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Consensus Tracking for Multi-Agent Systems with Nonlinear Dynamics under Fixed Communication Topologies

Guoguang Wen, Ahmed Rahmani and Yongguang Yu

Abstract—**This paper is devoted to the study of consensus tracking problems for multi-agent systems with nonlinear dynamics, in which dynamics of each follower consists of two terms: one is given by an inherent nonlinear dynamics and the other is a simple communication protocol relying only on the position of its neighbors. The consensus reference is taken as a virtual leader who gives only its position information to only its neighbors. In this paper, the consensus tracking problems are respectively considered under fixed undirected and directed communication topologies. It is shown that the consensus tracking can be achieved in finite time under only the position measurements of the followers and the virtual leader. Simulation examples are finally given to demonstrate the effectiveness of the theoretical results.**

Index Terms—**Nonlinear inherent dynamics, Finite-time consensus tracking, Multi-agent system, virtual leader.**

I. INTRODUCTION

IN recent years, consensus problem of multi-agent systems has attracted much attention among researchers in the fields of biology, physics, computer science and control engineering. This is partly due to its broad applications in many areas such as formation control of unmanned air vehicles, design of distributed sensors networks, cooperative control of mobile robots, and so on(see, for example, the survey paper [1] and references therein). For a cooperative multi-agent system, consensus means that each agent updates its state based on local information of its neighbors, such that all agents reach an agreement on certain quantities of interest by negotiating with their neighbors.

In the past few years, some theoretical results have been established in [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], to name a few. In [2], Vicsek et al. proposed a discrete-time model to simulate a group of autonomous agents moving in the plane with the same speed but different headings. When the network topology is a connected graph, Adbababie et al. in [3] further studied the linear Vicsek model, and proved that all agents can became a stable state. Lately, the research of consensus problems for multi-agent systems was also extended to the case of directed topology ([4], [5], [6], [7], [8], [9], [10]). In recent literatures, a (virtual) leader-follower approach has been widely used to the consensus problem due to the fact that the state of virtual leader can represent the state of common interest for all other agents. The consensus with a (virtual) leader is usually called consensus tracking which means to design a network distributed control policy such that all agents can track the (virtual) leader. In [11], Cao et al. studied a consensus tracking of multi-agent systems for single-integrator dynamics and double-integrator dynamics under fixed and switching undirected topologies. Our objective is to consider the consensus tracking problems for multi-agent systems with nonlinear dynamics, in which dynamics of each follower consists of two terms: one is given by an inherent nonlinear dynamics and the other is a simple communication protocol relying only on the position of its neighbors. On the other hand, the communication can be undirected or directed. Therefore, we consider the consensus tracking problem under fixed undirected topology and fixed directed topology. We show that the consensus tracking can be achieved in finite time under only the position measurements of the followers and the virtual leader.

This paper is organized as follows. Section II gives some preliminaries on algebraic graph theory and formally state of the consensus problem. In sections III and IV, consensus tracking under fixed undirected and directed communication topologies are investigated. In section V, some examples are simulated to verify the theoretical analysis. Finally conclusions and future research directions are given in section VI.

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II. PRELIMINARIES AND PROBLEM STATEMENT

In this section, we first give a brief introduction to the algebraic graph theory, and then state our problem.

A. Algebraic graph theory

The communication topology among agents is represented by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, with the set of nodes $\mathcal{V} = \{v_1, ..., v_n\}$, the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = (a_{ij})_{n\times n}$. Here, each node v_i in $\mathcal{V}$ corresponds to an agent i, and each edge $(v_i, v_j) \in \mathcal{E}$ in a weighted directed graph corresponds to an information link from agent j to agent i, which means that agent i can receive information from agent j. In contrast, pairs of nodes in weighted undirected graph are unordered, where an edge $(v_j, v_i) \in \mathcal{E}$ denotes that agent i and j can receive information from each other. The weighted adjacency matrix $\mathcal{A}$ of a weighted directed graph is defined such that $a_{ii} = 0$ for any $v_i \in \mathcal{V}, a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The weighted adjacency matrix $\mathcal{A}$ of a weighted undirected graph is defined analogously except that $a_{ij} = a_{ji}, \forall i \neq j$, since $(v_i, v_j) \in \mathcal{E}$ implies $(v_j, v_i) \in \mathcal{E}$. We can say v_i is a neighbor vertex of v_j, if $(v_i, v_j) \in \mathcal{E}$.

