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Fast Algorithm for Maximum Likelihood DOA Estimation in MIMO Array

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Abstract—**Maximum Likelihood estimator (ML) has shown excellent performance of Direction Of Arrival (DOA) estimation in Multiple Input Multiple Output (MIMO) array. However, the computation burden of MIMO ML is very large. In order to resolve this problem, a novel MIMO Maximum Likelihood DOA Estimation based on Metropolis-Hasting Sampling (MIMO MHML) is proposed, which combines Markov Monte Carlo method with MIMO Maximum Likelihood DOA estimator. MIMO MHML regards the power of the MIMO ML spectrum function as a target distribution up to a constant scalar, and uses Metropolis-Hasting sampler to sample from it. Simulation results show that MIMO MHML provides similar performance to that achieved by the MIMO ML method, but its computational cost is reduced greatly.**

Index Terms—**Maximum Likelihood estimator (ML), Direction Of Arrival (DOA), multiple-input multiple-output (MIMO) array, Metropolis-Hasting (MH) Sampling, computational complexity**

I. INTRODUCTION

THE idea of Multiple Input Multiple Output (MIMO) has been recently become a hot research for its potential advantages. MIMO radar has been proposed as a new radar system with various applications[1][2][3]. According to the array configuration, MIMO radar is classified into two categories. The first one is called distributed MIMO radar[1][4]. In this scenario, all the transmit array elements are widely separated and radiate independent signals to different look-directions of the target, such that each of the components extracted by the matched filters at the receiver contains independent information about the target. The second category is called co-located MIMO radar[3][5], which is considered in this paper. All the transmit and receive array element are collocated and independent waveforms are transmitted obtain many advantages by exploiting waveform diversity. As a result of enhanced flexibility in the design of transmitting beampattern and waveform synthesis, the performance of multiple target detection and identification can be improved[6]. Furthermore, the maximum number of targets to be detected and located by the array is increased[5][7]. By extended array aperture with virtual sensors and narrower beams, high-resolution spatial spectrum estimation is also obtained[2].

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Some typical DOA estimate method for MIMO radar have been proposed, including iterative GLRT (iGLRT) and maximum likelihood (ML). In [8], the minimum variance method is applied to MIMO array to estimate DOAs of targets. In [9], several design methods for transmitting signal of MIMO radar are proposed to improve the performance of adaptive MIMO radar algorithm. In [10], signal detection based on iterative testing of generalized likelihood ratio and maximum likelihood (ML) DOA estimation based on sufficient statistics are discussed, where MIMO ML DOA estimation method demonstrates good performance. However, MIMO ML requires a multidimensional search, and the computational complexity increases exponentially with the dimension. The method is not feasible in practice. Therefore a method which performs same as as MIMO ML but more efficient in computation is desired.

In recent years, Markov Chain Monte Carlo (MCMC) method has been shown to be a very powerful numerical method in reducing computational complexity of parameter estimation[11][12]. Metropolis-Hastings(MH) sampling is a classical method of MCMC, it has attracted lots of research work in the area of signal processing because of its application in statistical signal and array signal processing.

This paper is organized as follows. The co-located MIMO array signal model is present in Section II. In Section III, we incorporate one sampling method of MCMC—Metropolis-Hastings sampling into MIMO ML method to form a novel fast DOA estimator called Maximum Likelihood DOA Estimator based on Metropolis-Hastings (MIMO MHML) which not only maintains the excellent performance which MIMO ML achieves, but also significantly reduces the computation complexity. The simulations and analysis of the results for the proposed method are given in Section VI. Finally, Section V concludes the paper.

II. SIGNAL MODEL

Consider a MIMO narrowband array system with M transmitting sensors and M receiving sensors. The system simultaneously transmits M orthogonal waveforms, denoted by $s_m(t) \in C^{N\times 1}$, where $m = 1, 2, \cdots, M$. Assume that there are D point targets located in far field of this array with directions $\theta_d (d = 1, 2, \cdots, D)$. Then, the received data can be expressed as

$$X = \sum_{d=1}^{D} a_r(\theta_d)\,\beta_d a_t^{\mathrm{T}}(\theta_d)\,S + W \tag{1}$$

where $a_r(\theta) \in C^{M\times 1}$ and $a_t(\theta) \in C^{M\times 1}$ are the receive and the transmit array response vectors respectively. $S = [s_1, s_2, \cdots, s_M]^T$, is the orthogonal transmitting data

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matrix. $X \in C^{M\times N}$ is the received data snapshots, where N is the number of snapshot. β_d stands for the complex amplitude of the received signal. $W \in C^{M\times N}$ denotes the additive Gaussian white noise and $(\cdot)^{\mathrm{T}}$ is the matrix transpose operator.

