

An Adaptive Tutoring System for Calculus Learning

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Abstract— One-on-one tutoring is known to be an effective model for calculus learning. Then implementing one-on-one tutoring function into calculus learning software is a natural thing to do. In this article, we describe research on how to diagnose a student's knowledge structure about calculus without asking many questions. Then we show how an adaptive tutoring system is implemented into our calculus learning software JCALC using a relative distance.

Index Terms— adaptive tutoring system, student's knowledge structure, relative distance.

I. INTRODUCTION

We have been giving placement exams to the incoming students to measure how well they are prepared for college level calculus for the last ten years. This is partly because most of the students entered universities in the last ten years were educated under the new government course guidelines made by the ministry of education [9] which cut school hours for mathematics and physics in junior and senior high school. Another reason is that many universities quit requiring calculus questions in entrance exams [11].

To speed up their learning, we set up the standard course which offers a lecture once a week and the intensive course which offers lectures twice a week. After assigning students to either standard course or intensive course supposedly suited for their ability according to their placement-exam scores, we teach them calculus for four months. Then at the end of the semester, we give students unified-calculus exams to see if they learned enough to go on to the next stage. In the last three years, about 20% of the students are failed to go on to the next stage. Teaching some of those students who failed to go on to the next stage, we are convinced that these students are having trouble in pre-calculus. So, we decided to set up the supporting rooms to answer students' questions. But later on, we found out that the students visited the supporting rooms are relatively well-prepared students.

Among those students with bad placement-exam scores, no matter what we do for class coordination or instructional designs, there are students without much improvement. When we examined these students more carefully by analyzing questionnaire survey results about their study hours at home, we found out that these students have almost no hours of study at home. They are not bad students, simply not accustomed to self-study. On the other hand, by examining students who have studied calculus using

mathematical learning software JCALC, we found out that these students have the lowest rate of withdrawal. From this, we speculate that JCALC is somehow encouraging students to get used to self-study.

Well known effective-educational model for less prepared students is one-on-one tutoring [1]. But, one-on-one tutoring is not a realistic solution for our school system because of cost. These factors lead us to hypothesize that if Intelligent Tutoring Systems (ITSs) with one-on-one tutoring is implemented into JCALC and blended learning is utilized, less prepared students will acquire self-study skill and achieve better performance in learning calculus. Even though ITSs are becoming popular in among students at pre-college level mathematics courses [4], [8], designing ITS which accurately diagnose students' knowledge structure, skills, and styles is not easy. To implement ITS, as in [6], [7], we look in detail at what human can do in the process of tutoring a calculus student using short-answer questions

In this article, we show how an adaptive tutoring system is implemented into JCALC. Then we verify whether the hypothesis is true or not by selecting 36 students for JCALC use out of 220 students who failed to pass the unified exam given in the spring semester. Then we examine the difference of the slope of linear regression of the group with JCALC and the one without JCALC and learned that the hypothesis is valid.

II. PRODUDING SELF-GRADED SHORT-ANSWER QUESTION

A. *Classifying short-answer questions*

To produce short-answer calculus questions, we classify as in [13] the questions appearing in calculus into four categories:

1. A simple sentence type question:
 - (a) Differentiate $y = 2xe^{3x}$.
 - (b) Find the integral of $x/(x^2 + 1)$
2. A compound sentence type question:
 - (a) Find an equation of the tangent line of $y = 2xe^{3x}$ at $(1, e)$.
 - (b) Find the area of the region bounded by $y = 2xe^{3x}$ and $y = x + 2$
3. A complex compound sentence type question:
 - (a) A straight line $px + qy = 1$ ($p > 0, q > 0$) is tangent to the unit circle $x^2 + y^2 = 1$. Let V be the volume of the solid generated by rotating the region bounded by this tangent line and both axes around y -axis. Find the minimum of V
4. Proof type question:
 - (a) Show there are infinitely many primes.

We then divide questions into methods and arguments. For example, "Differentiate $y = 2xe^{3x}$ " is divided into

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“Differentiate”, and “ $y = 2xe^{3x}$ ”. For a compound sentence type question, we note that a question is made up with two simple sentences. For example, “Find an equation of the tangent line of $y = 2xe^{3x}$ at $(1, e)$ ” can be divided into two questions: “Find the slope m of the tangent line of $y = 2xe^{3x}$ at $(1, e)$ ” and “Find the straight line go through $(1, e)$ with the slope m ”. Now we compose these two questions. Then the result of the first question can be passed into the second question as an argument. Similarly, we can express a complex compound sentence type question. In JCALC, we do not consider any proof type questions.