The Laplacian matrix $L = (l_{ij})_{n\times n}$ of graph $\mathcal{G}$ is defined by $l_{ij} = -a_{ij}$ for $i \neq j$, and $l_{ii} = \sum_{j=1,j\neq i}^{n} a_{ij}$, $i, j \in \{1, ..., n\}$. For an undirected graph, L is symmetric positive semi-definite. However, L is not necessarily symmetric for a directed graph.

For simplicity, we denote by I_n the $n \times n$ identity matrix, and $0_{m\times n}$ the $m \times n$ zero matrix. Let $\mathbf{1}_n = [1, 1, \ldots, 1]^T \in R^n$ ($\mathbf{1}$ for short, when there is no confusion). $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ are the smallest and the largest eigenvalues of the matrix A respectively.

Definition 1([7]): A directed path from node v_i to v_j is a sequence of edges $(v_i, v_{j1}), (v_{j1}, v_{j2}), ..., (v_{jl}, v_j)$ in a directed graph $\mathcal{G}$ with distinct nodes $v_{jk}, k = 1, ..., l$.

Definition 2([7]): The directed graph $\mathcal{G}$ is said to have a directed spanning tree if there is a node (called root node) that can reach all the other nodes following a directed path in graph $\mathcal{G}$.

***Lemma** 2.1:* ([12]): A undirected graph $\mathcal{G}$ is connected if and only if its Laplacian matrix L has a eigenvalue zero with multiplicity 1 and corresponding eigenvector $\mathbf{1}$, and all other eigenvalues have positive real parts.

***Lemma** 2.2:* ([7]): A directed graph $\mathcal{G}$ has a directed spanning tree if and only if its Laplacian matrix L has a eigenvalue zero with multiplicity 1 and corresponding eigenvector $\mathbf{1}$, and all other eigenvalues have positive real parts.

***Lemma** 2.3:* ([13]) Kronnecker product $\otimes$ has the following properties: for matrices A, B, C and D with appropriate dimensions,
(1) $(A + B) \otimes C = A \otimes C + B \otimes C$;
(2) $(A \otimes B)(C \otimes D) = AC \otimes BD$;
(3) $(A \otimes B)^T = A^T \otimes B^T$;
(4) $(\xi A) \otimes B = A \otimes (\xi B)$,where ξ is a constant.

B. Problem statement

In this paper, we consider a multi-agent system that is made up of one virtual leader (labeled as 0) and n agents (labeled as agent 1 to n and called followers hereafter). Let the graph G represent the communication topology of all followers.

The dynamics of each follower i ($i = 1, ..., n$) is given by

$$\dot{\xi}_i(t) = f(t, \xi_i(t)) + u_i(t) \tag{1}$$

where $\xi_i(t) \in R^m$ is the position vector, $f(t, \xi_i(t)) \in R^m$ is its inherent nonlinear dynamics, and $u_i(t)$ is the control input.

The dynamics of the virtual leader 0 is given by

$$\dot{\xi}_0(t) = f(t, \xi_0(t)), \tag{2}$$

where $\xi_0(t) \in R^m$ and $f(t, \xi_0(t)) \in R^m$ are, respectively, the position states and a nonlinear vector-valued continuous function to describe the dynamics of virtual leader. Suppose that $f(t, \xi_0(t)) \neq 0$ i.e. the velocity of the virtual leader (2) is time-varying.

The consensus tracking problem of the multi-agent system is to design control inputs $u_i(t), i = \{1, ..., n\}$ such that

$$\lim_{t\to\infty} \|\xi_i(t) - \xi_0(t)\|_2 = 0 \tag{3}$$

for any i and for any arbitrary initial position states.

We suppose that the virtual leader share the same nonlinear inherent dynamics with all followers, and these nonlinear inherent dynamics satisfy a Lipchitz-type condition given by the Definition 3.

Definition 3 $\forall \xi, \zeta \in R^m; \forall t \geq 0$, there exists a nonnegative constant l such that

$$\|f(t, \xi) - f(t, \zeta)\|_2 \leq l \|\xi - \zeta\|_2 \tag{4}$$

Here, Definition 3 is a Lipschitz-type condition, which is satisfied in many well-known systems.

In what follows, the consensus tracking problem under fixed undirected and fixed directed topologies are considered in sections III and IV respectively.

