Evaluation of Reliability Confidence Lower Limit for Electronic Stability Control systems

Shiwen Niu

Abstract—**This article describes a reliability confidence lower limit evaluation method for Electronic Stability Control (ESC) system utilizing test data from multi-stage and subsystems. This method provides estimation that can potentially reduce the amount of testing significantly, without sacrificing the one-sided confidence level of the reliability. This also allows quicker design verification and validation for ESC systems. The method is derived under the assumption that the reliability parameter was a random variable with a given distribution function, and that the product's reliability increases monotonously during the development process. This new method is applied to the study of an Electronic Stability Control task. The selected task is typical for users who conduct tests and analyze the reliability Electronic Stability Control systems.**

Index Terms—**Electronic Stability Control, multi-stage, one-sided confidence level, system reliability evaluation**

I. Introduction

Electronic Stability Control (ESC) system is a critical component in vehicles. It is a brake control system that uses pressure sensors, a yaw sensor, an acceleration sensor, wheel speed sensors, solenoids, a motor, and a microcontroller to electronically modulate individual wheel torque to improve the vehicle stability [1]. The development of ESC evolved from earlier brake control products such as Anti-Lock Brake Systems (ABS) and Traction Control Systems (TCS) [2]–[4]. It is now widely used as a safety enhancement system in vehicles. The U.S. government is requiring all passenger vehicles under 10,000 pounds to be equipped with ESC starting 2012. Many other countries are also creating similar requirements. It is estimated by the National Highway Traffic Safety Administration that 5,300-9,600 annual fatalities will be avoided due to the required installation of ESC systems [5]. Data from several studies show that single-vehicle crashes involving cars are reduced by about 30-50% and SUVs by about 50-70% [6]. There are efforts being made to further enhance the functionality of the ESC system, for example, Roll Stability Control [7], [8] is one of such systems, and more features can be expected to be added to ESC systems.

In today's world, for an ESC product to be successful, it must have high reliability, in addition to the desired performance. There are stringent government regulations on such systems [9]. Automotive companies that make ESC spend much of their effort in using statistical tools such as Six Sigma [10]–[12] to improve the quality of their product. The more tests they conduct, the more they can say about the reliability of the ESC system. However, testing ESC systems is a complicated, costly, and time-consuming process. How to achieve high levels of reliability with more efficient test design has become a critical issue. Simple reduction of the number of tests would cause the confidence level to be low, and therefore is not acceptable. There are several results in the literature [13] that provide a reduced number of tests without sacrificing the confidence level on the reliability estimation. Specifically, Lindstrom and Madden proposed to use the reliability of subsystems to estimate the system reliability with reduced numbers of tests. In this paper, the Lindstrom and Madden method and one new method will be applied to the reliability estimation of ESC systems.

In order to analyze the reliability of ESC systems, one needs to understand how the development process works for ESC systems. Typically, a conceptual design is developed first based on the past experience and "voice of customers" [11]. The conceptual design is then verified during the feasibility study, which includes calculation, modeling and simulation. System level requirements are derived, followed by subsystem and component level requirements. The main subsystems for ESC systems include: mechanical (mainly hydraulic), electronic, and software subsystems. The ESC system can also be divided into subsystems according its functionality. Typically an ESC system also has ABS, TCS, and other subsystems. The components are tested and then integrated into subsystems. After tests on subsystems are completed, they are integrated into a system. The ESC system then goes through tests in the laboratory and in vehicles. Parts of this process may be repeated more than one time if problems are found during any of the test stages.