B. Generating short-answer questions

In section 1.A, we saw that calculus questions except a proof type can be produced by composing methods and arguments. Thus for example, to produce a simple sentence type question, all we need to prepare is a list of methods and arguments. We list some of arguments available in JCALC: $\{x^{ax+b}, \sin(ax+b), \cos(ax+b), \tan(ax+b), \sec(ax+b), \csc(ax+b), \cot(ax+b), \sin^{-1}(ax+b), \cos^{-1}(ax+b), \tan^{-1}(ax+b), e^{ax+b}, \log(ax+b), \dots\}$ Then most of the elementary functions appearing in calc textbooks such as [2] and [12] can be generated by binary operation and composition of the list of arguments. We now show how JCALC produces a short-answer question like “Differentiate $y = 2xe^{3x}$ ”.

1. Once a student selects a subject named differentiation and item named a product rule, a form named product rule is called upon and displayed.
2. When a student pushes “Start” button, the string “differentiate” is displayed.
3. At the same time, a component named “generate problem” randomly generates values for d and f to get “ $dx + f$ ” and for a and b to get e^{ax+b} for a corresponding argument.
4. Then in the product rule form, these expressions are put together to produce $(dx + f)e^{ax+b}$

By changing the values of integers a, b, c, d in the range of $-5, 5$, we can create ten thousand distinct arguments. Examining several calculus books mentioned above, we put in JCALC twelve sections on functions, eight sections on derivatives, six sections on integrals, and four sections on partial derivatives and double integrals. Thus, JCALC can generate a several thousand short-answer questions.

C. Producing a correct answer

When JCALC generates a short-answer question like “Differentiate $y = 2xe^{3x}$ ”, JCALC also generates the sentence such as $d(2xe^{3x})/dx$ by translating “Differentiate” into “ d/dx ”. Assuming JCALC can execute the expression $d(2xe^{3x})/dx$, we see that producing a correct answer to the question is same as executing the expression $d(2xe^{3x})/dx$. Thus, an answer can be generated from the list of methods and the list of arguments. In other words, when a student chooses a product rule from the menu, JCALC makes a one-to-one correspondence between the item chosen and the method d/dx which in turn gives rise to a one-to-one correspondence between the method d/dx and the argument $2xe^{3x}$, and the result of execution of $d(2xe^{3x})/dx$. This way, we can generate not only short-answer questions but also corresponding answers.

III. TESTING ANSWER

In section II.C, we showed that how to generate a correct answer for a short-answer question. Now we ask how we determine whether a student’s answer is correct or not. In this section, we answer this question.

A. Mathematical expressions

Most of answers to calculus questions involve mathematical expressions. So our system must have an ability to read mathematical expressions and understand them. But understanding mathematical expressions is not easy thing to do by computer. To overcome this problem, we put some restriction on how answers be written by students. One of natural restriction is requiring students to write answers in TEX format since TEX is widely used.

As in [1], we also consider a handwriting input for mathematical expressions using a pen tablet. Since speech recognition is available, we also consider voice input for those having trouble writing in TEX format.

B. One-point criterion

As we said before, the method we explain in this section works for elementary functions which appear in calculus. Suppose a generated question is “Differentiate $x^2 + \sin 2x$.” and a student answer is $2(\cos^2 x + x - \sin^2 x)$. Suppose also a correct answer generated by JCALC is $2x + 2 \cos 2x$. Then JCALC selects a testing value, say $x = 1.3$ to evaluate the values of $2(\cos^2 x + x - \sin^2 x)$ and $2x + 2 \cos 2x$. In this case, the values of the two functions are both 0.886222, and JCALC can decide these two functions are equal. The reason why JCALC can conclude these two functions are equal is explained in section IV. We note that testing values are assigned for each function to satisfy the following two conditions:

1. An assigned value has to be in the domain of function.
2. An assigned value has to be a decimal number with odd last digit.

As you can see, a one-point criterion only involves a straight forward numerical calculation. Unlike other mathematical software, it does not require transformations of expressions into standard forms to test two functions are equal.

