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Sliding Mode Control for Flexible Joint using Uncertainty and Disturbance Estimation

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***Index Terms*—sliding mode control, uncertainty and disturbance estimation, flexible joint.**

***Abstract*—This paper proposes sliding mode control based on uncertainty and disturbance estimator (UDE), for trajectory tracking control of flexible joint robotic system. UDE is used to estimate plant uncertainty and disturbances. The controller does not requires knowledge of plant uncertainty and external disturbance. Reaching phase is eliminated for robustification. The perturbation is efficiently compensated by feedback of the estimated value. The proposed reference model is to track the plant states according to this model. The closed loop stability for this model with uncertainty and disturbance is also proposed.**

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I. Introduction

The problem of joint flexibility has received considerable attention as the major source of compliance in most present day manipulator designs. This joint flexibility typically arises due to gear elasticity, shaft windup, etc., and is important in the derivation of control law. Perhaps it is more critical to account for the joint flexibility when dealing with force control problems, than it is for pure position control. Joint flexibility must be taken into account in both modeling and control in order to achieve better tracking performance, for practical applications. Unwanted oscillations due to joint flexibility, imposes bandwidth limitations on all algorithm designs; based on rigid robots and may create stability problems for feedback controls that neglect joint flexibility. Control of flexible joint has been an important research topic and received considerable attention after 1990. The importance of joint flexibility in the modeling, control, and performance evaluation of robot manipulators has been established by several researchers. Spong used a singular perturbation model of the elastic joint manipulator dynamics and showed force control techniques developed for rigid manipulators can be extended to the flexible joint case [1]. A completely linear algorithm is proposed for composite robust control of flexible joint robots. Moreover, the robust stability of the closed loop system in presence of structured and unstructured uncertainties is analyzed. To introduce the idea, flexible joint robot with structured and unstructured uncertainties is modeled and converted into singular perturbation form [2].

In literature, a number of feedback control schemes have been proposed to address the issue of joint flexibility. A sliding mode control based strategy [3] is proposed that needs knowledge of the bounds of uncertainty and also the complete state vector for its implementation. A dynamic feedback controller for trajectory tracking control problem of robotic manipulators with flexible joints is proposed in [4]. The design requires position measurements on the link as well as the motor side and the velocities required in the controller are estimated through a reduced order observer. Further, robustness of the closed loop system is established by assuming that the uncertainties satisfy certain conditions. A singular perturbation approach is employed for the same task [5], wherein the controller needs measurements of position and elastic force. A nonlinear sliding mode state observer is used for estimating the link velocities and elastic force time derivatives. A Feedback Linearization (FL) based control law made implementable using extended state observer (ESO) is proposed for the trajectory tracking control of a flexible joint robotic system in [6]. Controller design based on the integral manifold formulation [7], adaptive control [8], adaptive sliding mode [9] and back-stepping approach [10] are some other approaches reported in the literature. Most of the schemes that appeared in literature have certain issues that require attention. Firstly many of them require measurements of all state variables or at least the position variables on link and motor side. Next robustness wherever guaranteed, is often highly model dependent. Also some need knowledge of certain characteristics of the uncertainties, such as its bounds. A variable structure observer that requires only measurement of link positions to estimate the full state of a flexible joint manipulator is proposed in [11]. Additionally a reduced adaptive observer that requires the measurement of link and motor positions is reported in [12] and a MIMO design for the strongly coupled joints in [13].

The design of robust, model following, sliding mode, load frequency controller for single area power system based on uncertainty and disturbance estimator (UDE) is discussed in [14]. The literature on UDE also mentions control of uncertain LTI systems [15], model following sliding mode control [16], Ackermann's formula for reaching phase elimination [17], robust model following based on UDE [18]. The control proposed does not require the knowledge of bounds of uncertainty and disturbance and is continuous.

In this paper, SMC is proposed to control flexible joint manipulator with uncertainty and disturbance. A nonlinear disturbance is considerd here and reaching phase is eliminated for robustification. The plant model is controlled to follow the desired states and the uncertainty and disturbance is estimated with UDE.

