# Comparison of Adaptive Kalman Filter Methods in State Estimation of a Nonlinear System Using Asynchronous Measurements

Vahid Fathabadi, Mehdi Shahbazian, Karim Salahshour, Lotfollah Jargani

**Abstract-This paper presents the state estimation problem for nonlinear industrial systems using asynchronous measurements to simulate the circumstances of real case studies. The well-known conventional Kalman filters give the optimal solution but require synchronous measurements, an accurate system model and exact stochastical noise characteristics. Thus, the Kalman filter with incomplete information and asynchronous sensors measurements may be degraded or even diverged. In order to reduce the effect of noise variance uncertainty, adaptive fading extended Kalman filter and adaptive unscented Kalman filter are proposed to overcome this drawback. On the other hand, received data to estimation nodes from multi-sensors have different communication delays and various sampling rates. In this paper, conventional Kalman filter has been modified in a way to be workable for state estimation in plants with different communication delays in their sensors. Also decentralized multi sensor fusion has been used to estimate states in presence of multi-rate sensors. The feasibility and effectiveness of the presented methods are demonstrated through simulation studies on a continuous stirred tank reactor (CSTR) benchmark problem.**

***Keywords:*** **Multi sensor fusion, Decentralized data fusion, Extended Kalman filter, Adaptive Fading EKF, Adaptive fading UKF, State estimation.**

## I. INTRODUCTION

For a particular industrial process application, there might be plenty of associated sensor measurements located at different operational levels and having various accuracy and reliability specifications. One of the key issues in developing a MSDF system is the question of how can the multi-sensor measurements be fused or combined to overcome uncertainty associated with individual data sources and obtain an accurate joint estimate of the system state vector. There exist various approaches to resolve this MSDF problem, of which the KF or its information form is one of the most significant and applicable candidate solutions. In nonlinear systems, the EKF use the first order Taylor series to transform nonlinear system to linear system, and it is a used widely in nonlinear system.

A recent improvement to the EKF is the unscented Kalman filter (UKF) [1]. The UKF approximates the probability density resulting from the non-linear transformation of a random variable instead of approximating the nonlinear functions with a Taylor series expansion. The classical Kalman filter is a centralized fusion filter that assumes all observation coming *synchronously* to a control computing facility. The case where all the sensors operate synchronously has been widely studied in the literature for the linear cases [2]-[5] and for the nonlinear cases [6]-[8], among others. In the time delays context, a common approach is the PDE (partial differential equation) see Kwakernaak [9], Richard [10], Zhang, Zhang, and Xie [11]-[12] and references therein. This approach is usually related to solving a partial differential equation and boundary condition equations which do not have an explicit solution in general. For the case of discrete-time systems, the problem has been investigated via system augmentation and standard Kalman filtering, see Kailath et al. [13] and Anderson and Moore [14] or the polynomial approach [15]. The polynomial approach only addresses the steady-state filtering problem and it requires solving a much higher order of spectral factorization for systems with delays. An efficient method to deal with different sensors delay has been introduced in this paper. The main idea of this methodology is to recalculate Kalman filter in the delay time period. Furthermore, a decentralized state vector fusion is utilized in the case when sensors with different sampling rates are distributed on system. Simulation results depict the efficiency of the fusion approach for this purpose.

Basically, the conventional Kalman filter methodology hinge on prior knowledge about statistical characteristics of measurement and process noises. But when these are unknown, using adaptive Kalman filter strategy is imperative for state estimation purpose, [16]-[17]. In order to reduce the effect of prior measurement, fading memory algorithm has been applied in this work. Combining adaptive technique and proposed asynchronous method, we come into a solution for the problem of model-process mismatch and sensors noise uncertainty.

This paper is organized as follows: Section 2 derives state estimation procedure on the basis of EKF, UKF and Adaptive fading method. Section 3 shows how the KF algorithm can be changed in order to accommodate latency in the measurements

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Vahid Fathabadi is with the Petroleum University of Technology, Department of Instrumentation and Automation, P.O.Box 12333, Tehran, Iran. (Corresponding Author: Vahid Fathabadi, Phone: +989125799831; Fax: +982188742940; E-mail: vahidfathabadi@gmail.com).
Mehdi Shahbazian, Karim Salahshour and Lotfollah Jargani are also with the Petroluem University of Technology, Department of Instrumentation and Automation, P.O.Box 12333, Tehran, Iran. (shahbazian_m@yahoo.com, salahshoor@put.ir, pooria_jargani@yahoo.com )

because of both communication delay and multi-rate sensors. In section 4, CSTR industrial plant will be simulated. Simulation results are presented in section 5. Finally, section 6 summarizes the main conclusions.

## II. Proposed Methodology

### A. Extended Kalman filter

The Kalman filter in its various forms is clearly established as a fundamental tool for analyzing and solving a broad class of estimation problems[1].