Alternatively, (1) can be rewritten as

$$X = \sum_{d=1}^{D} \beta_d A(\theta_d) S + W \tag{2}$$

where

$$A(\theta_d) = a_r(\theta_d)\, a_t^{\mathrm{T}}(\theta_d) \tag{3}$$

III. MIMO ML Estimator based on Metropolis-Hastings Sampling

A. MIMO ML Estimator

As given in [10], the model in (2) can be rewritten in matrix form

$$\hat{\eta} = G(\Theta)\,\Xi + W \tag{4}$$

where $G(\Theta) = [d(\theta_1), d(\theta_2), \cdots, d(\theta_D)]$, $\Theta = [\theta_1, \theta_2, \cdots, \theta_D]$, $\Xi = [\beta_1, \beta_2, \cdots, \beta_D]^T$. $d(\theta_d)$ is the equivalent array response of size M^2 at the direction θ_d. Further, orthogonal signals, $d(\theta_d)$ is the product of the steering vectors in the receive mode and the steering vector in the transmit mode, $d(\theta_d) \triangleq \sqrt{N}\mathrm{vec}(A(\theta_d)I_M)$. Hence the MIMO ML estimator for target localization of the model in (4) is given by

$$\left(\hat{\Theta}, \hat{\Xi}\right)_{ML} = \arg\min_{\Theta,\Xi} \left\|\hat{\eta} - G(\Theta)\,\Xi\right\|^2 \tag{5}$$

After optimization with respect to Ξ, the ML estimator for Θ is given by

$$\hat{\Theta}_{ML} = \arg\max_{\Theta} L(\Theta) \tag{6}$$

$$L(\Theta) \triangleq \hat{\eta}^{\mathrm{H}} P_G(\Theta)\,\hat{\eta} \tag{7}$$

where $P_G(\Theta) \triangleq G(\Theta)\left(G^{\mathrm{H}}(\Theta)\,G(\Theta)\right)^{-1} G^{\mathrm{H}}(\Theta)$ is the projection matrix onto the subspace spanned by the columns of $G(\Theta)$. $(\cdot)^{\mathrm{H}}$ denotes the conjugate transpose operator. From (6) and (7), it can be noticed that the ML DOA estimation for MIMO array is D-dimensional search process. The computation burden of MIMO ML will become prohibitive when the number of sources increases.

B. Metropolis-Hastings Sampling

In order to resolve the problem of computational complexity, we resort to Metropolis-Hastings (MH) sampling, which is one of the sampling algorithms in Markov Monte Carlo methods. MH sampling is a typical MCMC method, whose application in statistical and array signal processing has attracted much attention in recent years. MH sampling is implemented by changing each component randomly. The changes are accepted or rejected based on the evaluation of their probability of state. This process can be regarded as establishing a Markov chain from a group of transition probabilities k_l, where $l = 1, 2, \cdots, n$.

C. MIMO MHML Estimator

Firstly, let $p(\Theta_x) = \frac{L(\Theta_x)}{\int L(\Theta_x)d\Theta}$, where Θ_x is regarded as a random variable. Then it is obvious that $p(\Theta_x)$ meets the requirement of probability density function (PDF). Thus $p(\Theta_x)$ can been seen as a pseudo-PDF. Assuming that Θ^i is the current state of the state space, the new state Φ will be produced via conditional distribution $q\left(\Phi|\Theta^i\right)$. With probability (8), we can produce the next state of Markov Chain $\Theta^{i+1} = \Phi$, otherwise $\Theta^{i+1} = \Theta$.

$$\alpha(\Theta^i, \Phi) = \min\left\{1, \frac{p(\Phi)\, q\left(\Theta^i, \Phi\right)}{p(\Theta^i) q\left(\Phi, \Theta^i\right)}\right\} \tag{8}$$

where α is the decision function.