The paper is organized as follows: Section II describes the problem statement. The mathematical model is explained in Section III and Section IV explains the stability analysis. A numerical example is explored in Section V. Simulation results and discussions are presented in Section VI and the paper concludes in Section VII.

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II. Problem Formulation

Reviewing the continuous plant defined as,

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + d(t) \qquad (1) \\ y &= Cx(t) \end{aligned}$$

$$A = A_{nc} + \Delta A \quad B = B_{nc} + \Delta B$$

Here 'nc' denotes the normal part of uncertain continuous time system. $x(t)$ is n-dimensional plant state vector, $u(t)$ is control input vector, $d(t)$ is external disturbance. ΔA and ΔB are the uncertainties in the system matrix.

Assumption 1: The uncertainties $\Delta A, \Delta B$ and disturbance $d(x,t)$ satisfy matching conditions given by,

$$\Delta A = BD \quad \Delta B = BE \quad d(x,t) = Bv(x,t) \qquad (2)$$

The system (2) can now be written as,

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Be(t) \qquad (3) \\ y &= Cx(t) \end{aligned}$$

where, $e(x,t) = Dx + Eu + v(x,t)$.

Reference model that generates desired trajectory as a LTI system can be defined as ,

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m u_m(t) \qquad (4) \\ y_m(t) &= C_m x_m(t) \end{aligned}$$

Assumption 2: The choice of model is such that,

$$A - A_m = BL \qquad (5)$$

$$B_m = BM \qquad (6)$$

Control objective is to find a control input 'u' that makes the states of the plant; asymptotically track the response of a reference model (4).

Assumption 3: The lumped uncertainty $e(x,t)$ is such that,

$$\dot{e} \neq 0, \qquad \text{for} \quad i = 1, 2, \ldots, (r-1) \qquad (7)$$

$$\dot{e} = 0, \qquad \text{for} \quad i = r \qquad (8)$$

where, r is any positive integer.

A. Design of Control

The main objective of this controller is to eliminate uncertainty and disturbance in the system and command a desired tip angle position.

In this section, a model following control is designed with help of method suggested in [16].

Define a sliding surface [17]

$$\sigma = b^T x + z \qquad (9)$$

where,

$$\dot{z} = -b^T A_m x - b^T b_m u_m \quad z(0) = -b^T x(0) \qquad (10)$$

Equation (10) for the auxiliary variable z defined here is different from that given in [17]. By virtue of the choice of the initial condition on z, $\sigma = 0$ at $t = 0$. If a control u can be designed ensuring sliding, then $\dot{\sigma} = 0$ implies;

$$\dot{x} = A_m x + b_m u_m \qquad (11)$$

and hence fulfills the objective of the model following.

Using (4), (9) and (10) gives,

$$\begin{aligned} \dot{\sigma} &= b^T Ax + b^T bu + b^T be(x,t) - b^T A_m x - b^T b_m u_m \qquad (12) \\ &= b^T bLx - b^T bMu_m + b^T bu + b^T be(x,t) \end{aligned}$$

Let the required control be expressed as,

$$u = u_n + u_{eq} \qquad (13)$$

Selecting,

$$u_{eq} = -Lx + Mu_m - (b^T b)^{-1} k\sigma \qquad (14)$$

where, k is a positive constant.

From (12) and (14) we get,

$$\dot{\sigma} = b^T bu_n + b^T be(x,t) - k\sigma \qquad (15)$$

The lumped uncertainty $e(x,t)$ can be estimated; as given in [15]. Rewriting this equation,

$$e(x,t) = (b^T b)^{-1}(\dot{\sigma} + k\sigma) - u_n \qquad (16)$$

It can be seen that lumped uncertainty $e(x,t)$ can be computed from (16), which cannot be done directly.