C. Testing student answer without generating a correct answer

If we look at what we showed in section IIIB a little more carefully, then since one-point criterion requires only the value of function at some point, JCALC does not have to generate a correct answer. Since we can approximate the value of the derivative of function by using a numerical formula, instead of generating a correct answer, we can directly get the value of derivative of the required function. We illustrate what we are saying here by using an example. Suppose the generated question is “Differentiate $x^2 + \sin 2x$ ” Then JCALC execute $d(x^2 + \sin 2x)/dx|_{x=1.3}$. When JCALC reads a string of the form $d(x^2 + \sin 2x)/dx$, JCALC considers the function $f(x) = x^2 + \sin 2x$. Then form a difference quotient $[f(1.3 + h) - f(1.3)]/h$ and evaluate the value of the difference quotient by using three-point formula or five-point formula for $h \rightarrow 0$. This limiting value is assigned as the correct answer value to the question of $d(x^2 + \sin 2x)/dx$. Then JCALC compares the values of student’s answer and this limiting value to see the student’s answer is correct. Thus, knowing what to do

with d/dx , we not only can produce a correct answer value to any differentiation questions but we do not have to provide a correct answer for each question.

D. Why one-point criterion works.

We first show why one-point criterion works for the case of polynomials. Suppose we have two functions:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

$$g(x) = b_n x^n + b_{n-1} x^{n-1} + \dots + b_1 x + b_0,$$

where $-10^i \leq a_i, b_i \leq 1$ for all $i, 0 \leq i < n$ and $l \leq m$. Let the testing value be denoted by $d(m) = 1.d_1 d_2 \dots d_m$ with $\gcd(d_j, 10) = 1$ for all j . Suppose also that the values of these two functions at the testing value are equal. Then we have $f(d(m)) = g(d(m))$. Now compare the last digit of $a_n d(m)^n$ and $b_n d(m)^n$. Then the last digits of $a_n d(m)^n$ and $b_n d(m)^n$ do not get any influence from other digits. Thus they are equal to each other. Furthermore, the last digit of $a_n d(m)^n$ is relatively prime to 10 by the choice of $d(m)$. Thus, we can conclude $a_n = b_n$. Similar argument can show $a_i = b_i$ for all $i, 0 \leq i < n$. This shows that one-point criterion works for polynomials.

To show one-point criterion works for other functions, we first note that all elementary functions can be expressed using Taylor expansions. Now if we chop off terms of Taylor expansions after the n th term, then it is known that the error caused by this process is much smaller than 10^{-n} . This shows that if the values of two functions are equal up to n th decimal digit, the probability of their not equal is less than 10^{-n} . If we choose n to be 16, then the error size is the same as the size of the round-off error caused by 64 bit machine.

E. Displaying a correct answer

By one-point criterion, to test a student's answer is correct or not, we saw that it is not necessary to provide a correct answer. But to display a correct answer, we certainly need to generate a correct answer. To overcome this problem, we only have to develop formulas to generate correct answers for all questions. For example, if we want to find a derivative of rational function such as $(ax^2 + bx + c)/(dx^2 + ex + f)$, then by using the quotient rule for differentiation, we only need to provide the following formula as a right answer.

$$\frac{(ae - bd)x^2 + (2af - 2cd)x + bf - ce}{(dx^2 + ex + f)^2}$$

This is the key step in making a correct answer. By changing parameters a, b, c, d, e, f , we can generate a correct answer for each question.

IV. ADAPTIVE TUTORING SYSTEM

A. How to tell a right answer from a wrong answer

The field we are in is college level calculus. The major component of educational process here is solving quantitative problems. This requires the students to be able to manipulate the followings:

- 1) Solving algebraic equations.
- 2) Evaluating elementary functions such as trig functions and inverse trig functions.
- 3) Finding the limit of sequences and the limit of functions.
- 4) Differentiating elementary functions and composite functions by using logarithmic differentiation.
- 5) Evaluating definite integrals by using u-substitution, partial fraction, and by parts.

- 6) Setting up integrals to express an area or a volume of the region bounded by simple curves.