In this paper, we assume that the DOAs are independent, and the states of D-dimensional state space are independent too, then we can get

$$\begin{cases} q\left(\Theta^i, \Phi\right) = q\left(\Theta^i\right) q(\Phi) \\ q\left(\Phi, \Theta^i\right) = q(\Phi)\, q\left(\Theta^i\right) \end{cases} \tag{9}$$

Substituting (9) into (8), the decision function can be rewritten as

$$\begin{aligned} \alpha\left(\Theta^i, \Phi\right) &= \min\left\{1, \frac{p(\Phi)\, q\left(\Theta^i\right) q(\Phi)}{p\left(\Theta^i\right) q(\Phi)\, q\left(\Theta^i\right)}\right\} \\ &= \min\left\{1, \frac{p(\Phi)}{p\left(\Theta^i\right)}\right\} \\ &= \min\left\{1, \frac{L(\Phi)}{L\left(\Theta^i\right)}\right\} \end{aligned} \tag{10}$$

According to the inverse theory of Markov Monte Carlo method, we have

$$p(\Phi)\, k(\Phi, \Theta) = p(\Theta)\, k(\Theta, \Phi) \tag{11}$$

where $k(\Phi, \Theta)$ is the transfer probability from state Φ to state Θ. Here, assume that Θ is the state vector near the spectrum peak and Φ is far away from the spectrum peak, then $p(\Theta) > p(\Phi)$. From (11), we can get $k(\Theta, \Phi) < k(\Phi, \Theta)$. Therefore, if the Markov chain moves to Θ, it will stay for a while near Θ. The time of stay is proportional to the sharpness of the spectrum peak.

Note that the likelihood function is directly used as a posteriori probability density function and the global maximum is not prominent local maximum . All of these result in the increase of the MH sampling convergence time and calculation time. In order to increase the time of Markov chain linger in the small neighborhood of desired state for enough time, we need to make the peaks pf likelihood function narrower. Thus we modify (10) to

$$\begin{aligned} \alpha(\Theta^i, \Phi) &= \min\left\{1, \left(\frac{p(\Phi)}{p(\Theta^i)}\right)^{\gamma}\right\} \\ &= \min\left\{1, \left(\frac{L(\Phi)}{L(\Theta^i)}\right)^{\gamma}\right\} \end{aligned} \tag{12}$$

where, γ is a large positive number.

Then the steps of MIMO MHML can be summarized as follow:

(1) Choose the initial state $\Theta = (\theta_1, \theta_2, \cdots, \theta_D)$.

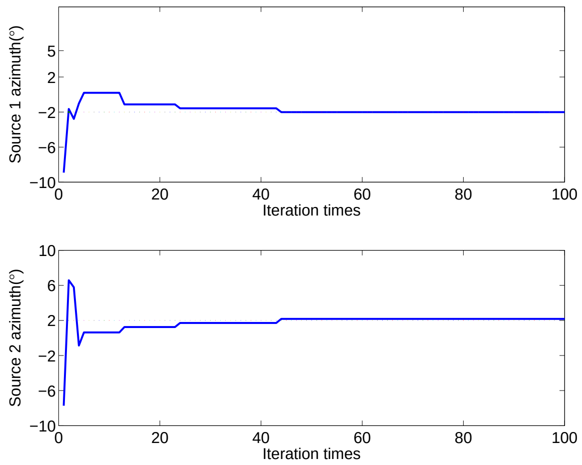

Fig. 1. The processing of sampling

(2) Do iteration operation, until $\| h \|_\infty < \varepsilon$, $h = (h_1, h_2, \cdots, h_D)$, where

$$\theta_d^0 = \frac{1}{R}\sum_{r=0}^{R-1}\theta_d^{i-r}, \qquad d = 1, 2, \cdots, D \tag{13}$$

$$h_d = \frac{1}{R}\sum_{r=0}^{R-1}|\theta_d^{i-r} - \theta_d^0|, \qquad d = 1, 2, \cdots, D \tag{14}$$

where R is a positive integer and $R < i$.

(3) Now we can get the estimates of DOAs from (13) and (14).

In order to further improve the computational speed of this algorithm, two different sampling state domain can be used to sample alternative.