Let the estimate of the uncertainty be defined as,

$$\hat{e}(x,t) = e(x,t)\ G_f(s) \qquad (17)$$

Using (16) and (17)

$$\hat{e}(x,t) = [(b^T b)^{-1}(\dot{\sigma} + k\sigma) - u_n]G_f(s) \qquad (18)$$

where, $G_f(s)$ is strictly proper order, low pass filter, with unity gain and enough bandwidth. With such a filter,

$$\hat{e}(x,t) \cong e(x,t) \qquad (19)$$

Error in the estimation is,

$$\widetilde{e}(x,t) = e(x,t) - \hat{e}(x,t) \qquad (20)$$

B. UDE with first order filter

If $G_f(s)$ is proper first order, low pass filter, with unity gain defined as,

$$G_f(s) = \frac{1}{\tau s + 1} \qquad (21)$$

where, τ is small positive constant.

With the above $G_f(s)$ and in view of (16), (18) and (20),

$$\begin{aligned} \widetilde{e}(x,t) &= (1 - G_f(s))[(b^T b)^{-1}(\dot{\sigma} + k\sigma) - u_n] \qquad (22) \\ &= \tau\dot{e}(x,t)G_f(s) \end{aligned}$$

The error in estimation varies with τ, enabling design of u_n as,

$$u_n = -\hat{e}(x,t) \qquad (23)$$

Combining (23) and (18)

$$u_n = -(b^T b)^{-1}(\dot{\sigma} + k)G_f(s) + G_f(s)u_n \qquad (24)$$

Solving for u_n gives,

$$u_n = \frac{(b^T b)^{-1}}{\tau}(\sigma + \frac{k\sigma}{s}) \qquad (25)$$

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III. Mathematical Model

We consider a single link manipulator, with revolute joint actuated by DC motor and model the elasticity of the joint as a linear torsional spring with stiffness K. The equations of motion for this system as taken from [19] are,

$$\begin{aligned} I\ddot{q_1} + MgL\sin(q_1) + K(q_1 - q_2) &= 0 \\ J\ddot{q_2} - K(q_1 - q_2) &= u \end{aligned} \tag{26}$$

where, q_1 and q_2 are the link and motor angles respectively, I is the link inertia, J being the inertia of motor, K is the spring stiffness, u is the input torque, and M and L are the mass and length of link respectively. The tracking problem for the system of (26) is to find a control which ensures $q_1^\star(t) - q_1(t) = 0$ for given initial states where $q_1^\star(t)$ is desired trajectory for $q_1(t)$.

The equations of motion for the Quanser's Flexible Joint module as given in [20] are,

$$\begin{aligned} \ddot{\theta} + F_1\dot{\theta} - \frac{K_{\text{stiff}}}{J_{\text{eq}}}\alpha &= F_2 V_m \\ \ddot{\theta} - F_1\dot{\theta} + \frac{K_{\text{stiff}}(J_{\text{eq}} + J_{\text{arm}})}{J_{\text{eq}}J_{\text{arm}}}\alpha &= -F_2 V_m \end{aligned} \tag{27}$$

where,

$$\begin{aligned} F_1 &\triangleq \frac{\eta_m\eta_g K_t K_m K_g^2 + B_{\text{eq}} R_m}{J_{\text{eq}} R_m} \quad \text{and} \\ F_2 &\triangleq \frac{\eta_m \eta_g K_t K_m}{J_{\text{eq}} R_m} \end{aligned}$$

The parameters are : θ is motor load angle, α is link joint deflection, η_m is the motor efficiency, η_g is the gearbox efficiency, K_t is the motor torque constant, K_m is the back EMF constant, K_g is the gearbox ratio, B_{eq} is the viscous damping coefficient, R_m is the armature resistance, J_{eq} is the gear inertia, K_{stiff} is the spring stiffness, J_{arm} is the link inertia, and V_m is the motor control voltage.

Considering the output of the system as $y = \theta + \alpha$, the dynamics (27) in terms of y and θ is re-written as,

$$\ddot{y} = \frac{K_{\text{stiff}}}{J_{\text{eq}}} F_3 y - \frac{K_{\text{stiff}}}{J_{\text{eq}}} F_3 \theta \tag{28}$$

$$\ddot{\theta} = \frac{K_{\text{stiff}}}{J_{\text{eq}}} y - \frac{K_{\text{stiff}}}{J_{\text{eq}}}\theta - F_1\dot{\theta} + F_2 V_m \tag{29}$$

where $F_3 \triangleq \left(1 - \frac{J_{\text{eq}} + J_{\text{arm}}}{J_{\text{arm}}}\right)$.