III. Consensus tracking under fixed undirected topology

In this section, we consider the fixed undirected topology case. To satisfy the equation (3), we consider the following control input (5) which has been used in [11] to solve consensus problem for single-integrator dynamics case,

$$\begin{aligned} u_i(t) = &- \alpha \sum_{j=0}^{n} a_{ij}(\xi_i(t) - \xi_j(t)) \\ &- \beta \mathrm{sgn}(\sum_{j=0}^{n} a_{ij}(\xi_i(t) - \xi_j(t))) \end{aligned} \tag{5}$$

where α is a nonnegative constant, β is a positive constant, $\mathrm{sgn}(\cdot)$ is the signum function, and $a_{ij}, i, j = 1, ...n$, is the (i, j)th entry of the adjacency matrix $\mathcal{A}$ associated to G. Note that $a_{i0} > 0 (i = 1, ..., n)$ if the virtual leader's position is avaulable to follower i, and $a_{i0} = 0$ otherwise.

Using (5), (1)can be rewritten as

$$\begin{aligned} \dot{\xi}_i(t) = &f(t, \xi_i(t)) - \alpha \sum_{j=0}^{n} a_{ij}(\xi_i(t) - \xi_j(t)) \\ &- \beta \mathrm{sgn}(\sum_{j=0}^{n} a_{ij}(\xi_i(t) - \xi_j(t))) \end{aligned} \tag{6}$$

A. Main result

Let $M = L + diag(a_{10}, ..., a_{n0})$, where L is the Laplacian matrix of $\mathcal{G}$.

***Theorem** 3.1:* Suppose that the fixed undirected graph $\mathcal{G}$ is connected and at least one $a_{i0} > 0$. If $\alpha > \frac{l\lambda_{\max}(M)}{\lambda^2_{\min}(M)}$, then the system (6) satisfies $\|\xi_i(t) - \xi_0(t)\|_2 = 0$ in finite time.

Proof: Let $\tilde{\xi}_i(t) = \xi_i(t) - \xi_0(t), i = \{1, ..., n\}$. Form (2) and (6),

$$\begin{aligned} \dot{\tilde{\xi}}_i(t) = &- \alpha \sum_{j=0}^{n} a_{ij}(\tilde{\xi}_i(t) - \tilde{\xi}_j(t)) \\ &- \beta \mathrm{sgn}(\sum_{j=0}^{n} a_{ij}(\tilde{\xi}_i(t) - \tilde{\xi}_j(t))) \\ &+ f(t, \xi_i(t)) - f(t, \xi_0(t)), \end{aligned} \tag{7}$$

Let

$$\begin{aligned} \tilde{\xi}(t) &= [\tilde{\xi}_1^T(t), \tilde{\xi}_2^T(t), ..., \tilde{\xi}_n^T(t)]^T, \\ F(t, \tilde{\xi}(t)) &= [(f(t, \xi_1(t)) - f(t, \xi_0(t)))^T, ..., (f(t, \xi_n(t)) \\ &- f(t, \xi_0(t)))^T]^T. \end{aligned}$$

Rewrite (7) in the matrix form as

$$\begin{aligned} \dot{\tilde{\xi}}(t) = &- \alpha(M \otimes I_m)\tilde{\xi}(t) - \beta \mathrm{sgn}((M \otimes I_m)\tilde{\xi}(t)) \\ &+ F(t, \tilde{\xi}(t)), \end{aligned} \tag{8}$$

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where $\otimes$ stands for Kronecker product.

Because the fixed undirected graph $\mathcal{G}$ is connected and at least one a_{i0} is positive, M is symmetric positive definite.

Consider the following Lyapunov function candidate

$$V(t) = \frac{1}{2}\tilde{\xi}^T(t)(M \otimes I_m)\tilde{\xi}(t). \tag{9}$$

Tracking the time derivative of $V(t)$ along the trajectory (8) yields

$$\begin{aligned}\dot{V}(t) =& \frac{1}{2}\tilde{\xi}^T(t)(M \otimes I_m)\dot{\tilde{\xi}}(t) + \frac{1}{2}\dot{\tilde{\xi}}^T(t)(M \otimes I_m)\tilde{\xi}(t)\\ =& \frac{1}{2}\tilde{\xi}^T(t)(M \otimes I_m)[-\alpha(M \otimes I_m)\tilde{\xi}(t)\\ & - \beta\mathrm{sgn}((M \otimes I_m)\tilde{\xi}(t)) + F(t, \tilde{\xi}(t))]\\ & + \frac{1}{2}[-\alpha\tilde{\xi}^T(t)(M \otimes I_m) - \beta\mathrm{sgn}(\tilde{\xi}^T(t)(M \otimes I_m))\\ & + F^T(t, \tilde{\xi}(t))](M \otimes I_m)\tilde{\xi}(t)\\ \leq& - \alpha\tilde{\xi}^T(t)(M \otimes I_m)^2\tilde{\xi}(t)\\ & - \beta\left\|(M \otimes I_m)\tilde{\xi}(t)\right\|_1\\ & + l\lambda_{\max}(M)\left\|\tilde{\xi}(t)\right\|_2^2\\ \leq& - \alpha\lambda^2_{\min}(M)\left\|\tilde{\xi}(t)\right\|_2^2 + l\lambda_{\max}(M)\left\|\tilde{\xi}(t)\right\|_2^2\\ =& - (\alpha - \frac{l\lambda_{\max}(M)}{\lambda^2_{\min}(M)})\lambda^2_{\min}(M)\left\|\tilde{\xi}(t)\right\|_2^2.\end{aligned} \tag{10}$$