Currently, the reliability data is derived based on the test data from the final system, both from laboratory and vehicle tests. This calculation does not take advantage of the vast test data available between conceptual design and final system testing. For instance, during the feasibility study, system performance and failure modes are evaluated via modeling and simulation using software tools such as MATLAB and LabVIEW [14]–[17]; The component level and subsystem level tests are usually conducted with hardware-in-the-loop tests; system level test data from earlier designs are also available. The statistical design of experiments (DOE) method [10] is widely used to reduce the number of tests during the entire product development cycle. This data can provide significant amounts of information in calculating reliability data, but are currently not being used. Therefore, there are many opportunities in reducing the amount of tests without sacrificing the confidence level of the reliability

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estimation. First, subsystem reliability can be calculated based on the component reliability provided by the suppliers and additional test data collected during the component level testing. If the design goes through several revisions, then test data from earlier versions of the design can be taken into consideration. Second, the calculated subsystem reliability can be used in addition to the system level test data for system level reliability calculation. Similar to the subsystem level reliability, test data from earlier versions of the design can be used to further reduce the system level testing. The challenge is how to use this data to enhance the reliability calculation, and potentially reduce the design verification and validation time.

System reliability growth modeling and evaluation have been studied by a number of researchers. Fries [18], [19] presented a discrete reliability growth model, derived from the learning-curve property, to describe reliability growth. Hall and Mosleh [20] introduced a framework for the evaluation of reliability growth for one-shot systems. Bayes approaches were widely used to evaluate system reliability, especially in small sample size situations [21]–[24]. Mazzuchi and Soyer [25], [26] adopted the ordered Dirichlet distribution to incorporate prior opinion into the analysis. Calabria *et al.* [27], [28] introduced the Power-Law process to describe the failure pattern of systems. However, these approaches only yield reliability estimates of the current stage after it is finished, and cannot provide formalism for incorporation of information from the development process. Therefore, it is desirable to incorporate prior information as much as possible into the analysis during the development process of the system using data acquired from the smaller sample size test.

The rest of the paper is organized as follows: Section II introduces a reliability confidence lower limit evaluation method using data from earlier design stages in addition to the final test data in combination with the basic theory of Lindstrom and Madden; Section III contains a simple example; An example of ESC system is used to illustrate the application of the theoretical results in Sections II; Some discussion and conclusion are given in Section IV.

II. FORMULATION

For research and development cycle of the product with n stage experiments, let R_i be the estimation value of one-sided reliability confidence lower limit of the *i*-th stage of the product, which relies only on the experiment data from the *i*-th stage. As the product development process evolves, it is reasonable to assume that R_i is independent random variable satisfying the following constraint:

$$R_1 \le R_2 \le \ldots \le R_i \le \ldots \le R_{n-1} \le R_n, \tag{1}$$

where R_n is the reliability of the final experiment stage derived from the final experiment data. Since R_n does not make use of the prior $n-1$ phases' experimental data and reliability information, the evaluation result is usually too conservative. In real development process of the product, an evaluation method that can utilize the prior $n-1$ stages experimental data and reliability information is needed.

Denote the probability distribution of reliability R_i by $F_i(R_i)$ and the joint probability distributions of R_i of all n phases by

$$F(R_1,\ldots,R_i,\ldots,R_n) = P(R_1 = r_1,\ldots,R_i = r_i,\ldots,R_n = r_n), \tag{2}$$

where r_i is the observed value of the random variable R_i. Then, the joint probability distribution $F(R_1,\ldots,R_i,\ldots,R_n)$ relies on test data and information from all n phases.

To derive the probability distributions of the final phase reliability R_n, the reliability probability distributions of product in the final phase is marginal probability distribution with regard to R_n of the joint probability distribution $F(R_1,\ldots,R_i,\ldots,R_n)$, it is assumed to satisfy constraint defined in (1).