Defining the state variables as, $x_1 = y$, $x_2 = \dot{y} = \dot{x}_1$, $x_3 = \theta$, $x_4 = \dot{\theta} = \dot{x}_3$, the dynamics (28)–(29) become,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{K_{\text{stiff}}}{J_{\text{eq}}} F_3 (x_1 - x_3) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{K_{\text{stiff}}}{J_{\text{eq}}}(x_1 - x_3) - F_1 x_4 + F_2 V_m \end{aligned} \tag{30}$$

The state space form for (30) can be written as,

$$\dot{x} = A\,x + B\,V_m \tag{31}$$

where, $\dot{x} = [\dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3 \quad \dot{x}_4]^T$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{K_{\text{stiff}}}{J_{\text{eq}}}F_3 & 0 & -\frac{K_{\text{stiff}}}{J_{\text{eq}}}F_3 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_{\text{stiff}}}{J_{\text{eq}}}F_3 & 0 & -\frac{K_{\text{stiff}}}{J_{\text{eq}}}F_3 & -F_1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ F_2 \end{bmatrix}$$

For the desired output, the relative output y can be differentiated in proper manner. In order to satisfy the model following conditions, the above system (31) is converted to phase variable form by using the transformation,

$$Z = Tx$$

Then the Eq. (31) can be written as [6],

$$\dot{z} = A\,z + B\,V_m \tag{32}$$

where, $\dot{z} = [\dot{z}_1 \quad \dot{z}_2 \quad \dot{z}_3 \quad \dot{z}_4]^T$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{K_{\text{stiff}}F_1}{J_{\text{arm}}} & -\frac{K_{\text{stiff}}(J_{\text{eq}} + J_{\text{arm}})}{J_{\text{eq}}J_{\text{arm}}} & -F_1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_{\text{stiff}}F_2}{J_{\text{arm}}} \end{bmatrix}$$

IV. Stability Analysis

Using Eq. (17)

$$\hat{e}(x,t) = e(x,t)\;G_f(s) \tag{33}$$

Choosing $G_f(s)$ from Eq. (21)

$$\begin{aligned} \hat{e}(x,t) &= e(x,t)\,\frac{1}{\tau s + 1} \\ \hat{e}(x,t)(\tau s + 1) &= e(x,t) \\ \hat{e}(x,t)(\tau s) + \hat{e}(x,t) &= e(x,t) \end{aligned} \tag{34}$$

Simplifying Eq. (34) and using Eq. (20)

$$\tau\dot{\hat{e}} + \hat{e} = e$$

Adding and subtracting $\tau\dot{e}$ on LHS and simplifying,

$$\begin{aligned} \tau\dot{\hat{e}} + \tau\dot{e} - \tau\dot{e} &= e - \hat{e} \\ -\tau\dot{\tilde{e}} &= \tilde{e} - \tau\dot{e} \\ \dot{\tilde{e}} &= -\frac{1}{\tau}\tilde{e} + \dot{e} \end{aligned} \tag{35}$$

Using Eq. (15), (20) and (23)

$$\begin{aligned} \dot{\sigma} &= -b^T b\,\hat{e} + b^T b\,e - k\sigma \\ &= b^T b\,\tilde{e} - k\sigma \end{aligned} \tag{36}$$

The dynamics of the flexible joint can be represented in state space form using equations (35) and (36) as,

$$\begin{bmatrix} \dot{\sigma} \\ \dot{\tilde{e}} \end{bmatrix} = \begin{bmatrix} -k & (b^T b) \\ 0 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} \sigma \\ \tilde{e} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \dot{e} \tag{37}$$

This satisfies the separation principle. The eigen values are decided by constant k and filter constant τ, thus ensuring stability. The appropriate choice of k and τ ensures sliding variable σ and lumped uncertainty e tend to zero.