Kalman filter use to estimate the state $x \in R^{nx}$ of a discrete-time controlled process that is governed by the linear stochastic difference equation

$$x_k = F_{k-1}x_{k-1} + G_{k-1}u_{k-1} + w_{k-1} \quad (1)$$
$$y_k = H_k x_k + v_k \quad (2)$$

The noise processes $\{w_k\}$ and $\{v_k\}$ are white, zero-mean, uncorrelated, and have known covariance matrices $Q_k$ and $R_k$, respectively:

$$w_k \sim (0, Q_k) \quad (3)$$
$$v_k \sim (0, R_k) \quad (4)$$
$$E\left[w_k w_k^T\right] = Q_k \delta_{k-j} \quad (5)$$
$$E\left[v_k v_k^T\right] = R_k \delta_{k-j} \quad (6)$$
$$E\left[v_k v_k^T\right] = 0 \quad (7)$$

To this point we have considered linear filters for linear systems. However, many practical systems are non-linear. Nonlinear filtering can be a difficult and complex problem. It is certainly not as mature, cohesive, or well understood as linear filtering. There is still a lot of room for advances and improvement in nonlinear estimation techniques. However, some nonlinear estimation methods are becoming widespread. These techniques include nonlinear extensions of the Kalman filter, unscented filtering, and particle filtering. Nonlinear systems can be linearized and then linear estimation techniques (such as the Kalman or H∞ filter) can be applied. This involves finding a linear system whose states represent the deviations from a nominal trajectory of a nonlinear system. We can then use the Kalman filter to estimate the deviations from the nominal trajectory, and hence obtain an estimate of the states of the nonlinear system. The derivation was based on linearizing the nonlinear system around a nominal state trajectory. The question that arises is, how do we know the nominal state trajectory? In some cases it may not be straightforward to find the nominal trajectory. However, since the Kalman filter estimates the state of the system, we can use the Kalman filter estimate as the nominal state trajectory. This is a sort of the bootstrap method. We linearize the nonlinear system around the Kalman filter estimate, and the Kalman filter estimate is based on the linearized system. This idea of the extended Kalman filter (EKF) was originally proposed by Stanley Schmidt so that the Kalman filter could be applied to nonlinear spacecraft navigation problems [19].

[1] Leonard McGee and Stanley Schmidt [18]

The nonlinear system equations obey the following non-linear relationships:

$$x_k = f_{k-1}(x_{k-1}, u_{k-1}) + w_{k-1} \quad (8)$$
$$y_k = h_k(x_k) + v_k \quad (9)$$
$$w_k \sim (0, Q_k) \quad (10)$$
$$v_k \sim (0, R_k) \quad (11)$$

Where $w_k$ and $v_k$ are process noise and measurement noise with variances of $Q_k$ and $R_k$ respectively.

A Taylor series expansion of the state equation will be performed around $x_{k-1} = \hat{x}_{k-1}^+$ and $w_{k-1} = 0$ to obtain the following:

$$\begin{aligned} x_k &= f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0) + \frac{\partial f_{k-1}}{\partial x}\Big|_{\hat{x}_{k-1}^+}(x_{k-1} - \hat{x}_{k-1}^+) + \frac{\partial f_{k-1}}{\partial w}\Big|_{\hat{x}_{k-1}^+} w_{k-1} \\ &= f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0) + F_{k-1}(x_{k-1} - \hat{x}_{k-1}^+) + L_{k-1}w_{k-1} \end{aligned} \quad (12)$$

Where $F_{k-1}$ represents $\frac{\partial f_{k-1}}{\partial x}$ and $L_{k-1}$ indicates $\frac{\partial f_{k-1}}{\partial w}$. Linearization the measurement equation around $x_k = \hat{x}_k^-$ and $v_k = 0$ lead to

$$\begin{aligned} y_k &= h_k(\hat{x}_k^-, 0) + \frac{\partial h_k}{\partial x}\Big|_{\hat{x}_k^-}(x_k - \hat{x}_k^-) + \frac{\partial h_k}{\partial v}\Big|_{\hat{x}_k^-} v_k \\ &= h_k(\hat{x}_k^-, 0) + H_k(x_k - \hat{x}_k^-) + M_k v_k \end{aligned} \quad (13)$$

Where $H_k$ represents $\frac{\partial h_k}{\partial x}$ and $M_k$ indicates $\frac{\partial h_k}{\partial v}$. A linear state space system and a linear measurement equation are in (12) and (13) respectively. It means that standard Kalman filter equations can be used to estimate the state. Thus, the following equations are named as the EKF equations:

$$\hat{x}_k^- = f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0) \quad (14)$$
$$P_k^- = F_{k-1}P_{k-1}^+F_{k-1}^T + L_{k-1}Q_{k-1}L_{k-1}^T \quad (15)$$
$$K_k = P_k^- H_k^T\left(H_k P_k^- H_k^T + M_k R_k M_k^T\right)^{-1} \quad (16)$$
$$\hat{x}_k^+ = \hat{x}_k^- + K_k(y_k - h_k(\hat{x}_k^-, 0)) \quad (17)$$
$$P_k^+ = (I - K_k H_k)P_k^- \quad (18)$$
$$P_k^+ = (I - K_k H_k)P_k^- \quad (19)$$

### B. Unscented Kalman Filter

Julier and Uhlman developed the UKF algorithm[1] and [20], they used the unscented transform to compute the statics character of states and measurements, for the unscented transform is second-order equal to the real statics character at least. An unscented transformation is based on two fundamental principles. First, it is easy to perform a nonlinear transformation on a single point (rather than an entire pdf). Second, it is not too hard to find a set of individual points in state space whose sample pdf approximates the true pdf of a state vector.