Most of steps in 1) through 6) and the solutions have no unique correct answer. For example, if a student writes $-2/(\tan(x/2) + 1)$ for the answer to the question of "differentiate $1/(1 + \sin x)$ ", it is a right answer. Also, if a student writes $\tan x - \sec x$ for the same question, then this too is a right answer. So, judging a student input is a right answer or not, we cannot simply list the right answers and compare words by words. For this reason, we note that any system which produces multiple choice questions and answers is not suited for college level calculus. This suggests that we must have a system which handles short-answer questions. Furthermore, according to [10], short-answer questions have the advantage of avoiding cueing rather than selecting or guessing from options supplied. Thus, in our system, a student must enter his/her answer in the form of mathematical expressions. This also suggests that our system must be able to read a student's input and be able to tell whether it is a right answer or not.

The expressions for right answers are not unique, rather unlimited. Thus, preparing all expressions for right answers in database, and checking whether students' answer is in database to decide the students answer is correct is not plausible. So, we must develop some other way to tell right answers from wrong answers. We first look at the values of the power of x , say at $x = 1.3$. Then we have $x^2 = 1.69, x^3 = 2.197, x^4 = 2.8561, \dots$. As you can see that the last digit's decimal place increases one place each time. This means that the counting the decimal places appear in expression, it is possible to tell which power of x the expression contains. Which in turn implies that two polynomial expressions are evaluated to be equal imply that they are exactly the same expressions. So to decide the student input is a right answer or not, we need to have a correct answer to check with. Noting that every elementary function can be approximated by the polynomial, we can determine that a student input is right answer or not by checking the value of a student input and a correct answer at some point.

B. Inferring Student knowledge structure

We have shown [13] that it is possible to determine the student input is right answer or not by evaluating the correct answer generated by JCALC and a student input at certain value. We studied this method carefully to notice that when the value of the correct answer and the student input are different, their difference or ratio has some tendency among group of students. Suppose that a displayed question is "find a derivative of $(x^3 + 2x^2 + 3)^4$ " and a student input is " $(x^3 + 2x^2 + 3)^3(3x^2 + 4x)$ ". Furthermore, the correct answer generated by JCALC is " $4(x^3 + 2x^2 + 3)^3(3x^2 + 4x)$ ". Looking at the student input, anyone with calculus teaching experience judges that this student has the knowledge of derivative of composite function because he/she has first took care of derivative of power function then worked inside function. We have gone through all types of questions appearing in calculus textbooks mentioned above, and the similar results are obtained.

Suppose this time that the student input is " $4(3x^2 + 4)^3$ ". Then again anyone with calculus teaching experience would say that this student did not master the rule of derivative of composite functions. It is because the derivative of the inside function is taken before the derivative of the power function.

This time it is not easy to design our system to judge the same way as the experienced educator. For students inputs vary many ways and it is impossible to cover all.

C. The relative distance

In section 1B, we showed that how we comprehend a student knowledge structure by reading the student's input and comparing the student's input with a correct answer. Then, we judge students' knowledge structure by the knowledge structure of educators. This is a quite common activity in ordinary teaching environment. Now we want to convey this idea into calculus learning software. If we can develop software capable of reading student input and understanding what is written, then we do not have to do anything. But right now there is not any software equipped with an artificial intelligence to understand mathematical expressions. So, we need to think of other way to assess students' knowledge structure. We look at the case that how experienced educators assess student if he/she knows the material. In this case, there are essentially two types to judge whether the student knows a material or not. One of the two is the case where a student input and a generated correct answer differ by constant. The other one is the case where the ratio of a student input and a generated correct answer differ by constant. Here, we define the ratio or difference of the evaluated values of the student's input and a correct answer. We first define the distance d : Let vin and vca be defined as follows:

vin = the value of the input evaluated at certain point.

vca = the value of the correct answer evaluated at certain point. Then we define

$$d = \begin{cases} \text{the difference of } vin \text{ and } vca \\ \text{the ratio of } vin \text{ and } vca \end{cases}$$

With this distance d , we can assess student knowledge as correctly as an experienced educator can.

Now we look at the case how experienced educators judge student if he/she does not know much about the material. In this case, even an experience educator cannot conclude whether a student knows a material a little or none. We only can say that a student making this type of mistakes needs more help. Here, we define the relative distance by

$$rd = \frac{d}{\text{evaluated value of a correct answer}}$$

We explain how the relative distance can be used to assess a student's knowledge structure. If the value rd is large, then d must be very large compared with the evaluated value of correct answer. Then it is quite natural to assume that students do not know anything about the material. On the other hand, if the value rd is small, we assume that students know the material a little.