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V. EXAMPLE

In order to illustrate the proposed control algorithm, a single flexible link with flexible joints is considered. Simulation is performed to demonstrate the effectiveness, of the proposed control algorithm. Numerical dynamic model of this system from [20] is as follows;

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -10007 & -837 & -28 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10007 \end{bmatrix} V_m \tag{38}$$

The initial conditions are $x(0) = [0 \quad 0 \quad 0 \quad 0]$. This nominal values of the various flexible joint parameters are from [20]: K_{stiff}=1.248 $N-m/rad$, η_m=0.69, η_g=0.9, K_t=0.00767 $N-m$, K_g = 70, J_{eq}=0.00258 $kg-m^2$, J_{arm}=0.00352 $kg-m^2$, R_m=2.6 Ω.

The model to be followed is assumed as;

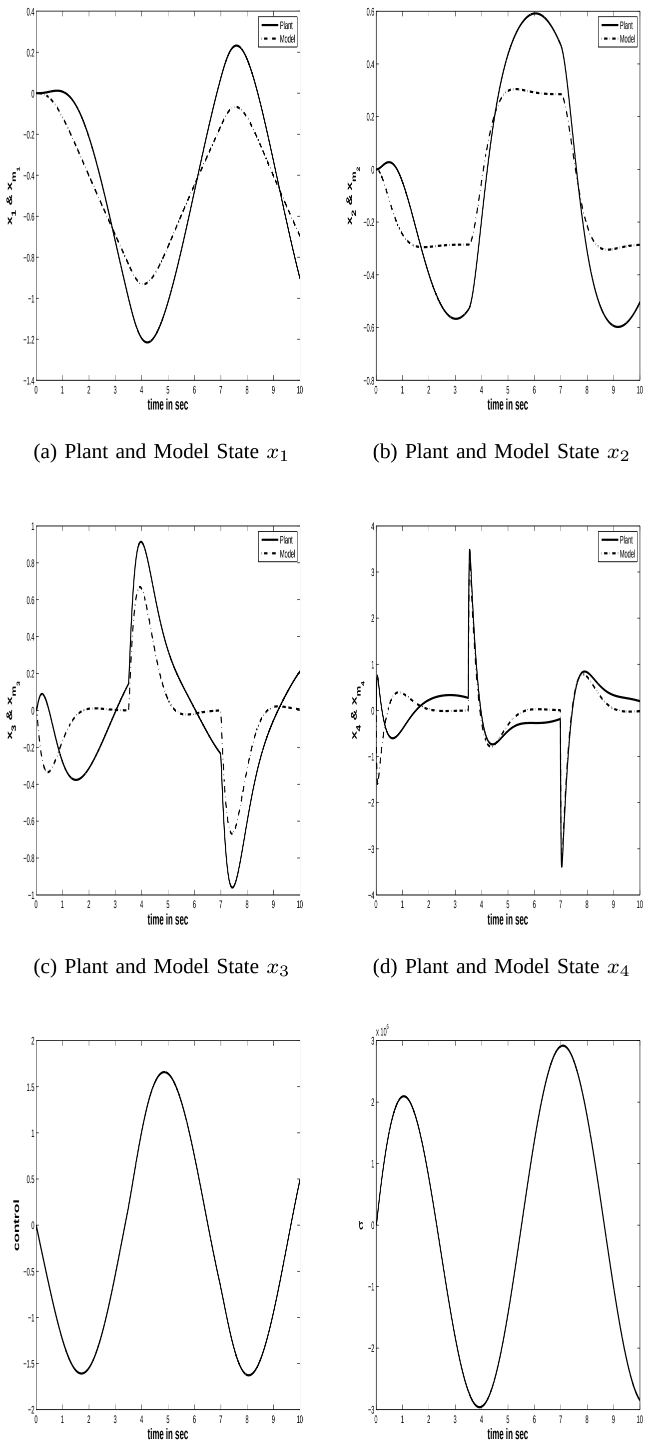
$$\begin{bmatrix} \dot{x}_{m_1} \\ \dot{x}_{m_2} \\ \dot{x}_{m_3} \\ \dot{x}_{m_4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -560 & -320 & -85 \end{bmatrix} \begin{bmatrix} x_{m_1} \\ x_{m_2} \\ x_{m_3} \\ x_{m_4} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 160 \end{bmatrix} V_m \tag{39}$$

with initial conditions are $x_m(0) = [0 \quad 0 \quad 0 \quad 0]$. The disturbance $d(t) = 2\sin(t)$, is sinusoidal with amplitude 2 and frequency 1 rad/sec and uncertainty in the plant is 40%.