Suppose that we know the mean $\bar{x}$ and covariance $P$ of a vector $x$. A set of deterministic vectors called sigma points whose ensemble mean and covariance are equal to $\bar{x}$ and $P$ can be find. We next apply our known nonlinear function $y = h(x)$ to each deterministic vector to obtain transformed vectors. The ensemble mean and covariance of the transformed vectors will give a good estimate of the true mean and covariance of $y$. This is the key to the unscented transformation. The UKF algorithm can be simply obtain by replacing the EKF equations with unscented transformations[21]-[24].

Suppose the nonlinear system equations obey the following non-linear relationships:

The UKF algorithm can be summarized as follows:

1- The algorithm will be started with some initial guesses for the state estimation ($x_0$) and the error covariance matrix ($P_0$), defined as:

$$\hat{x}_0^+ = E[x_0] \tag{20}$$

$$P_0^+ = E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T] \tag{21}$$

2- The following time update equations are used to propagate the state estimate and covariance from one measurement time to the next:

(a) Calculate a collection of sigma points, stored in the columns of the $n \times (2n+1)$ sigma point matrix $\hat{x}_{k-1}^+$ as:

$$\chi_{k-1} = \left[\hat{x}_{k-1}^+ \quad \hat{x}_{k-1}^+ + \sqrt{(n+\lambda)P_{k-1}^+} \quad \hat{x}_{k-1}^+ - \sqrt{(n+\lambda)P_{k-1}^+}\right] \tag{22}$$

The parameter is a scaling parameter defined as:

$$\lambda = \alpha^2(n + \kappa) - n \tag{23}$$

The constant $\alpha$ determines the spread of the sigma points around $\hat{x}_{k-1}^+$ and $\kappa$ is a secondary scaling parameter.

(b) Propagate each column of $\chi_{k-1}$ (i.e., $\chi_{k-1}^i$) through the nonlinear system dynamic equation to perform the prediction or time update step as:

$$\chi_{k-1}^{*,i} = f(\chi_{k-1}^i, u_{k-1}) \quad i = 0,1,\cdots,2n \tag{24}$$

$$\chi_{k-1}^* = f(\chi_{k-1}) \tag{25}$$

(c) Then a priori estimate values for state and error covariance are calculated as:

$$\hat{x}_k^- = \sum_{i=0}^{2n} W_i^{(m)} \chi_{k-1}^{*,i} \tag{26}$$

$$P_k^- = \sum_{i=0}^{2n} W_i^{(c)} (\chi_{k-1}^{*,i} - \hat{x}_k^-)(\chi_{k-1}^{*,i} - \hat{x}_k^-)^T + Q_{k-1} \tag{27}$$

Where $W_i^{(m)}$ and $W_i^{(c)}$ are sets of scalar weights defined by:

$$W_0^{(m)} = \frac{\lambda}{\lambda+n} \tag{28}$$

$$W_0^{(c)} = \frac{\lambda}{\lambda+n} + (1 - \alpha^2 + \beta) \tag{29}$$

$$W_i^{(m)} = W_i^{(c)} = \frac{1}{2(\lambda+n)} \quad i = 1,\cdots,2n \tag{30}$$

$\beta$ is a parameter used to incorporate any prior knowledge about the distribution of $x$.

3- Now that the time update equations are done, we implement the measurement-update equations.

(a) The sigma points are updated as:

$$\chi_{k|k-1} = \left[\hat{x}_k^- \quad \hat{x}_k^- + \sqrt{(n+\lambda)P_k^-} \quad \hat{x}_k^- - \sqrt{(n+\lambda)P_k^-}\right] \tag{31}$$

(b) Propagate each column of $\chi_{k|k-1}$ through the nonlinear system measurement equation to predict the measurement values as:

$$\hat{y}_k^{(i)} = h(\chi_{k|k-1}^i) \tag{32}$$

$$\hat{y}_k = \sum_{i=0}^{2n} W_i^{(m)} \hat{y}_k^{(i)} \tag{33}$$

(c) After the prediction step, the correction or measurement update step is performed to calculate the posterior estimate state as follows:

$$\hat{x}_k^+ = \hat{x}_k^- + K_k(y_k - \hat{y}_k) \tag{34}$$

Where $y_k$ is the actual measurement vector. The $y_k$ can be pre-filtered with a simple Kalman filter (KF) with identity gain and unit observation gain. This pre-filter removes the Gaussian noises much extent and helps the UKF algorithm for better convergence and less deviations in the final estimated states. $K_k$ is the Kalman gain defined by:

