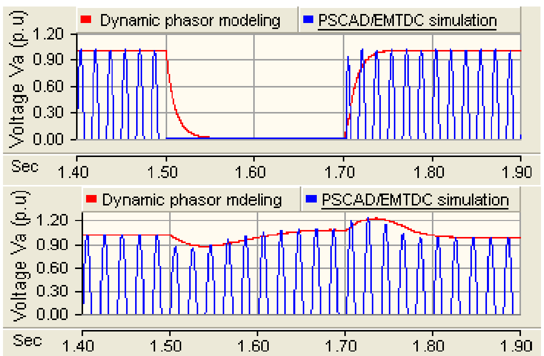

Fig. 12 Load voltage compensation capability using dynamic phasor model and PSCAD/EMTDC simulation a) without USSC b) with USSC

F. Power flow controller

The active and reactive power flow of test distribution system with or without USSC connected is investigated using the dynamic phasor model and PSCAD/EMTDC transient simulation. At fault condition, without USSC connected, both the active and reactive powers of the system are increased as shown in Fig 13 (a), in which dynamic phasor model and PSCAD/EMTDC simulation has good agreement between the waveforms. From the USSC operation point view, the increase in the reactive power requires an increase in the active power injection and an increase in the active power supplied by the shunt inverter of the USSC. The exchange of real power can be made in either direction between the series and shunt inverters of the USSC.

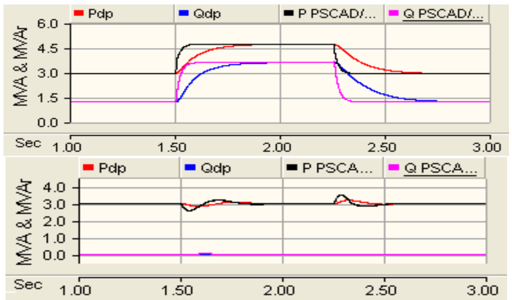

Fig. 13 Real and reactive power flow using dynamic phasor model and PSCAD/EMTDC simulation a) without USSC b) with USSC

With the USSC connected in the system, the reactive power of the system is reduced from 1.45 MVAr to zero in order to achieve a steady-state value of active power, as shown in Fig. 13(b). In here, it can be seen that the dynamic phasor model has good agreement with PSCAD/EMTDC simulation and it also provides more steady active power flow to the system. Thus, the active and reactive power flows can be controlled and maintain at a pre-fault levels.

V. Conclusion

Dynamic phasor model were used to model USSC including a simple test system to analysis the PQ issues using Matlab/Simulink and validated by comparing its results with PSCAD/EMTDC simulation. The test results shows that the dynamic phasor model achieve a very good accuracy and faster in speed compared with standard time-domain PSCAD/EMTDC simulation. In brief, it can be concluded that the proposed model is well suited for PQ mitigation analysis since it is accurate and performs faster dynamic simulation.

References

[1] N. G. Hingorani and L. Gyugyi, "*Understanding FACTS Concept and Technology of Flexible AC Transmission System"*, New York: IEEE Press, 2000.

[2] A. L. Olimpo and E. Acha, "Modeling and analysis of custom power systems by PSCAD/EMTDC," *IEEE Trans. Power Delivery*, vol. 17, no. 1, pp. 266–272, Jan. 2002.

[3] M. L. Crow, "Power quality enhancement using custom power devices", *IEEE Power and Energy Magazine*, Vol. 2, No. 2, pp. 50, Mar-Apr 2004.

[4] M. A. Hannan and Azah Mohamed, "PSCAD/EMTDC simulation of unified series-shunt compensator for power quality improvement", *IEEE Transactions on Power Delivery,* Vol. 20, No. 2, PP. 1650-1656, 2005.

[5] L. X. Bui, *G.* Mortin and J. Reeve, "EMTP TACS-FORTAN interface development for digital controls

modeling", IEEE *Trartsacfiom* of *Power Systems.* Vol. 7, No. l, pp. 314-319, 1992.

[6] P.C. Stefanov and A.M. Stankovic, ‘‘Modeling of UPFC operation udder unbalanced conditions with dynamic phasors", *IEEE Transactions on Power Systems,* Vol. 17, *No.* 2, pp: 395-403, 2002.

[7] M A Hannan and K W Chan, "Modern power systems transients studies using dynamic phasor models", *International Conference on Power System Technology* (POWERCON 2004), pp. 1469-1473, Singapore, 21-24 November 2004.

[8] Gilead Tabor, "On approximate phasor models in dissipative bilinear systems", *IEEE Transactions on Circuits und Systems-1: Fundumentul Theory and Applications,* Vol. 49, No. 8, pp. 1167-1179, 2002.

[9] Ruiwen He and Zexiang Cai, "Modeling and harmonic analysis of TCSC with dynamic phasors", *IEEE/PES Transmission and Distribution Conference & Exhibition: Asia and Pacific*, pp. 1-5, Dalian, China, 2005.

[10] E. Zhijun, K W Chan and F. Dazhong, "A practical dynamic phasor model of static VAR compensator", *2nd International Conference on Power Electronics Systems and Applications,* pp. 23-27, Hong Kong, China, 2006.

[11] S. R. Sanders, J. M. Noworolski, X. Z. Liu, and G. C. Verghese, "Generalized averaging method for power conversion circuits", *IEEE Transactions on Power Electronics,* Vol. 6*,* No. 2, pp. 251-259, 1991.