VI. SIMULATION

The simulation studies reveal the results as shown in Fig. 1 – Fig. 4. Fig. 1 and Fig. 2 show the results for $k = 1$ and $k = 5$ respectively, when the value of τ is 10 ms. Fig. 1(a)–1(d) are the plant states i.e. displacement, velocity, acceleration and jerk, when $k = 1$ and $\tau = 10$ ms. The plant and model states are plotted in this window. Fig. 1(e) shows the control torque required and the Fig. 4(f) shows sliding variable (σ). The uncertainty in the plant is considered 40% (in both i.e. state matrix A and input matrix B). The tracking performance is improved as the gain k is increased to 5. This is shown in Fig. 2(a)–2(f).

Fig. 3 and Fig. 4 shows the results for $k = 1$ and $k = 5$ respectively, when the value of τ is 1 ms. The figure reveals the ability of the controller, to drive the system to follow the reference model. It is easily observed that system is robust even in presence of parameter variations and external disturbance. Controller is able to force the plant to follow the given model inspite of parameter variations.



(a) Plant and Model State x_1 (b) Plant and Model State x_2

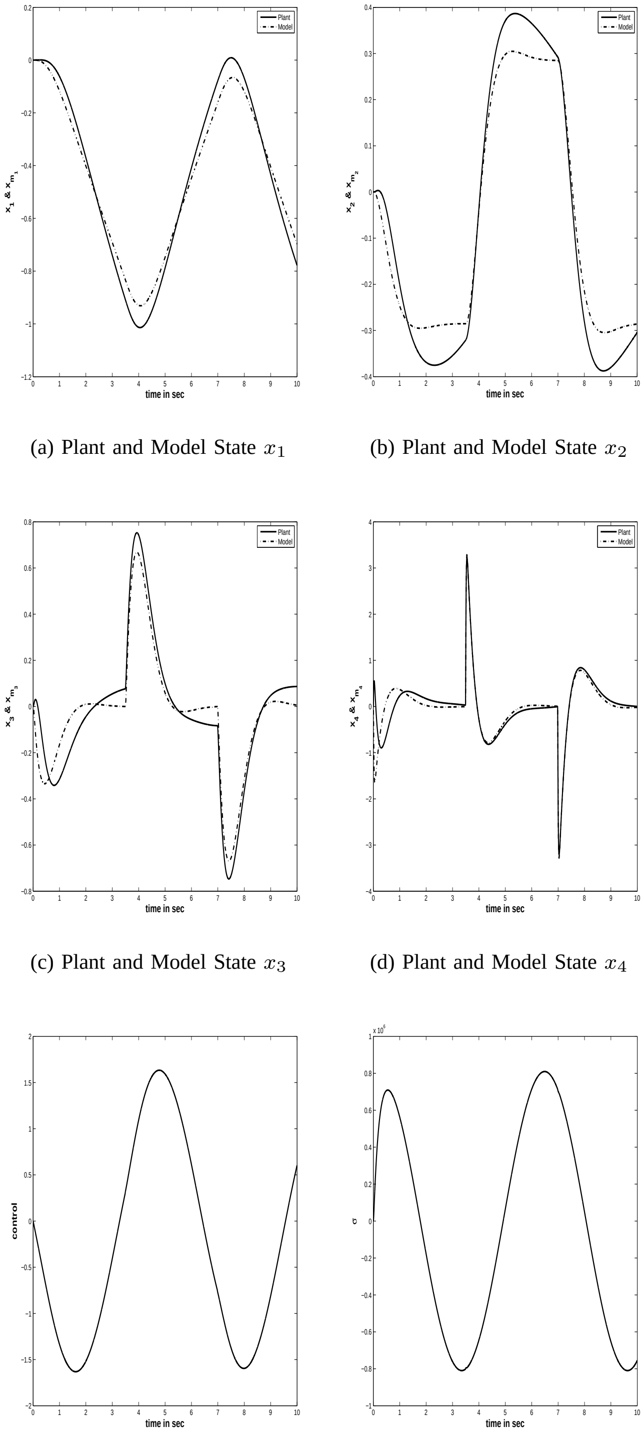
(c) Plant and Model State x_3 (d) Plant and Model State x_4