Then the steps of alternative sampling are given as follow:

(1). Sampling

a). The global state space sampling. It means that the new candidate state of Markov chain is generated from the uniform distribution $U_D^{[\theta_L,\theta_H]}$ of D-dimensional state space, where $[\theta_L, \theta_H]$ is the angle searching scope.

b). The local state space sampling, which means that the new candidate state of Markov chain is got from the Gaussian distribution of D-dimensional state space, where the mean of this Gaussian distribution is the current state Θ^i.

(2). Get the decision function α from (12).

(3). Produce $u \sim U_{[0,1]}$, where $U_{[0,1]}$ is the uniform distribution between 0 and 1. If u is smaller than α, $\Theta^{i+1} = \Phi$, otherwise $\Theta^{i+1} = \Theta^i$.

It's noticed that the difference between global state space sampling and local state space sampling lies only in the way how candidate state is produced.

IV. Performance Analysis

In this section, we compare the performances of MIMO ML and MIMO MHML through simulations.

A. DOA Estimation

Consider an Uniform Linear Array (ULA) of 12 monostatic sensors and the sensors are spaced half wavelength apart. Two targets are at $-2°$ and $2°$ with the sampling

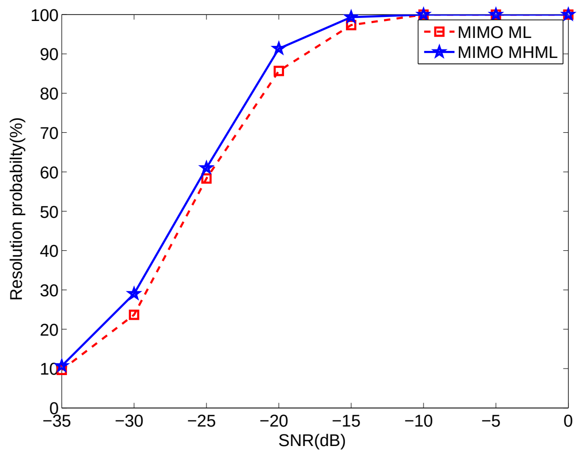

Fig. 2. Resolution probability

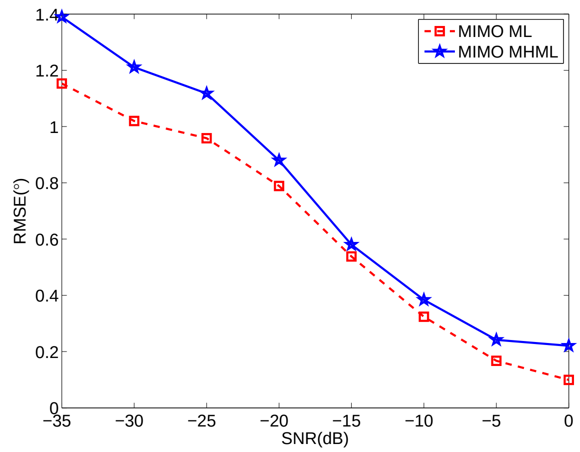

Fig. 3. RMSE of Source 1 versus SNR

frequency and the signal frequency being $30kHz$ and $7kHz$ respectively.

Figure 1 shows the sampling process when the SNR is $5dB$. From this figure, we can see that the chain moves to the real azimuth after 40 iterations and the estimator will "stay" at the true azimuth. Then we can get the DOA estimate from the mean of "stay".

Figure 2, 3 and 4 illustrate the resolution probability and the RMSE of these algorithms. Two sources with DOA θ_1 and θ_1 are considered to be resolved/detected if both $|\hat{\theta}_1 - \theta_1|$ and $|\hat{\theta}_2 - \theta_2|$ are less than $|\theta_1 - \theta_2|/2$. Results are given from 100 Monte Carlo experiments at each SNR. From these diagrams, it is obvious that the resolution probability of MIMO MHML is almost the same as MIMO ML. Although the RMSE of MIMO MHML is slightly higher than MIMO ML, the resolution probability of MIMO MHML is slightly higher than MIMO ML in the low SNR. In summary, MIMO MHML method remains the estimation performance of MIMO ML in the sense of similar RMSE and resolution probability.