$$K_k = P_{xy} P_y^{-1} \tag{35}$$

Where

$$P_{xy} = \sum_{i=0}^{2n} W_i^{(c)} (\chi_{k|k-1}^i - \hat{x}_k^-)(\hat{y}_k^{(i)} - \hat{y}_k)^T \tag{36}$$

$$P_y = \sum_{i=0}^{2n} W_i^{(c)} (\hat{y}_k^{(i)} - \hat{y}_k)(\hat{y}_k^{(i)} - \hat{y}_k)^T + R_k \tag{37}$$

$$P_k^+ = P_k^- - K_k P_y K_k^T \tag{38}$$

### C. *Adaptive Fading*

The Kalman filter formulation assumes complete a priori knowledge of the process and measurement noise covariance matrices $Q_k$ and $R_k$. However, in most practical applications these matrices are initially estimated or, in fact, are unknown. The problem here is that the optimality of the estimation algorithm in the Kalman filters setting is closely connected to the quality of the a priori noise statistics [25]. It has been shown how poor estimates of the input noise statistics may seriously degrade the Kalman filter performance, and even provokes the divergence of the filter [26]-[27]. From this point of view it can be expected that an adaptive formulation of the extended Kalman filter will result in a better performance or will prevent filter divergence.

In this case, the covariance of the adaptive fading algorithm is[28]:

$$C_k = E[\eta_k \eta_k^T] = H_k P_k^- H_k^T + R_k \tag{39}$$

Where $\eta_k = y_k - h_k(\hat{x}_k^-)$, $P_k^-$ and $R_k$ are innovations, a predicted error covariance and a measurement covariance of the KF, respectively. $C_k$ is referred to as the calculated innovation covariance in this paper. In general, the innovation of the filter is easily affected by unaccounted errors, such as an unknown fault bias, an un-modeled dynamic, or an unknown initial condition. Also, the innovation covariance shows the effect of any unaccounted errors, as they are directly involved in the computations of the innovation.

For example, if we know an exact dynamic equation, the innovation covariance is equal to $C_k$. But sometimes, the exact dynamic equation of a nonlinear stochastic system is not available. Then, an estimation error and a predicted error covariance may increase by the effect of the unknown information. In (39), if $P_k^-$ is increased, then $C_k$ is also increased. Similarly, sometimes, the exact measurement equation of a nonlinear stochastic system is not available. Then, an innovation covariance $C_k$ may be also increased by the effect of an unknown information. In this case, an innovation covariance $C_k$ is increased by an increased measurement covariance $R_k$ in (39). As a result, the change of an innovation covariance can be used for an adaptive filter. The increased innovation covariance can be estimated as

$$\bar{C}_k = \frac{1}{M-1}\sum_{i=k-M+1}^{k}\eta_i\eta_i^T \quad (40)$$

where M is a window size. We call $\bar{C}_k$ an estimated innovation covariance in this paper.

What is the method to decrease the effect by unaccounted errors? To account for the effect of the unaccounted system model errors, Kim [29] proposed the AFEKF. In this section, we summarize the structure of the AFEKF. We assume that we do not know the exact dynamic or measurement equation for the system. The relation between $C_k$ and $\bar{C}_k$ is defined as $\bar{C}_k = \alpha_k C_k$. Then, the scalar variable $\alpha_k$ can be estimated by

$$\alpha_k = max\left\{1, \frac{1}{m}tr(\bar{C}_k C_k^{-1})\right\} \quad (41)$$

Where m is the dimension of $z_k$, which is the m×1 measurement vector. When the innovation covariance is increased by unaccounted errors, an estimated innovation covariance $\bar{C}_k$ shows the estimate of the true innovation covariance.

We consider the first case, in which the dynamic equation is not known exactly. Generally, the effects of incomplete information in the dynamic equation can be compensated by the increase of the magnitude of $P_k^-$. Thus a predicted error covariance must be increased to compensate the effect of an inexact dynamic equation as $\bar{P}_k^-$ where $\bar{P}_k^- = \lambda_k P_k^-$. Here $\lambda_k$ is called a forgetting factor and $\lambda_k \geq 1$. Then $\bar{C}_k$ can be represented by

$$\bar{C}_k = H_k\bar{P}_k^- H_k^T + R_k = H_k(\lambda_k P_k^-)H_k^T + R_k \quad (42)$$

In (42), we can obtain the following equations

$$\alpha_k[H_k P_k^- H_k^T + R_k] = \lambda_k H_k P_k^- H_k^T + R_k \quad (43)$$

$$\lambda_k \approx \frac{tr(\alpha_k H_k P_k^- H_k^T + (\alpha_k - 1)R_k)}{tr(H_k P_k^- H_k^T)} \quad (44)$$

Here, (44) gives an approximate value of $\lambda_k$. But the measurement equation does not have unaccounted errors in the first case. As a matter of fact, an innovation covariance is increased by not the measurement covariance but the increased predicted error covariance. This indicates that the ratio of innovation covariances $\alpha_k$ is mainly generated by $\lambda_k$.