Note that if $\alpha > \frac{l\lambda_{\max}(M)}{\lambda^2_{\min}(M)}$, then $\dot{V}(t) < 0$.

Next, we prove that $V(t)$ will decrease to zero in finite time. From (10), when $\alpha > \frac{l\lambda_{\max}(M)}{\lambda^2_{\min}(M)}$, $\dot{V}(t)$ will also satisfy that

$$\begin{aligned}\dot{V}(t) &\leq -\beta\left\|(M \otimes I_m)\tilde{\xi}(t)\right\|_1\\ &\leq -\beta\lambda_{\min}(M)\left\|\tilde{\xi}(t)\right\|_2\\ &\leq -\beta\frac{\sqrt{2}\lambda_{\min}(M)\sqrt{V(t)}}{\sqrt{\lambda_{\max}(M)}}.\end{aligned} \tag{11}$$

From (11),

$$\sqrt{V(t)} \leq \sqrt{V(0)} - \frac{\sqrt{2}}{2}\beta\frac{\lambda_{\min}(M)}{\sqrt{\lambda_{\max}(M)}}t. \tag{12}$$

Let $\sqrt{V(0)} - \frac{\sqrt{2}}{2}\beta\frac{\lambda_{\min}(M)}{\sqrt{\lambda_{\max}(M)}}t^* = 0$, then

$$t^* = \frac{\sqrt{2\tilde{\xi}^T(0)(M \otimes I_m)\tilde{\xi}(0)}\sqrt{\lambda_{\max}(M)}}{\beta\lambda_{\min}(M)}. \tag{13}$$

Therefore, when $t > t^*$, we have $V(t) = 0$. The proof is completed.

B. Determination of the control parameters

1) Determination of α: α is chosen such that it satisfies the condition of Theorem 3.1. Indeed, the calculation of the expression $\frac{l\lambda_{\max}(M)}{\lambda^2_{\min}(M)}$ can be used to select α.

2) Determination of β: β is chosen according to the bound of time t^*, see (13). Indeed, first choosing the time t^*, β can be determined using the following expression

$$\beta = \frac{\sqrt{2\tilde{\xi}^T(0)(M \otimes I_m)\tilde{\xi}(0)}\sqrt{\lambda_{\max}(M)}}{t^*\lambda_{\min}(M)}.$$

IV. Consensus tracking under directed fixed topology

In this section, we consider the consensus tracking for multi-agent systems with nonlinear inherent dynamics under fixed directed topology. We still consider the control input (5).

Since the graph $\mathcal{G}$ is a fixed directed graph, generally speaking, the matrix M may be asymmetric.

Let the graph $\tilde{\mathcal{G}}$ represent the communication topology of all the followers and the virtual leader. Assume that the virtual leader has no information about its followers, and has independent motion. It implies that if the fixed directed graph $\tilde{\mathcal{G}}$ contains a directed spanning tree, then the node corresponding to the virtual leader 0 is the root node.

***Lemma** 4.1:* Suppose the fixed directed graph $\tilde{\mathcal{G}}$ contains a directed spanning tree, then there exists a symmetric positive definite matrix S such that the matrix $SM+M^TS$ is also symmetric positive definite.

Proof: Consider the following Laplacian matrix of $\tilde{G}$,

$$\tilde{L} = \begin{bmatrix} M & -\mathbf{a}_0 \\ \mathbf{0}_n^T & 0 \end{bmatrix}, \tag{14}$$

where $\mathbf{a}_0 = [a_{10}, ..., a_{n0}]^T$.

