Effect of Semi conduction on the Reflection of Electron Wave From Rigidly Fixed Surface of a Half Space

J. N. Sharma and A. Sharma

Abstract- The paper concentrates on the study of reflection characteristics of acoustodiffusive waves from the surface of a semiconductor halfspace which is subjected to rigidly fixed, isoconcentrated and impermeable conditions. The amplitude ratios (reflection coefficients) of reflected waves to that of incident wave are obtained for incident electron- (N) wave cases. The special cases of normal and grazing incidence a5re also derived and discussed. Finally, the numerical computations of reflection coefficients are carried out with the help of Gauss elimination method by using MATLAB programming software.

The computer simulated results in respect of rigidly fixed conditions have been plotted graphically for germanium (Ge) semiconductors.

The study may be useful in semiconductors and surface acoustic wave (SAW) devices in addition to geophysical applications.

Ke*ywords:* critical angle; electron wave, n- type semiconductors Snell's Law.

1. Introduction

Lyamshev and Shevyakhor [1] investigated the reflection of transverse waves in a dielectric-piezoelectric semiconductor structure with current. Biot [2] developed the coupled theory of thermo elasticity. Hussain and Ogden [3] studied the reflection and transmission of a wave at a shear- twin interface. Abdala [4] studied the relaxation effect on relaxation effect on reflection of generalized magneto thermoelastic waves. The general equations for the problems of reflection and refraction of plane wave from plane stress free boundaries are derived by Knott [5]. The reflection and refraction phenomenon of elastic waves in solids under different situations have been treated in detail as reported in books [6-7].

J.N.Sharma, V.Kumar and D. Chand [8] studied the reflection of generalized thermo elastic waves from the boundary of a transversely isotropic half space. Maruszewski [9] presented theoretical considerations of the simultaneous interactions of elastic, thermal and diffusion of charge carrier fields in order to study surface waves in semiconductors.

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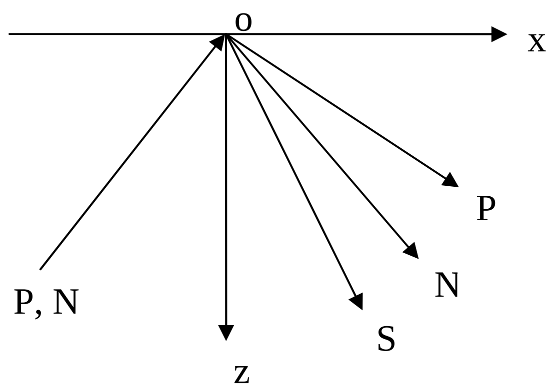

Fig. 1: Geometry of the problem.

The present paper is an attempt to explore the reflection of elastic waves from the rigidly fixed isoconcentrated and rigidly fixed impermeable boundaries of semiconductor (n- type) half space.

The mathematical model consisting of governing partial differential equations of motion and charge carriers' diffusion in n- type semiconductors has been solved both analytically and numerically in the study. The computer simulated results so obtained have been illustrated graphically in case of germanium (Ge) semiconductor half space.

II. Formulation of the Problem

Let the origin of rectangular Cartesian co-ordinate system $oxyz$ be fixed at a point on the boundary of the semiconductor half space with positive z- axis directed normally inside the medium and x-axis along the horizontal direction, the y-axis is taken in the direction of the line of intersection of the plane wave front with the plane surface as shown in Fig 1. If we restrict our analysis to plain strain in the $xz-$ plane, then all the field variables may be taken as function of x, z and t only.

The basic governing equations of motion and diffusion of charge carrier fields for a homogeneous isotropic, elastic n- type semiconductor, in the absence of body forces and electromagnetic forces, are given as [9]

For n- type

$$\mu\nabla^2\vec{u} + (\lambda+\mu)\nabla\nabla.\vec{u} - \lambda^n N = \rho\ddot{\vec{u}} \qquad (1)$$

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$$\rho D^n \nabla^2 N - a_2^n T_0 \lambda^T \nabla.\dot{\vec{u}} + \rho\left[\frac{1}{t_n^+} - \left(1 - \frac{t^n}{t_n^+}\right)\frac{\partial}{\partial t} - t^n \frac{\partial^2}{\partial t^2}\right]N = 0 \qquad (2)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$ is the Laplacian operator, $N(x, z, t) = n - n_0$ is the electron concentration change and $\vec{u}(x, z, t) = (u, 0, w)$ is the displacement vector. Here λ, μ are Lame parameters; ρ is the density of the semiconductor; $\lambda^n = (3\lambda + 2\mu)\alpha_N$ is the elastodiffusive constants of electrons; α_N is the coefficients of linear electron concentration expansions. D^n is the diffusion coefficients of electron carriers; t_n and t_n^+ are the relaxation and life times of the electron charge carriers respectively; n and n_0 are respectively, the non-equilibrium and equilibrium values of electrons and holes concentrations of the semiconductors; T_0 and $\lambda^T = (3\lambda + 2\mu)\alpha_T$ are the uniform temperature and adiabatic thermo mechanical coupling constant respectively; a_2^n, is the flux like parameters. The superposed dot represents differentiation with respect to time.