B. Computational Complexity

Under the same simulation conditions mentioned in the previous subsection, from 100 Monte Carlo trials, Table

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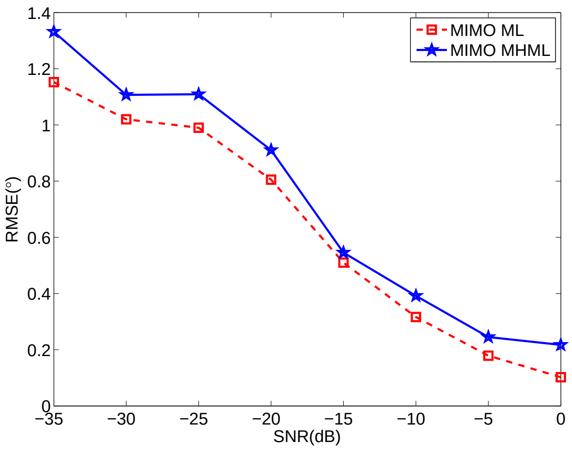

Fig. 4. RMSE of Source 2 versus SNR

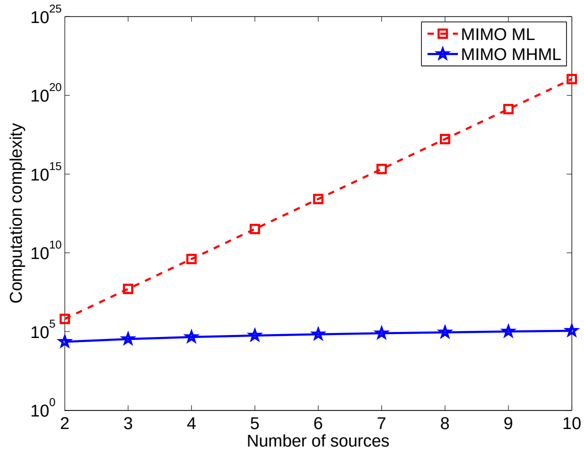

Fig. 5. Comparison of computation complexity

TABLE I
AVERAGE ITERATION TIMES AT DIFFERENT SNRS

SNR(dB)	-20	-15	-10	-5	0	5
Average iteration times V	198	157	134	112	97	41

1 shows the average iterations V for MIMO MHML to converge to the ture DOAs at different SNRs.

Table 1 shows that the convergence of MIMO MHML algorithm is accelerated with the increase of SNR. For simplicity of comparison, assume that L is the computational cost of one peak search for MIMO ML. According the simulation condition, the computation complexity of these two algorithms can be given as:

$$J_{MIMO\ ML} = ((\theta_H - \theta_L)/step)^D \times L = 6400L \quad (15)$$

$$J_{MIMO\ MHML} = V \times (L \times D) = 224L \quad (16)$$

where the SNR is $-5dB$ and the $step$ of the research is $0.25°$.

Obviously, the computational complexity of MIMO MHML is roughly $1/28$ of MIMO ML, which means that the proposed algorithm is a promising method in reducing computational burden for MIMO ML. Assume that $SNR = -5dB$, $L = 100$, $V = 112$, $\theta_L = -10°$, $\theta_H = 10°$ and $step = 0.25°$. The computational comparison of these algorithms is shown in figure 5. From figure 5, it can be seen that the computation complexity of MIMO ML increases exponentially with D, but that of MIMO MHML increases linearly with D. As shown in (15), when the search step size of MIMO ML becomes smaller, the advantage of MIMO MHNL over MIMO ML becomes more significantly. In a word, MIMO MHML method maintains the excellent performance of MIMO ML and significantly reduces the amount of calculation at the same time.

V. CONCLUSION

In order to reduce the prohibitive computation complexity of MIMO ML due to the multidimensional search, a new fast algorithm for MIMO ML is described. The proposed algorithm explores the merits of Markov Monte Carlo methods and MIMO ML estimator. It is called MIMO Maximum Likelihood DOA Estimator based on Metropolis-Hasting Sampling (MIMO MHML). MIMO MHML regards the power of the MIMO ML spectrum function as a probability distribution function for signal, and uses Metropolis-Hasting sampler to sample from this probability distribution function. The whole process of theoretical deduction is given and the way to choose the judgement function is discussed. Simulation results show that MIMO MHML not only keeps the excellent performance of the original MIMO ML but also reduces the computational complexity greatly, especially when the number of sources is large. This new method provides a promising way to implement MIMO ML in engineering practice.

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