In order to get marginal distribution function $F_{R_n}(R_n)$, marginal probability density function $f_{R_n}(R_n)$ should be derived first. Since R_i, *i =1,2,…,n*, are independent, the joint probability density function of R_i is given by

$$f_1(R_1)\cdot f_2(R_2)\cdot\cdots\cdot f_i(R_i)\cdot\cdots\cdot f_{n-1}(R_{n-1})\cdot f_n(R_n). \tag{3}$$

Under the constraint (1), the marginal probability density function can be written as

$$f_{R_n}(R_n) = \iiint\limits_{R_1\le\ldots R_i\le\ldots R_n} f_1(R_1)\cdot\cdots\cdot f_n(R_n)\,d(R_1\ldots R_{n-1}). \tag{4}$$

As a result, the marginal probability distributions function of R_n is:

$$F_{R_n}(R_n) = \int_0^{R_n} f_{R_n}(R_n)\,dR_n. \tag{5}$$

Denote the probability of constraint (1) by $P(R_1 \le \ldots \le R_i \le \ldots \le R_n)$. The joint probability distribution value of all experiment phases under the constraint of $R_1 \le \ldots \le R_i \le \ldots \le R_n$, is the integration of the joint probability distribution within the boundary specified by the constraint

$$P(R_1 \le \ldots \le R_i \le \ldots \le R_n) = \iiint\limits_{R_1\le\ldots\le R_i\le\ldots\le R_n} f_1(R_1)\cdot\cdots\cdot f_n(R_n)\,d(R_1\ldots R_{n-1}), \tag{6}$$

where $0 < R_i < 1$. It follows that

$$P(R_1 \le \ldots \le R_i \le \ldots \le R_n) = \iiint\limits_{R_1\le\ldots\le R_i\le\ldots\le R_n,\,0<R_i<1} f_1(R_1)\cdot\cdots\cdot f_n(R_n)\,d(R_1\ldots R_{n-1}). \tag{7}$$

The reliability interval estimation solved in this article is under the constraint (1), which is relatively easy to satisfy.

According to the conditional probability formula:

$$P(A\mid B) = \frac{P(AB)}{P(B)}, \tag{8}$$

the final distribution function of reliability R_n can be written as

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$$F_{final}(R_n) = \frac{F_{R_n}(R_n)}{P(R_1 \le \ldots \le R_n)} = \frac{\int_0^{R_n} f_{R_n}(R_n)dR_n}{\iiint_{R_1 \le \ldots R_i \le \ldots \le R_n, 0<R_i<1} f_1(R_1)\cdot \cdots \cdot f_n(R_n)d(R_{1\ldots}R_{n-1})}, \quad (9)$$

The α fractile of $F_{final}(R_n)$ is the reliability lower limit with a confidence of $1-\alpha$.

This method is a process which transforms multiple integrals to step integrals. If the research and development cycle of a product has n (n>2) experiment phases, the test data of the first and second phase should be analyzed by this method. The reliability of the first and second phase, R_1, R_2, can be calculated from the test data of these phases. Simultaneously, the probability distribution function and the probability density function of the first and second phases, $F_1(R_1), F_2(R_2)$, $f_1(R_1), f_2(R_2)$, could be obtained, so the joint probability density is given by

$$f(R_1, R_2) = f_1(R_1) f_2(R_2). \quad (10)$$

Based on the above discussion, the probability density function of the real reliability of product in the second phase is the marginal probability density function with regard to R_2 of the joint probability distributions $F(R_1, R_2)$ under the constraint $R_1 \le R_2$:

$$f_{R_2}(R_2) = \int_0^{R_2} f_1(R_1) f_2(R_2)dR_1 = f_2(R_2)F_1(R_2). \quad (11)$$

The marginal probability distribution function of the reliability in the second phase on the constraint is

$$F_{R_2}(R_2) = \int_0^{R_2} f_2(R_2)F_1(R_2)dR_2. \quad (12)$$

The probability distribution of the real reliability is

$$F_{final}(R_2) = \frac{F_{R_2}(R_2)}{P(R_2 \ge R_1)}, \quad (13)$$

where

$$P(R_2 \ge R_1) = P(R_2 - R_1 \ge 0) = \iint_{R_2 - R_1 \ge 0, 1 > R_{1,2} > 0} f_1(R_1) f_2(R_2)d(R_1 R_2), \quad (14)$$

and since the integral range is $\begin{cases} R_2 - R_1 \ge 0 \\ 1 > R_{1,2} > 0 \end{cases}$, the following is true

$$P(R_2 \ge R_1) = \int_0^1 dR_2 \int_0^{R_2} f_1(R_1) f_2(R_2)dR_1 = \int_0^1 f_2(R_2)F_1(R_2)dR_2 = F_{R_2}(1), \quad (15)$$

as shown in Fig. 1.