The non- vanishing components of stress tensor in the semiconductor are given by

n-type: $$\tau_{zz} = (\lambda + 2\mu)\frac{\partial w}{\partial z} + \lambda \frac{\partial u}{\partial x} - \lambda^n N,$$

$$\tau_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \qquad (3)$$

We define the quantities

$$x' = \frac{\omega^* x}{c_1}, \quad z' = \frac{\omega^* z}{c_1}, \quad t' = \omega^* t,$$

$$N' = \frac{N}{n_0}, \quad w' = \frac{\rho \omega^* c_1}{\lambda^n n_0} w$$

$$u' = \frac{\rho\, \omega^* c_1}{\lambda^n n_0} u \quad \tau'_{ij} = \frac{\tau_{ij}}{\lambda^n n_0},$$

$$t^{n'} = t^n \omega^*, \quad t_n^{+'} = t_n^+ \omega^*$$

$$\delta^2 = \frac{c_2^2}{c_1^2}, \quad \omega^* = \frac{c_1^2}{D^n}, \quad c_1^2 = \frac{\lambda + 2\mu}{\rho},$$

$$c_2^2 = \frac{\mu}{\rho}, \varepsilon_n = \frac{a_2^n T_0 \lambda^T \lambda^n}{\rho(\lambda + 2\mu)} \qquad (4)$$

where ω^* is the elastodiffusive characteristic frequency and c_1, c_2 are respectively, the longitudinal and shear wave velocities. Upon using quantities (4) in equations (1)-(3), we obtain

$$\delta^2 \nabla^2 \vec{u} + (1 - \delta^2)\nabla\nabla.\vec{u} - \nabla N = \ddot{\vec{u}} \qquad (5)$$

$$\nabla^2 N - \varepsilon_n \nabla \cdot \dot{\vec{u}} - \left[t^n \frac{\partial^2}{\partial t^2} + \left(1 - \frac{t^n}{t_n^+}\right)\frac{\partial}{\partial t} - \frac{1}{t_n^+}\right]N = 0 \qquad (6)$$

$$\tau_{xz} = \delta^2\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right),$$

$$\tau_{zz} = (1 - 2\delta^2)\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} - N \qquad (7)$$

We introduce the elastic potential functions ϕ and ψ through the relations

$$u = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial z}, \quad w = \frac{\partial \phi}{\partial z} - \frac{\partial \psi}{\partial x} \qquad (8)$$

Upon introducing equation (8) in equations (5) – (6), we get

$$\nabla^2 \phi - \ddot{\phi} - N = 0 \qquad (9)$$

$$\nabla^2 N - \varepsilon_n \nabla^2 \dot{\phi} - \left[t^n \frac{\partial^2}{\partial t^2} + \left(1 - \frac{t^n}{t_n^+}\right)\frac{\partial}{\partial t} - \frac{1}{t_n^+}\right]N = 0 \qquad (10)$$

$$\nabla^2 \psi = \frac{\ddot{\psi}}{\delta^2} \qquad (11)$$

In case the semiconductors are of relaxation type, the life time and relaxation time become comparable to each other $(t^n \cong t_n^+)$ and consequently, the equations (10) and (11) get simplified. The stresses (7) in terms of potential functions ϕ and ψ with the help of equations (9) and (10) become

$$\tau_{,zz} = \ddot{\phi} - 2\delta^2(\phi,_{xx} + \psi,_{xz}),$$

$$\tau_{,xz} = \ddot{\psi} + 2\delta^2(\phi,_{xz} - \psi,_{xx})$$

III. Boundary Conditions

Following two sets of boundary conditions are assumed to hold at the surface $z = 0$ of the semiconductor halfspace, which is mechanically assumed to be rigidly fixed isoconcentrated and rigidly fixed impermeable.

Set 1: Rigidly fixed and isoconcentrated surface

$$u = 0, \quad w = 0, \quad N = 0 \qquad (13)$$

Set 2: Rigidly fixed and impermeable surface

$$u = 0, \quad w = 0, \quad N_{,z} = 0 \qquad (14)$$

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IV. Solution of the Problem

We assume plane wave solution of the form

$$(\phi, \psi, N) = (A, B, C)\exp\{\iota k(x\sin\theta - z\cos\theta - \upsilon t)\} \quad (15)$$

$\upsilon = \dfrac{\omega}{k}$, ω is circular frequency and k is the wave number.