(e) Control u (f) Sliding Variable σ

Fig. 1: State tracking, Control and Sliding variable for $\tau = 10$ ms and $k = 1$

VII. CONCLUSION

In this paper, a trajectory tracking controller for flexible joint system, based on uncertainty and disturbance estimation (UDE) is proposed. The uncertainties and disturbance is estimated and compensated in the system performance. This



(a) Plant and Model State x_1

(b) Plant and Model State x_2

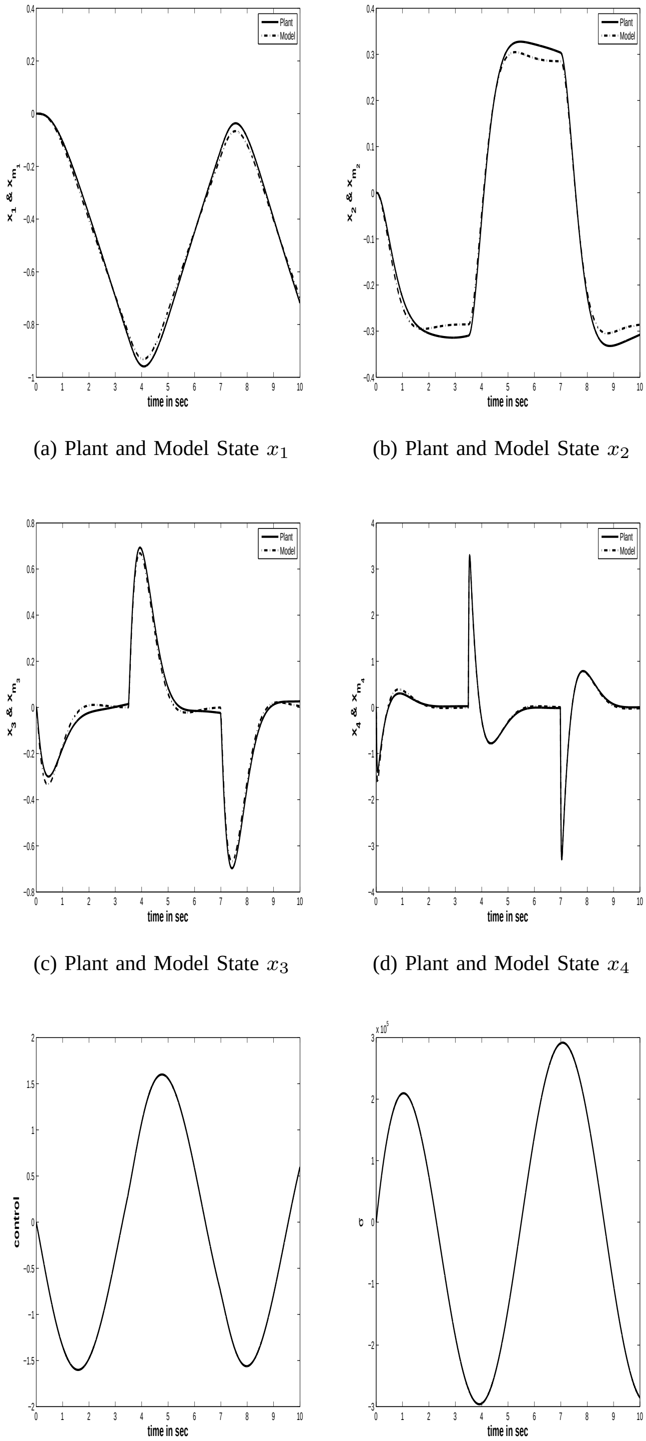
(c) Plant and Model State x_3

(d) Plant and Model State x_4

(e) Control u

(f) Sliding Variable σ

Fig. 2: State tracking, Control and Sliding variable for $\tau = 10$ ms and $k = 5$