Therefore we can assume that $\alpha_k$ is almost equal to $\lambda_k$. If we assume $\lambda_k = \alpha_k$, then the error covariance is $\bar{P}_k^- = \alpha_k P_k^-$. The AFEKF using this concept is denoted in this paper as "the AFEKF with rescaling-$P_k$."

Next, we consider the second case, in which the measurement equation is not known exactly. The estimation error and the innovation covariance may be also increased by the effect of the unknown information, as they were in the first case. Here, the dynamic equation does not have unaccounted errors in the second case. So, an innovation covariance is increased by not a predicted error covariance but an increased measurement covariance. The effects of incomplete information in the measurement equation can be compensated by the decrease of the magnitude of $K_k$. We set $\lambda_k = 1$ because the predicted error covariance is unchanged as $\bar{P}_k^- = P_k^-$. And we use the Kalman gain that is decreased by $1/\alpha_k$. The decrease of the Kalman gain magnitude means to depend less on measurement information. The AFEKF using this concept is denoted in this paper as "the AFEKF with rescaling-$K_k$." From these results, we propose the following filter [29].

DEFINITION 1 A discrete-time adaptive fading EKF is given by the following coupled difference equations when the information of a nonlinear stochastic system is partially known

$$\hat{x}_k^- = f_{k-1}(\hat{x}_{k-1}^+) \quad (45)$$

$$\bar{P}_k^- = \lambda_k[F_{k-1}\bar{P}_{k-1}^+ F_{k-1}^T + Q_{k-1}] \quad (46)$$

$$\bar{K}_k = \frac{\lambda_k}{\alpha_k}\bar{P}_k^- H_k^T[H_k\bar{P}_k^- H_k^T + R_k]^{-1} \quad (47)$$

$$\bar{P}_k^+ = (I - \bar{K}_k H_k)\bar{P}_k^- \quad (48)$$

$$\hat{x}_k^+ = \hat{x}_k^- + \bar{K}_k[z_k - h_k(\hat{x}_k^-)] \quad (49)$$

Where $\lambda_k \geq 1$, $\eta_k = z_k - h_k(\hat{x}_k^-)$, $\bar{C}_k = \alpha_k C_k$, $C_k = E[\eta_k\eta_k^T] = H_k P_k^- H_k^T + R_k$,
$\bar{C}_k = \frac{1}{M-1}\sum_{i=k-M+1}^{k}\eta_i\eta_i^T$ and $\alpha_k = max\left\{1, \frac{1}{m}tr(\bar{C}_k C_k^{-1})\right\}$

## III. ASYNCHRONOUS KALMAN FILTER

In the previous section, it was assumed that all the sensor measurements are synchronously available at each sampling instant. This unrealistic assumption must be disregarded in according to the communication delay or the different sensor sampling rates that affect the multi-sensor data fusion procedure.

### A. *Asynchronous communication delay*

A nonlinear discrete system observed by non-delayed measurements where both process and measurements are influenced by additive Gaussian noise can be put in state space form in (8) and (9).

Furthermore, if this system has an output that is delayed n samples, for instance due to a slow sensor or a long processing time of the sensor data, there will be a second output equation:

$$z_k^* = h^*(x_s, s) + v_k^* \quad (50)$$

where

$$s = k - N$$

The delayed measurement cannot be fused using the normal extended Kalman filter equations but requires some modifications in the structure of the filter.

*Recalculation Method :*

If only a few measurements are fused in the delay period or if the computational burden of the filter is uncritical, an optimal filter estimate can be obtained simply by recalculating the filter through the delay period .As the measurement are not available in the time interval t=S to t=K ,it is suggested to update state and covariance without measurement update in this time interval .As soon as measurement of time t=s is received with delay at t=K ,estimation procedure begins with time update and measurement update again from t=S ,and will be proceed to time t=K+1 using only time update equations .The repeated manner of this procedure imposes high computational burden. But, it is not a big deal in industrial plant that time constant of systems is high.

### B. *Different sampling rate*

Assume that the m sensors are geographically distributed on an industrial plant. Estimation procedure should have the ability to deal with the amount of data that will be received by the estimation node at different times. In this paper decentralized data fusion will be used to estimate states from asynchronous different sampling rate sensors.

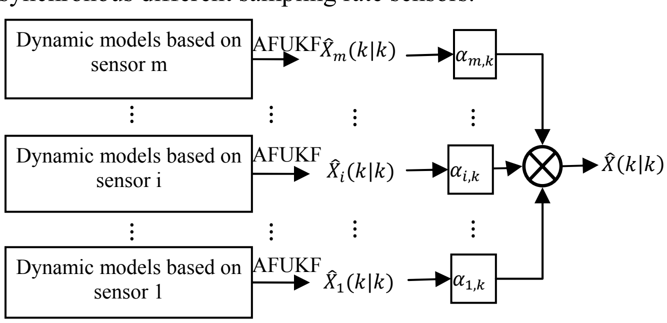


Figure 1. Decentralized data fusion schematic.