Upon using equations (15) in equations (9)-(11), we obtain a system of two coupled equations in unknowns A and C and one equation in B. The condition for the existence of nontrivial solution of these systems of equations provides us

$$k^4 - \omega^2\left(1 + \alpha_n^* + \iota\omega^{-1}\varepsilon_n\right)k^2 + \alpha_n^*\omega^4 = 0,$$

$$k^2 = \frac{\omega^2}{\delta^2} \quad (16)$$

where $\alpha_n^* = \dfrac{1}{t_n^+\omega^2} + \iota\omega^{-1}\left(1 - \dfrac{t^n}{t_n^+}\right) + t^n$, (17)

The equations (16) provide us

$$k_1^2 = a_1^2\omega^2, \quad k_2^2 = a_2^2\omega^2, \quad k_3^2 = \frac{\omega^2}{\delta^2}$$

where $a_1^2 + a_2^2 = 1 + \alpha_n^* + \iota\omega^{-1}\varepsilon_n$, $\quad a_1^2 a_2^2 = \alpha_n^*$

In the absence of electron field $(N = 0 = \varepsilon_n)$ we have

$$a_1^2 = 1, \; a_2^2 = \alpha_n^* \quad (20)$$

CASE 1: *N wave incidence at a rigidly fixed surface*
We now consider the reflection of a plane N wave for rigidly fixed boundary.

For N wave incidence, we have

$$\phi = \phi_r = A_i \exp\{ik_1(x\sin\theta - z\cos\theta)\}$$
$$+ A_1 \exp\{ik_1(x\sin\theta_1 + z\cos\theta_1)\}$$
$$+ A_2 \exp\{ik_2(x\sin\theta_2 + z\cos\theta_2)\}$$
$$\psi = \psi_r = A_3 \exp\{ik_3(x\sin\theta_3 + z\cos\theta_3)\}$$
$$N = N_i + N_r = S_2 A_i \exp\{\iota k_2(x\sin\theta - z\cos\theta)\}$$
$$+ \sum_{j=1}^{2} S_j A_j \exp\{\iota k_j(x\sin\theta_j + z\cos\theta_j)\} \quad (21)$$

Using solution (21) in the boundary conditions (13)-(14) at the surface $z = 0$ and assuming that all the incident or reflected waves are in phase at this surface for all values of x and t, we have

$$k_2\sin\theta = k_1\sin\theta_1 = k_2\sin\theta_2 = k_3\sin\theta_3$$

This relation implies that

$$\theta = \theta_2, \quad a_1\sin\theta_1 = a_2\sin\theta_2 = \frac{1}{\delta}\sin\theta_3$$

This is the modified form of Snell's law for the considered material in this case.
The amplitude ratios (reflection coefficients) in this case are obtained as:
For Set 1

$$R_1^n = \frac{S_2\delta\left[a_2\cos(\theta_2 - \theta_3) + a_1\cos(\theta_2 + \theta_3)\right]}{\Delta},$$

$$R_2^n = \frac{-a_1\delta\left[S_1\cos(\theta_1 + \theta_3) + S_2\cos(\theta_1 - \theta_3)\right]}{\Delta},$$

$$R_3^n = \frac{a_1 a_2\delta^2\left[S_1\sin 2\theta_2 - S_2\sin(\theta_1 - \theta_2) - a_1^2 S_2\sin(\theta_1 + \theta_2)\right]}{\Delta} \quad (23)$$

For Set 2

$$R_1^n = \frac{a_1 a_2\delta\cos\theta_2 S_2\left[\cos(\theta_2 + \theta_3) - \cos(\theta_2 - \theta_3)\right]}{\Delta^*},$$

$$R_2^n = \frac{a_1^2\delta\left[S_2\cos\theta_2\cos(\theta_1 - \theta_3) - S_1\cos\theta_1\cos(\theta_2 + \theta_3)\right]}{\Delta^*}$$

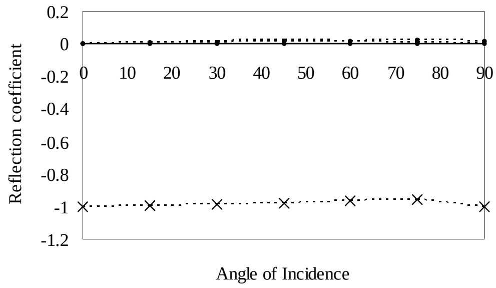

Fig. 2. N-wave incidence at rigidly fixed impermeable Boundary

$$R_3^n = \frac{2a_1^2 a_2\delta^2\cos\theta_1\cos\theta_2\sin\theta_2\left(S_1 - S_2\right)}{\Delta^*} \quad (24)$$

where $R_k^n = \dfrac{A_k}{A_i}(k = 1, 2, 3)$ are the reflection coefficients. Here

$$\Delta = \delta\left[a_1 S_2\cos(\theta_1 - \theta_3) - a_2 S_1\cos(\theta_2 - \theta_3)\right]$$
$$\Delta^* = a_1 a_2\delta\left[S_2\cos\theta_2\cos(\theta_1 - \theta_3) - S_1\cos\theta_1\cos(\theta_2 - \theta_3)\right]$$

.

In the absence of electron field, we have

$$R_1^n = \frac{\cos(\theta_2 + \theta_3)}{\cos(\theta_1 - \theta_3)}, \; R_2^n = -1, \quad R_3^n = 0$$

This result shows that reflected electron wave annihilates the incident electron wave.

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V. CONCLUSIONS

The study may find application in semiconductor and surface acoustic wave (SAW) devices and geophysical field

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