(a) Plant and Model State x_1

(b) Plant and Model State x_2

(c) Plant and Model State x_3

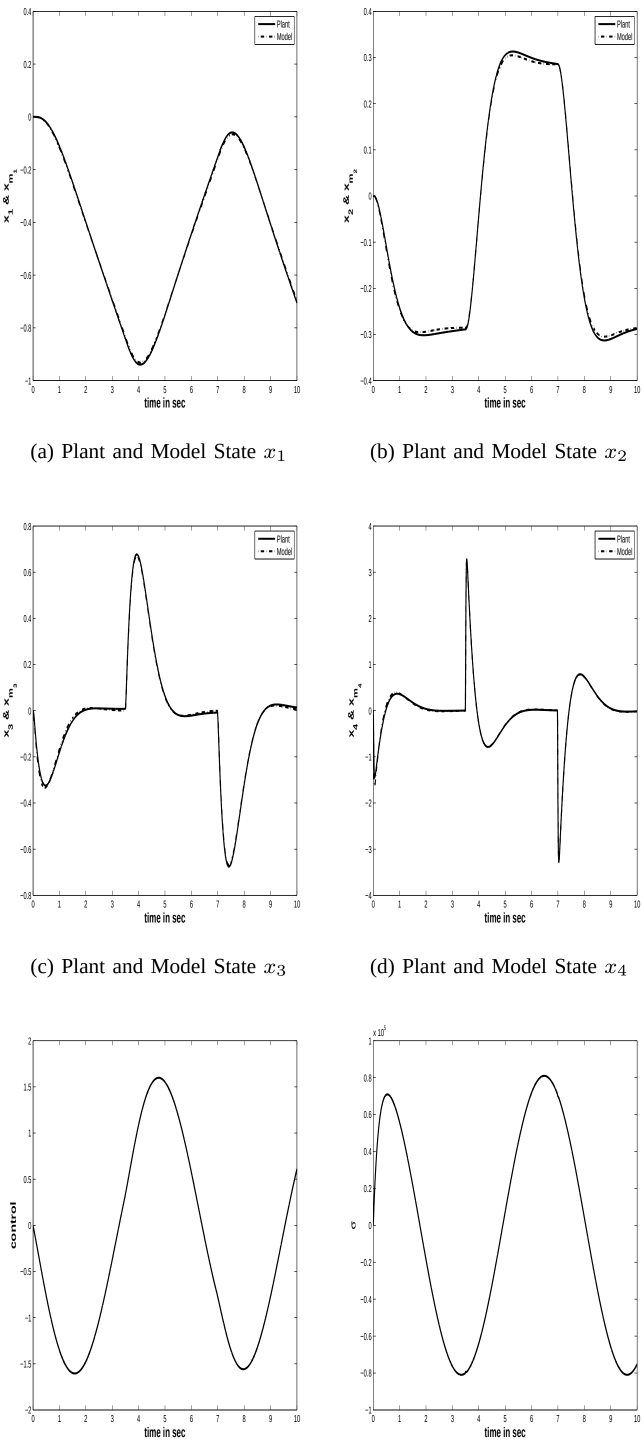
(d) Plant and Model State x_4

(e) Control u

(f) Sliding Variable σ

Fig. 3: State tracking, Control and Sliding variable for $\tau = 1$ ms and $k = 1$

is demonstrated through simulation. The control strategy includes Ackerman's method, which eliminates reaching phase to robustify the system. UDE is used to estimate the uncertainties and disturbance. The model is decided and control is designed to force the plant trajectories to track the model states. The plant follows the model states, even in the presence of uncertainties and disturbance. The results prove that the system performance is robust to parameter variations and external disturbances. The tracking performance is improved as the filter time constant τ becomes small. The performance also shows marked improvement as the value of k is increased. The performance can be further improved by using a higher order filter.



(a) Plant and Model State x_1 (b) Plant and Model State x_2

(c) Plant and Model State x_3 (d) Plant and Model State x_4

(e) Control u (f) Sliding Variable σ

Fig. 4: State tracking, Control and Sliding variable for $\tau = 1$ ms and $k = 5$

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