In order to deal with different sampling rate sensors, Alouani suggestion [30] is to fuse received measurement values at the end of pre-selected time interval. In contrast, the author's suggestion is to fuse data of multi-rate sensors at any time in which a measurement is received. At this step, the fused value of states and covariances are sent back to estimation nodes as the information of data of time t=K and the procedure will be repeated to estimate the desired values of time t=K+1. As the sensors are multi-rate, naturally measured values of all sensors cannot be accessible at each arbitrary time-step. Thus, for those sensor nodes that measured value are not available, they send state and covariances which have been computed by only time-update equation to fusion node.

For i = 1, 2, . . . , m, suppose that that $\hat{X}_i(k)$ and $P_i(k)$ are the state estimates and the estimation error covariance matrices of $X_m(k)$ by Kalman filtering based on model (8) and (9) respectively, which are independent of each other, then the optimal fused estimate in the sense of linear minimum covariance is given by

$$\hat{X}(k|k) = \sum_{i=1}^{m} \alpha_{i,k}\, \hat{X}_{i,m}(k|k) \quad (51)$$

Where

$$\alpha_{i,k} = \left(\sum_{j=1}^{m} P_{j,m}^{-1}(k|k)\right)^{-1} P_{i,m}^{-1}(k|k) \quad (52)$$

and the corresponding estimation error covariance matrix is

$$P(k|k) = \left(\sum_{j=1}^{m} P_{j,m}^{-1}(k|k)\right)^{-1} \quad (53)$$

In addition, it can be shown

$$P(k|k) \le P_{i,m}(k|k) \qquad (i = 1,2,\cdots,m) \quad (54)$$

## IV. Mathematical Model of CSTR

An irreversible and exothermic reaction A→B takes place inside the jacket CSTR that is shown in Figure 2 [31]. The reaction is operated by two proportional controllers that are used to regulate the outlet temperature and the tank level. A cooling jacket surrounds the reactor and the coolant is water in this case. Negligible heat losses, constant densities, perfect mixing inside the tank and uniform temperature in the jacket are assumed.

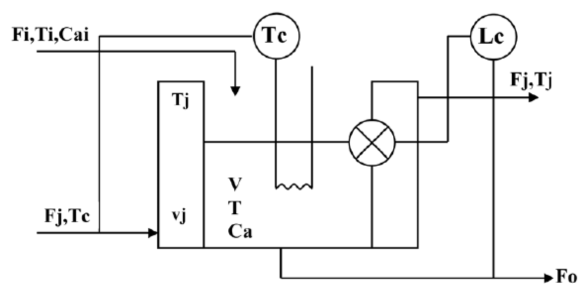


Figure 2. Schematic diagram of the process.

Table 1: Non isothermal CSTR parameter

| Notation | Variable | Steady state values |
|---|---|---|
| $F_o$ | Outlet flow rate | 40 $ft^3$/h |
| $Ca_i$ | Inlet reactant concentration | 0.5 lb. mol of A/$ft^3$ |
| T | Reactor temperature | 600°R |
| $F_j$ | Coolant flow rate | 49.9 $ft^3$/h |
| V | Volume of reactor | 48 $ft^3$ |
| Ca | Reactant concentration in reactor | 0.245 lb.mol of A/$ft^3$ |
| $T_j$ | Jacket temperature | 594.6°R |
| $T_i$ | Inlet feed temperature | 530°R |
| **Notation** | **Variable** | **Parameter values** |
| $V_j$ | Volume of jacket | 3.85 $ft^3$ |
| $E_a$ | Activation energy | 30000Btu/lb.mol |

| | | |
|---|---|---|
| U | Heat-transfer coefficient | 150 Btu/h $ft^2$ °R |
| $T_c$ | Inlet feed temperature | 530°R |
| $c_p$ | Heat capacity (process side) | 0.75 Btu/lbm°R |
| ρ | Density of process mixture | 50 lbm/$ft^3$ |
| $k_0$ | Frequency factor | 7.08×1010$h^{-1}$ |
| R | Universal gas constant | 1.99Btu/lb.mol°R |
| $a_0$ | Heat-transfer area | 250 $ft^2$ |
| ΔH | Heat of reaction | -30000 Btu/lb.mol |
| $C_j$ | Heat capacity (coolant side) | 1.0 Btu/lbm°R |
| $\rho_j$ | Density of coolant | 62.3lbm/$ft^3$ |

The dynamic equations describing the system are given by [32]:

$$\frac{dV}{dt} = F_i - F_o \tag{55}$$

$$\frac{d(VCa)}{dt} = F_i Ca_i - F_o Ca - V\left(k_0 \exp\left(\frac{E_a}{RT}\right)\right)Ca \tag{56}$$

$$\rho c_p \frac{d(VT)}{dt} = \rho c_p (F_i T_i - F_o T) - \Delta H V\left(k_0 \exp\left(\frac{E_a}{RT}\right)\right)Ca - Ua_0 (T - T_j) \tag{57}$$

$$\rho_j V_j c_j \frac{dT_j}{dt} = \rho_j c_j F_j (T_c - T_j) + Ua_0 (T - T_j) \tag{58}$$

$$F_o = 40 - 10(48 - V) \text{ (Level controller)} \tag{59}$$

$$F_j = 49.9 - 4(600 - T) \text{ (Temperature controller)} \tag{60}$$

Table 1 gives values of process parameters and steady state conditions.