Since the fixed directed graph $\tilde{\mathcal{G}}$ has a directed spanning tree, by Lemma 2.2, $\tilde{L}$ has a simple eigenvalue zero with an associated eigenvector **1**, and all other eigenvalues have positive real parts. Note from (14) that, each element in the row $n+1$ is zero, which implies that all eigenvalues of the sub-matrix M are not zero. Moreover, note that each row sum of the matrix $\tilde{L}$ is zero. Using the elementary transformation to (14), we can find a matrix $P = \begin{bmatrix} I_n & \mathbf{1}_n \\ \mathbf{0}_n^T & 1 \end{bmatrix}$, such that

$$P\tilde{L}P^{-1} = \begin{bmatrix} M & \mathbf{0}_n \\ \mathbf{0}_n^T & 0 \end{bmatrix}. \tag{15}$$

It implies that all eigenvalues of M are the eigenvalues of $\tilde{L}$. Hence all eigenvalues of M have positive real parts. Using Theorem 1.2 in [14] , there exists a symmetric positive definite matrix S such that $SM + M^TS$ is also symmetric positive definite. The proof is completed.

A. Main result

***Theorem** 4.2:* Suppose that the fixed directed graph $\tilde{\mathcal{G}}$ contains a directed spanning tree. Using (5) for (1), if $\alpha > \frac{2l\lambda_{\max}(S)}{\lambda_{\min}(SM+M^TS)}$, then $\|\xi_i(t) - \xi_0(t)\|_2 = 0$ in finite time.

Proof: Using the same operation as in the proof of Theorem 3.1, we can still obtain the equation (8). Consider the Lyapunov function candidate

$$V(t) = \tilde{\xi}^T(t)(S \otimes I_m)\tilde{\xi}(t). \tag{16}$$

Tracking the time derivative of $V(t)$ along the trajectory of

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(8) gives

$$\begin{aligned}
\dot{V}(t) =&\tilde{\xi}^T(t)(S\otimes I_m)\dot{\tilde{\xi}}(t)+\dot{\tilde{\xi}}^T(t)(S\otimes I_m)\tilde{\xi}(t)\\
=&-\alpha\tilde{\xi}^T(t)(S\otimes I_m)(M\otimes I_m)\tilde{\xi}(t)\\
&-\beta\tilde{\xi}^T(t)(S\otimes I_m)\mathrm{sgn}((M\otimes I_m)\tilde{\xi}(t))\\
&+\tilde{\xi}^T(t)(S\otimes I_m)F(t,\tilde{\xi}(t))\\
&-\alpha[(M\otimes I_m)\tilde{\xi}(t)]^T(S\otimes I_m)\tilde{\xi}(t)\\
&-\beta\mathrm{sgn}\{[(M\otimes I_m)\tilde{\xi}(t)]^T\}(S\otimes I_m)\tilde{\xi}(t)\\
&+F^T(t,\tilde{\xi}(t))(S\otimes I_m)\tilde{\xi}(t)\\
=&-\alpha\tilde{\xi}^T(t)[(SM+M^TS)\otimes I_m)]\tilde{\xi}(t)\\
&-2\beta\tilde{\xi}^T(t)(S\otimes I_m)\mathrm{sgn}[(M\otimes I_m)\tilde{\xi}(t)]\\
&+2\tilde{\xi}^T(t)(S\otimes I_m)F(t,\tilde{\xi}(t))\\
\le&-\alpha\lambda_{\min}(SM+M^TS)\left\|\tilde{\xi}(t)\right\|_2^2\\
&+2l\lambda_{\max}(S)\left\|\tilde{\xi}(t)\right\|_2^2-2\beta\lambda_{\min}(S)\left\|\tilde{\xi}(t)\right\|_1\\
=&-(\alpha-\frac{2l\lambda_{\max}(S)}{\lambda_{\min}(SM+M^TS)})\lambda_{\min}(SM\\
&+M^TS)\left\|\tilde{\xi}(t)\right\|_2^2.
\end{aligned} \tag{17}$$

Note that when $\alpha > \frac{2l\lambda_{\max}(S)}{\lambda_{\min}(SM+M^TS)}$, $\dot{V}(t)<0$.

Next, we prove that $V(t)$ will decrease to zero in finite time. From (17), when $\alpha > \frac{2l\lambda_{\max}(S)}{\lambda_{\min}(SM+M^TS)}$, $\dot{V}(t)$ will satisfy that

$$\begin{aligned}
\dot{V}(t)&\le -2\beta\left\|(S\otimes I_m)\tilde{\xi}(t)\right\|_1\\
&\le -2\beta\lambda_{\min}(S)\left\|\tilde{\xi}(t)\right\|_2\\
&\le -2\beta\frac{\lambda_{\min}(S)\sqrt{V(t)}}{\sqrt{\lambda_{\max}(S)}}.
\end{aligned} \tag{18}$$

From (18),

$$\sqrt{V(t)}\le\sqrt{V(0)}-2\beta\frac{\lambda_{\min}(S)}{\sqrt{\lambda_{\max}(S)}}t. \tag{19}$$

Therefore, when

$$t>t^*=\frac{\sqrt{\tilde{\xi}^T(0)(S\otimes I_m)\tilde{\xi}(0)}\sqrt{\lambda_{\max}(S)}}{2\beta\lambda_{\min}(S)}, \tag{20}$$

we have $V(t)=0,$ where t^* is given by (19). The proof is completed.