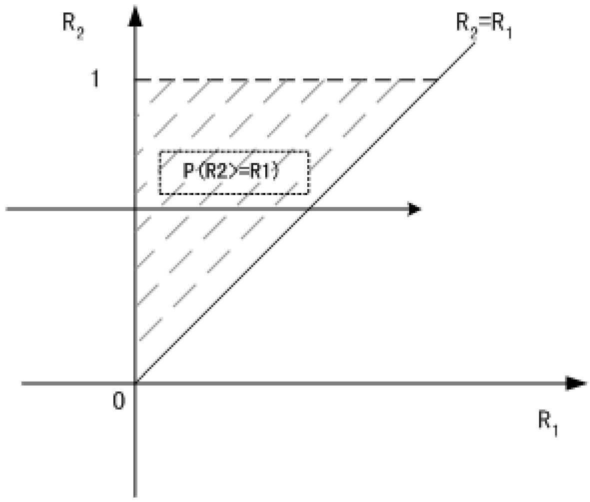

Fig. 1. Restriction range.

Therefore, at the end of the second phase, the probability distribution of reliability is

$$F_{final}(R_2) = \frac{F_{R_2}(R_2)}{F_{R_2}(1)}. \quad (16)$$

Likewise, the test data of the third phase can be calculated, using the result above. $F_{final}(R_2)$ and $F_3(R_3)$ can form a new two phase model to calculate and analyze the probability distribution function of reliability in the third phase. This iterative process can be continued until the probability distribution function of reliability of the last phase $F_{final}(R_n)$ is obtained. So, the α fractile of $F_{final}(R_n)$ is the reliability lower limit for the product with a confidence level of $1-\alpha$.

The Lindstrom and Madden evaluation formula

This method was originated by Lindstrom and Madden; it is applicable to systems consisting of k subsystems in series connection. The test data of the i-th subsystem is (n_i, s_i), which is known. Here, n_i is the time of trials of the i-th subsystem, s_i is the time of successful tests of the i-th subsystem. Denote the times of failure tests of the i-th subsystem by f_i, it follows that $n_i = s_i + f_i, i = 1,2,..., k$. Equivalent times of successful trials of the system N and the equivalent times of failure trials of the system F are given by

$$\left.\begin{aligned} N &= \min\{n_1, n_2, \cdots, n_i\} \\ F &= N\left(1 - \prod_{i=1}^{k} \frac{n_i - f_i}{n_i}\right) \end{aligned}\right\} \quad (17)$$

If the confidence of the series connection system is γ, the reliability lower confidence limit R_L can be written as

$$\sum_{X=0}^{F} \begin{bmatrix} N \\ X \end{bmatrix} R_L^{N-1}(1-R_L)^X = 1-\gamma. \quad (18)$$

Test data transformation to success or failure equipment

Based on the derivation of the formulas, the Lindstrom and Madden method can only be applied in systems consisting of success or failure subsystem. However, the test data of non-success or failure subsystem can be transformed to the data of success or failure.

According to the test data of devices and the reliability evaluation method for unit-level devices, reliability point estimation value $\hat{R}_i$ and reliability lower confidence limit $R_{iL}(\gamma)$ can be obtained, from which the test data of devices can be transformed to the data of success or failure (n^*, f^*). The formula of transformation is give as follows

$$\left.\begin{aligned} & s^* = n^* \hat{R}_i \\ & \frac{\int_0^{R_{iL}(\gamma)} x^{S_i^*-1}(1-x)^{n_i^*-S_i^*}\,dx}{B(S^*, n^*-S^*+1)} = 1-\gamma \end{aligned}\right\}, \quad (18)$$

where $\hat{R}$ can be substituted by $R_{iL}(0.5)$ with $\gamma = 0.5$.