D. Feedback

It is noted in [3] that any educational software needs to give a quick feedback to encourage a student to study more. In our system, after a student's input is read, interpreted, and the relative distance is calculated, depending on the value of rd and the number of wrong input, two different types of hints will be displayed. Since the value of rd itself cannot assess a student's knowledge structure completely, the number of mistakes made adds more valuable information about how much student knows.

Now we show how hints are generated and displayed. We checked all types of questions in calculus textbooks except

the proof type. Then we divided short-answer questions into 44 groups depending on the techniques used to solve these questions. For example, when we want to generate a question of differentiating a product of functions, we search a collection of techniques which gives a product rule. Then using the function generated, we display a hint explaining what to do to solve this question. If rd is less than 1 and the number of wrong input is 1, then using the displayed question, hint for which experienced educator give is displayed.

V. IMPLEMENTING JCALC

A. Design

We design JCALC to handle human processes of tutoring as shown in steps 1 - 9.

- 1) Choose a subject to study.
- 2) Set the amount of time to spend, Set the number of questions to answer.
- 3) Enter student ID to start system.
- 4) A short-answer question is generated and displayed.
- 5) Type an answer in TEX format.
- 6) Read the student input and translate it into a form which will be evaluated at some point.
- 7) If the answer is correct, then the next question will appear. Otherwise, we measure the distance between the right answer and a submitted answer.
- 8) If the distance is integer value, then return the message saying "Be more careful" and ask a student to answer the question again. If the distance is not integer and small, then return hint and ask a student to answer the question again. If the distance is not integer and large, return more detailed hint and ask a student to answer the question again.
- 9) When either the time expires or the questions are exhausted, the score will appear and all data will be recorded into data base.

B. Implementing Steps 1-9

In this section, we show how we managed to implement steps 1-9. We have created the following classes, components, web services, and main form shown in Fig1 by using C#.NET, .Net Framework, web-service, and database server. Now we explain how we achieved steps 1-9 by showing snapshots of CALC.

To achieve step1, we create a menu as shown in Fig2. Menu bars contain 7 main menus. Each individual menu then has submenus. For example, the menu SUBJECTS contain the submenus PRECALC and CALCULUS. Furthermore, as you can see in Fig2, CALCULUS contains 6 submenus.

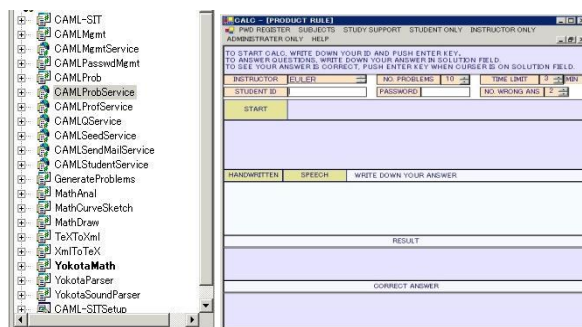


Fig1 Interface

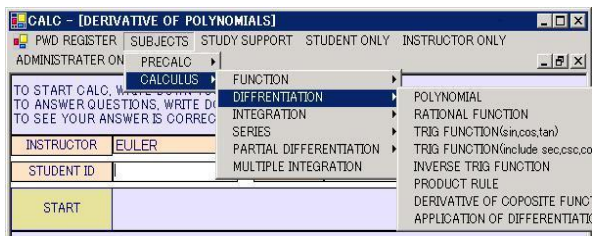


Fig2 Menu

To achieve step2, we used a NumericUpDown from a tool box provided in C#. To achieve step3, we make JCALC not start unless student ID is inputted. To achieve step4, we create a form for each sub items. These forms inherit every component from the main form except one label. Using the “generate problem” component, JCALC generates questions and a main form shown in Fig1 to display the resulting questions as shown in Fig5. To achieve step5, we create a textbox to accept student input written in TEX format. To achieve step 6, suppose for example we create differentiation form. Then JCALC uses one-point method to approximate a derivative of a given function. This way we can implement the process explained in the section IIIB into an individual form for each sub items. To achieve step7, we create a parser to read the student’s input and to evaluate a student’s input and to evaluate a student’s input. To achieve step8, we utilize the distance d . To achieve step8, we create a table in data base server to store student ID, Subjects selected, in classroom use or self study, instructor’s name, score, no. of questioned tried, date.