B. Determination of the control parameters

1) Determination of α: Similar to the case in section III.B, α is chosen such that it satisfies the condition of Theorem 4.2. Indeed, the calculation of the expression $\frac{2l\lambda_{\max}(S)}{\lambda_{\min}(SM+M^TS)}$ can be used to select α.

2) Determination of β: β is chosen according to the bound of time t^*. Indeed, choosing the time t^*, β can be determined using the following expression

$$\beta=\frac{\sqrt{\tilde{\xi}^T(0)(S\otimes I_m)\tilde{\xi}(0)}\sqrt{\lambda_{\max}(S)}}{2t^*\lambda_{\min}(S)}.$$

V. Simulations

Let us consider a multi-agent system consisting of one virtual leader indexed by 0 and seven followers indexed by 1 to 7, respectively.

For simplicity, we consider that $a_{ij}=1$ if agent i can receive information from agent j, $a_{ij}=0$ otherwise, $i\in\{1,...,7\}$ and $j\in\{0,1,...,7\}$.

A. Case of undirected fixed topology

1) Dynamics of agents: The communication topology is given in Fig. 1. Suppose that the dynamics of the follower i ($i=1,...,7$) is described by the equation (21)

$$\begin{aligned}
\dot{\xi}_i(t)=&\sin(\xi_i(t))-\alpha\sum_{j=0}^{n}a_{ij}(\xi_i(t)-\xi_j(t))\\
&-\beta\mathrm{sgn}(\sum_{j=0}^{n}a_{ij}(\xi_i(t)-\xi_j(t))),
\end{aligned} \tag{21}$$

and their initial x positions and y positions are given by $\xi_x(0)=[\ -3\ \ -2.5\ \ 2\ \ -1\ \ 2.5\ \ -2\ \ 1\ ]$ and $\xi_y(0)=[\ 3\ \ 2\ \ -1\ \ -2\ \ 1.5\ \ 1\ \ 2\ ]$ respectively.

The dynamics of the virtual leader 0 is given by

$$\dot{\xi}_0(t)=\sin(\xi_0(t)), \tag{22}$$

and its initial and desired positions are given in Fig. 2.

In addition, there are obstacles between the initial position and desired position of the virtual leader. It is necessary to choose the trajectory of the virtual leader, so that it can reach the desired position by avoiding obstacles (Fig. 2). Here, the trajectory function of virtual leader is chosen to be $\xi_0(t)=[t+\sin(t),\sin(\frac{1}{3}\pi\mathrm{t})]^T$.

2) Determination of α: The matrix M can be derived from the topology given in Fig.1.

$$M=\begin{bmatrix}
3 & -1 & -1 & 0 & 0 & 0 & 0\\
-1 & 3 & 0 & -1 & -1 & 0 & 0\\
-1 & 0 & 3 & 0 & 0 & -1 & -1\\
0 & -1 & 0 & 1 & 0 & 0 & 0\\
0 & -1 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & -1 & 0 & 0 & 1 & 0\\
0 & 0 & -1 & 0 & 0 & 0 & 1
\end{bmatrix}.$$

Using Theorem 3.1, it is easy to obtain that when $\alpha > \frac{l\lambda_{\max}(M)}{\lambda^2_{\min}(M)}=502.0042$, the consensus tracking can be achieved. Here we choose α = 510.

3) Determination of β: We aim to design the simulation such that the consensus tracking can be achieved in at most t^* = 10 s. In order to do that, we have to choose β to be bigger than 28.09, which can be calculated from (13). Let's take $\beta=29$.

4) Simulation results: Fig.3 shows the trajectories of the virtual leader and the followers. Figs. 4 and 5 show the position tracking errors of x positions and y positions respectively. It is easy to see from Figs. 3, 4 and 5 that the consensus tracking can be achieved in about 0.4 seconds. This value is remarkably less than the desired bound 10 seconds.