## V. Simulation Studies

For computer simulation, the CSTR nonlinear model is implemented using s-function and SIMIULINK facilities in MATLAB. The basic time unit is hours (hr) and the sampling time is taken to be equal to 0.005 hr.

As it is clear from Fig.2, the outputs of the system are volume and temperature of product, concentration of A, and temperature of CSTR jacket. For the simulation studies, measurements (V,T) have been assumed as the observed values in order to estimate of all states of the system (V,T,Ca,Tj ).

The simulation studies on this case study have been conducted to investigate the performance of the discussed methods in different situations.

### A. *Asynchronous communication delay*

The simulation results of implemented methods in section 2 on the case study are depicted in figures 3-6. In order to investigate the capability of proposed methods in estimation of system states according to realistic settings in which *asynchronous* sensor data are corrupted with *unknown noise*, both incorrect values of noise variance assumptions and different communication delays are embedded in the simulation. The ratios of the incorrect values of noise variance to the correct corresponding values are 0.1 in Fig 3-6. These approaches should compensate for the effect of lack of data about the noise statistical characteristics. The capabilities of presented methods in extracting real values are clearly illustrated via figures and root mean square error criteria (RMSE).

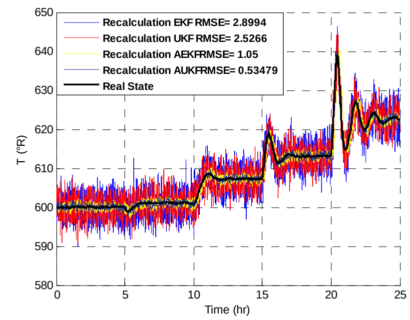


Figure 3. Estimation of Reactor temperature when number of delay is 30.

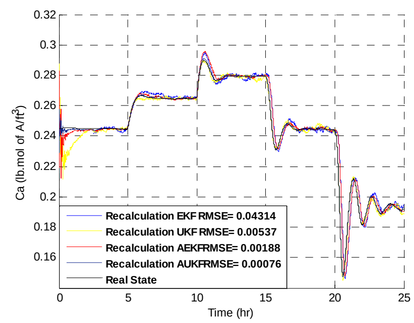


Figure 4. Estimation of Reactant concentration when number Of delay is 30.

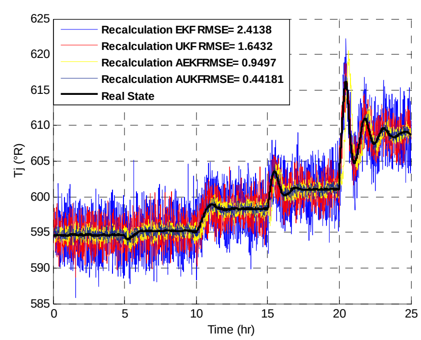


Figure 5. Estimation of Reactor jacket temperature when number of delay is 30.

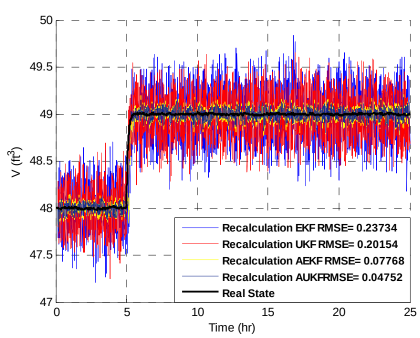


Figure 6. Estimation of Reactor volume when number of delay in measurement of V is 30.

Table 2. RMSE of different methods in estimation of CSTR volume when data of sensor V received after different values of delay .The measurement noise variance used in estimation is artificially set as 0.1 actual noise variance

| RMSE \ Number of delay | 2 | 5 | 10 | 15 | 20 | 30 | 50 | 80 | 100 |
|---|---|---|---|---|---|---|---|---|---|
| **EKF Recalculation Method** | 0.1843 | 0.18447 | 0.18484 | 0.18598 | 0.18735 | 0.19128 | 0.20058 | 0.21454 | 0.22326 |
| **UKF Recalculation Method** | 0.09196 | 0.09202 | 0.09205 | 0.09227 | 0.09249 | 0.09261 | 0.09337 | 0.09708 | 0.10144 |
| **AFEKF Recalculation Method** | 0.05548 | 0.05607 | 0.05819 | 0.06175 | 0.06608 | 0.0759 | 0.09595 | 0.12206 | 0.13671 |
| **AFUKF Recalculation Method** | 0.04525 | 0.04539 | 0.04554 | 0.04592 | 0.04633 | 0.04667 | 0.04712 | 0.04839 | 0.04942 |