VI. SELF-STUDY WITH JCALC

A. LMS

For students to continue studying by using calculus-learning software, we have to implement not only hint giving function but learning management system. Since this system is mainly used at home and the school we work has very strict network security, we cannot implement database server into JCALC. Instead, we implemented JCALC an access database so that everything students inputted by the key strokes can be saved into the database. We also prepared installer for students to install JCALC to their PC. Once students went through questions, they can check to see how well they did by pushing the study-log button, and the same time, when they push send button, all data in the access database can be uploaded to the server.

A student can push the study-log button to displays the date, the subject, the number of questions tried, and the number of correct answer. For educators, if they access the server, they can retrieve all expressions students inputted to get right answers. So, by looking at these expressions, educators can find out what is causing a problem or where to put emphasis in class.

B BLENDED LEARNING

The data uploaded to the server contains so much information about students. They contain what students inputted, how many times they made mistakes to get to the right answer, how long they took to solve questions, and the rate of correctness. An educator can obtain these data by accessing the server. So, we check through all these data, especially what they inputted and count the number of mistakes made by students. We sort the number of mistakes students made and then we look at the question for which the

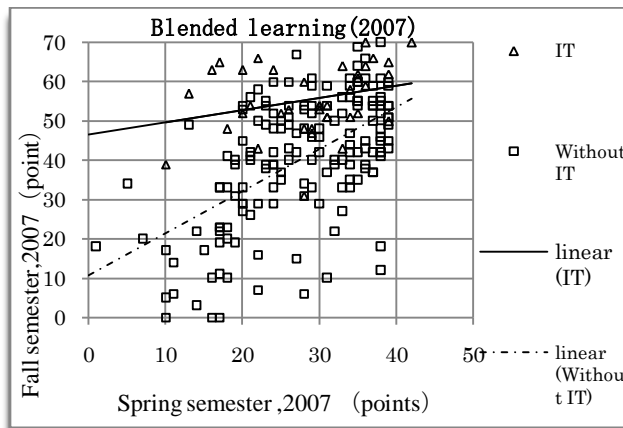


Fig 3 Comparison between the JCALC use and without JCALC largest number of mistakes made. Once we pick up a few questions for which the number of mistakes is large, we go over these questions in the class next time we meet.

We selected 36 students out of 208 students who failed in the unified exam given at the spring semester, and compared the unified-exam scores given at the fall semester and the ones at the spring semester.

The equation of the linear regression line for blended study with JCALC is given by $y = 0.313x + 46.52$, and the equation of the linear regression line without JCALC is given by $y = 1.074x + 10.64$. Now the equation $y = 0.313x + 46.52$ implies that students studied with JCALC can score around 46 points even though they scored low in the spring semester unified exam. The equation $y = 1.074x + 10.64$ has the slope close to 1 and the y-intercept 46.52. Thus we can say that students without JCALC improved their score by 10.

Now we test the hypothesis which states that blended learning with JCALC is effective for improving the self-study skill. To test this, we test the slope whether the slope and y-intercept has 5% significance difference. If there is not enough statistical significance, the slope divided by residual must be almost the same. So, we make this as a null hypothesis H_0 . Now let the slope of the regression line be b_1, b_2 and the accumulated standard deviation is calculated according to the following formula $s_{b_1, b_2} = \sqrt{S_{b_1}^2 + S_{b_2}^2} / 2$. Then under H_0 , we have $T = (1.074 - 0.313) / s_{b_1, b_2} = 6.569$ Also, 5% statistical significance for student t value is $t(199 - 2, 0.05) = 1.973$. Thus, using two-sided test, we can conclude that there is 5% statistical significance in the slopes. Similarly, we can conclude that there is a 5% statistical significance in y-intercept.

VII. CONCLUSION

JCALC has introduced new capabilities for not only creating differential and integral calculus questions but also determining submitted answers are correct or not without comparing right answers saved in the data base. In addition, JCALC has made two significant change compared to other known systems. Prior to JCALC, many tutoring systems involved with algebraic equations to judge the correctness of a submitted equation. JCALC can determine the correctness of submitted answers by simple evaluation of the equation without actually finding a derivation. Other change is how JCALC returns feedback. JCALC measures the user’s

understandings by the distance between the right answer and a submitted answer. These are not infallible methods. But these methods allow creating more complex self-graded short answer questions in much less time.

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