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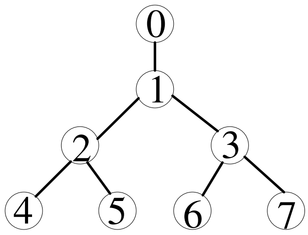

Fig. 1. The fixed undirected topology of followers and virtual leader

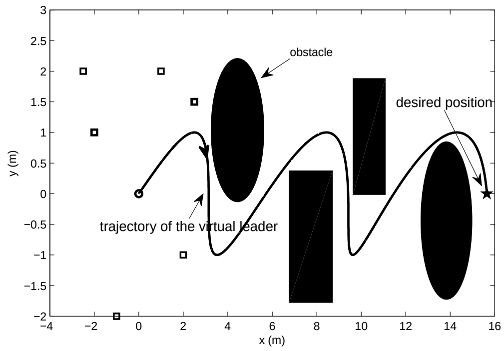

Fig. 2. Example scenario of obstacle avoidance for all agents

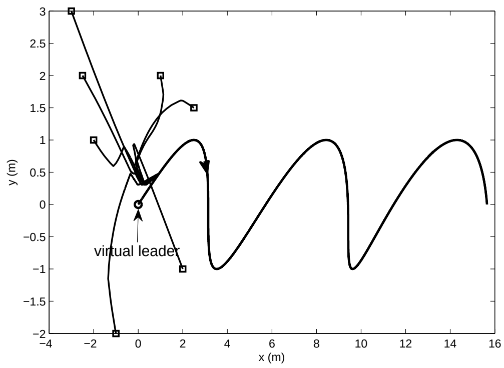

Fig. 3. Trajectories of the virtual leader and the followers under (21) and (22). The circle denotes the initial position of the virtual leader, while the squares denote the initial positions of the followers.

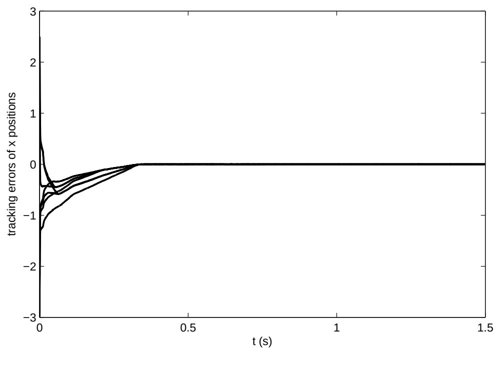

Fig. 4. Position tracking errors of x positions under (21) and (22).

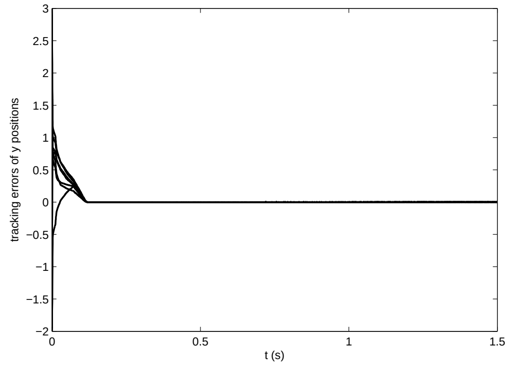

Fig. 5. Position tracking errors of y positions under (21) and (22).

B. Case of undirected fixed topology

1) Dynamics of agents: The communication topology is given in Fig. 6. Suppose the dynamics of each follower is described by

$$\begin{aligned}\dot{\xi}_i(t) = &\sin(\xi_i(t) + \frac{\pi}{4}) - \alpha \sum_{j \in \bar{N}_i(t)} b_{ij}(\xi_i(t) - \xi_j(t)) \\ &- \beta \mathrm{sgn}(\sum_{j \in \bar{N}_i(t)} b_{ij}(\xi_i(t) - \xi_j(t))),\end{aligned} \tag{23}$$

and their initial x positions and y positions are given by $\xi_x(0) = [\ -2.5 \quad -3 \quad -2.8 \quad -4 \quad -2 \quad -1 \quad -3.5\ ]$ and $\xi_y(0) = [\ 0.5 \quad -1.5 \quad 1 \quad -0.5 \quad 1.5 \quad 2 \quad 0.8\ ]$ respectively.

The dynamics of the virtual leader is given by

$$\dot{\xi}_0(t) = \sin(\xi_0(t) + \frac{\pi}{4}) \tag{24}$$

and its initial position is given in Fig. 7.

Our aim is that the virtual leader moves along an ellipse. So the trajectories of the virtual leader is chosen to be $\xi_0(t) = [-2cos(t), sin(t)]^T$.