III. Implementation

Consider a pass or fail Electronic Stability Control system with three subsystems, whose development cycle can be divided into four phases. It is assumed that in each phase, exposed faults have been amended. The original test data are shown in Table 1:

Table 1 The original test data of different phases

Phase i	Times Of system test n_i	Times Of successful system test s_i	Subsystem1 (n_{1i}, s_{1i})	Subsystem2 (n_{2i}, s_{2i})	Subsystem3 (n_{3i}, s_{3i})
1	6	3	(5,3)	(3,2)	(3,2)
2	4	3	(5,4)	(4,3)	(4,3)
3	7	6	(6,5)	(5,5)	(5,5)
4	4	4	(6,6)	(5,5)	(4,4)

Confidence level $1-\alpha$ =0.7 is chosen to evaluate the one-sided confidence lower limit of reliability R_4 in the final phase.

If the qualification of one-sided reliability confidence lower limit of this system is 0.9, which means that in the verification test before manufacturing, this system must either pass 12 consecutive tests, only fail 1 time out of 25 tests, only fail 2 times out of 37 tests, or only fail 3 times out of 50 tests and so forth. According to the test data (Table 1), this system has passed 4 consecutive tests (the one-sided reliability confidence lower limit is 0.74) in the final stage. If the general test and evaluation method is used, this system must have at least 8 more successful tests before being approved.

Table 2 Comprehensive test data of system

Phase i	Equivalent times of system test n_i	Equivalent times of successful system test s_i	Comprehensive times of system test n_i'	Comprehensive times of successful system test s_i'
1	3	1	9	4
2	4	2	8	5
3	5	5	12	11
4	4	4	8	8

Applying the L-M evaluation method, equivalent system test data (shown as the column 2 and 3 in table 2) can be used based on the subsystems test data. Comprehensive test data of system derived from all development phases can be obtained (shown as the column 4 and 5 in table 2).

According to the comprehensive test data derived from all phases (shown as the column 4 and 5 in Table 2), the one-sided reliability confidence lower limit of each phase is calculated in column 4 of Table 3. Since $R_1 \le R_2 \le R_3 \le R_4$, the constraint $R_1 \le R_2 \le \ldots \le R_i \le \ldots \le R_{n-1} \le R_n$ is satisfied.

Table 3 One-sided reliability confidence lower limit of each phase

Phase i	Comprehensive times of system test n_i'	Comprehensive times of successful system test s_i'	One-sided reliability confidence lower limit of phase R_i
1	9	4	0.3127
2	8	5	0.4701
3	12	11	0.8086
4	8	8	0.8603

Using the reliability calculation method discussed in section II , the one-sided reliability confidence lower limit R_4 of the final stage is calculated as: R_{4L} =0.9035, which is higher than the qualification threshold of 0.9. Therefore, this Electronic Stability Control system can be approved without additional tests. This amounts to a saving of at least 8 successful verification tests, and the development cycle is shortened because of the reduced test time.

IV. Conclusion

Based on the discussion in Section III, the evaluation of the reliability confidence lower limit for the Electronic Stability Control systems can significantly reduce the amount of tests without sacrificing the one-sided confidence level of the reliability. Since the amounts of tests are reduced, the research expenses will be reduced and the development cycle will be shortened.

It can be seen from the result of the example that the estimation of the reliability parameter is lower if we perform reliability estimates using only the test data of single phase or system level. This is because the one-phase test is inadequate, and the result cannot reflect the present reliability level exactly. The approach in this paper is suitable for practical systems, so that the astringency of interval result gets better and closer to the true value of the product reliability of the last development phase. The method can be applied as long as the test data of product phase has an increasing trend and data from the product development cycle are available.

This evaluation method may be also used in other engineering areas such as semiconductor testing.

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