Table 3. RMSE of estimation with different sampling rates.(NH corresponds to temperature sampling rate as high-rate sensor and NL is for volume as low rate sensor; NH/NL is the ratio of two quantities.)

| RMSE \ NH/NL | 2 | 5 | 8 | 10 | 15 | 20 | 30 | 40 | 50 | 80 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Low Rate** | 0.02443 | 0.02852 | 0.03523 | 0.04144 | 0.05606 | 0.07351 | 0.10703 | 0.14301 | 0.17768 | 0.28633 | 0.35603 |
| **High Rate** | 0.02597 | 0.03172 | 0.03927 | 0.04578 | 0.06091 | 0.07852 | 0.12213 | 0.16822 | 0.21284 | 0.33154 | 0.44128 |
| **Fusion** | 0.02427 | 0.02847 | 0.03519 | 0.04141 | 0.05603 | 0.07349 | 0.103 | 0.14298 | 0.17766 | 0.28531 | 0.3401 |

Classical Kalman filter strategies fundamentally rely on known model and noise information. Consequently, as it is obvious in figures, they cannot compensate the effect of model-process mismatch and noise uncertainty. As discussed in section 2, UKF is comparatively more acceptable for estimation of states in nonlinear plants than EKF. Subsequently, adaptive recalculation UKF methodology can also estimate states with less error than adaptive recalculation of EKF method. In order to provide a more comprehensive comparison between these methods, the RMSE values of each method in terms of different delays from sensor V is presented in table 2. Consequently, table 2 also certifies that the modified adaptive fading UKF method by recalculation procedure expose more accurate results, compared to other methods.

*B. Sensors with different sampling rate*

Figures 7-10 illustrate the results of the proposed method in section 2.2.2 in which data generation rate of sensor T(as a high rate sensor) is 30 times of sensor V(as a low rate sensor) .

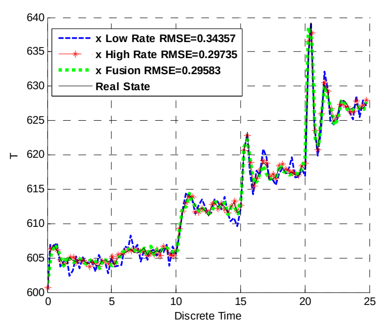


Figure 7. Estimation of Reactor temperature.

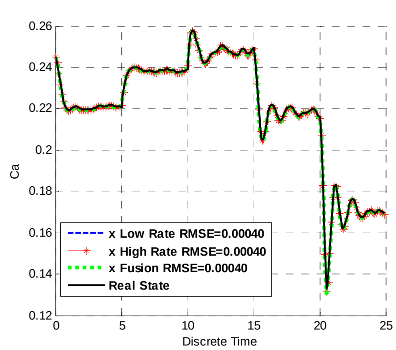


Figure 8. Estimation of Reactant concentration.

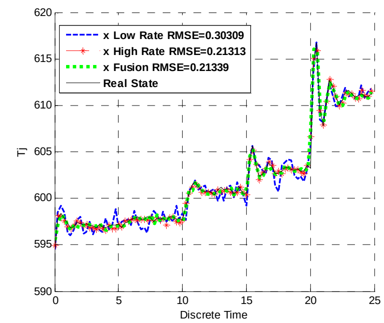


Figure 9. Estimation of Reactor temperature.

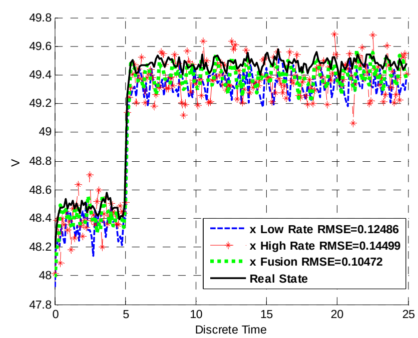


Figure 10. Estimation of Reactor volume.

Presented data in table3 is calculated by MATLAB software in which sampling rate of sensor T as a high rate is NH/NL times of sensor V as a low rate one.

## VI. Conclusion

The ability to incorporate data from a large, possibly changing variety of sources and to accommodate sensing delays without derogating estimation is extremely valuable in systems. The inaccurate estimation of states in EKF and UKF methods due to a lack of solid consideration of existing model uncertainty and also a nonrealistic pre-assumption on noise distribution matrices has been alleviated by incorporation of an *adaptive fading method* into KF strategies. In order to consider real assumption, i.e. sensors with inherent or communication delays, recalculation method has been embedded to AFEKF and AFUKF in order to enable them for performance monitoring of asynchronous measurements. Moreover, the simulation results denote the improvement of nonlinear state estimation using UKF that approximates the probability density from the non-linear transformation of a random variable, compared to EKF method in which approximation is done employing Taylor series expansion.

Besides, an asynchronous decentralized data fusion platform for an estimation of multi-sampled signals based on AFUKF has been studied. The fused estimate shows better performance than Kalman filtering based on each single sensor's information.

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