2) Determination of α: Based on the Fig. 6, the matrix M is obtained as follows

$$M = \begin{bmatrix} 2 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Let $SM + M^T S = I_n$. Then we obtain

$$S = \begin{bmatrix} 0.7227 & 0.9454 & 1.6176 & 0.0714 & 0.0714 & 0.0357 & 0.0357 \\ 0.9454 & 2.0210 & 3.1134 & 0.2143 & 0.2143 & 0.1071 & 0.1071 \\ 1.6176 & 3.1134 & 5.8739 & 0.1429 & 0.1429 & 0.3214 & 0.3214 \\ 0.0714 & 0.2143 & 0.1429 & 0.5000 & 0 & 0 & 0 \\ 0.0714 & 0.2143 & 0.1429 & 0 & 0.5000 & 0 & 0 \\ 0.0357 & 0.1071 & 0.3214 & 0 & 0 & 0.5000 & 0 \\ 0.0357 & 0.1071 & 0.3214 & 0 & 0 & 0 & 0.5000 \end{bmatrix}$$

Using Theorem 4.1, $\alpha > \frac{2l\lambda_{\max}(S)}{\lambda_{\min}(SM+M^T S)} = 16.2584$. Here we choose α = 18.

3) Determination of β: we aim to design the simulation such that the consensus tracking can be achieved in at most t^* = 10 s. β must to be bigger than 3.5217 using equetion(13). Let's take $\beta = 4$.

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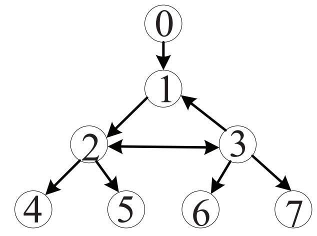

Fig. 6. The fixed directed topology of followers and virtual leader

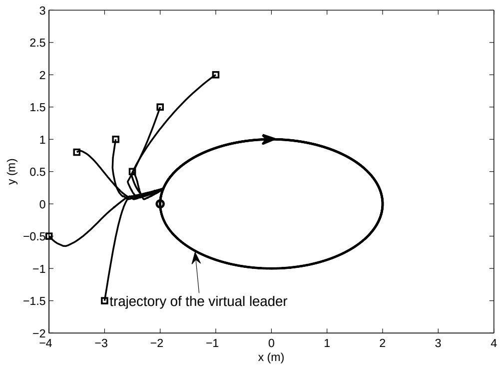

Fig. 7. Trajectories of the virtual leader and the followers under (23) and (24). The circle denotes the initial position of the virtual leader, while the squares denote the initial positions of the followers.

4) Simulation results: Fig.7 shows the trajectories of the virtual leader and the followers. Figs. 8 and 9 show the position tracking errors of x positions and y positions respectively. It is easy to see from Figs. 7, 8 and 9 that the consensus tracking can be achieved in about 0.2 seconds. This value is remarkably less than the desired bound 10 seconds.

VI. CONCLUSIONS AND FUTURE WORKS

In this paper, we consider the consensus tracking for multi-agent systems with nonlinear inherent dynamics under fixed undirected and directed communication topologies. It is shown that the consensus tracking can be achieved in finite time using appropriate control laws. Some simulation examples are finally given to validate the theoretical results.

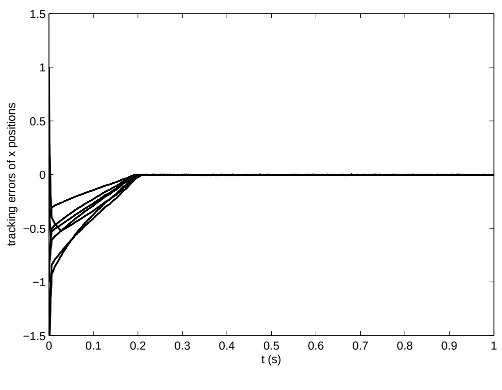

Fig. 8. Position tracking errors of x positions under (21) and (22).

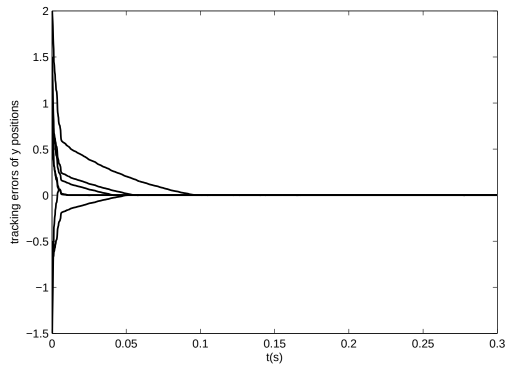

Fig. 9. Position tracking errors of y positions under (21) and (22).

As further works, we will consider the second-order consensus tracking for multi-agent systems with nonlinear inherent dynamics. On the other hand, since the operation of most multi-agent systems is naturally delayed, the consensus tracking with time delay will be also investigated.

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