

Robust PID Controller Design for an UAV Flight Control System

B. Kada, Y. Ghazzawi

Abstract—The present paper describes the structure and the design aspects of a robust PID controller for higher-order systems. A design scheme that combines deadbeat response, robust control, and model reduction techniques to enhance the performances and robustness of PID controller is presented. Unlike conventional deadbeat controllers, the tuning parameters are reduced to one cascade gain which yields a practical tuning method. The design scheme is illustrated on a pitch-axis autopilot design of an Unmanned Aerial Vehicle (UAV). Computer simulations show that the proposed method improves time-domain response performances and exhibits stronger robustness property to conquer system uncertainties.

Index Terms— deadbeat controller, higher-order systems, robust control, UAV flight control system

I. INTRODUCTION

THE integration of an autopilot in the control loop of an UAV airframe provides complete set of avionics which enable the UAV to autonomously complete its mission. This electronic system is generally designed to provide intelligent and autonomous flight Navigation and Control (N&C) system to conventional airframes (stable fixed wing with aileron-elevator-rudder control surfaces and engine throttle control) for autonomous navigation between predefined waypoints. Most of modern autopilots incorporate control law algorithms to meet the demanding requirements of flight maneuvers with high performance and to successfully accomplish the mission of autonomous flight.

In recent years, considerable research into the design of algorithms for UAV autopilots using modern control theory has been completed. A large number of control algorithms have been developed for onboard N&C systems. Most of these systems include some nonlinear terms [1-3], evolutionary algorithms [4-8], or optimization techniques [9]. Despite their success, only a small number of implementations of these systems have been reported and it appears that there is not much enthusiasm to use them due to their complexity, nonlinear nature, and computation cost.

Manuscript received July 26, 2011, revised August 16, 2011. This work was supported in part by the Deanship of Scientific Research under grant T95/429 and the Aeronautical Engineering Department KAU.

B. Kada is with Department of Aeronautical Engineering, Faculty of Engineering, King Abdul-Aziz University, PO Box 80204, Jeddah 21589, KSA (corresponding author phone: 966-2-640-2000 Ext. 68729; fax: 966-2-695-2944; e-mail: bkada@kau.edu.sa).

Y. Ghazzawi is with Department of Aeronautical Engineering, Faculty of Engineering, King Abdul-Aziz University, PO Box 80204, Jeddah 21589, KSA (e-mail: yghazzawi@hotmail.com).

On the other hand, PID autopilots have been successfully integrated as real-time control and online navigation systems for UAVs. This is not only due to their simple structure and easy implementation, but also to their adequate performances. However, for successful implementation of such controllers, and without requiring complex mathematical development, parameter adjustment or tuning procedure is needed if enhanced performance is to be achieved through the operating envelope.

The tuning process, whereby the optimum values for the controller parameters are obtained, is a critical challenge. Many studies were conducted to find the best way for tuning PID parameters in order to get adequate performances such as fast response, zero steady-state error, and minimum overshoot/undershoot [10,11]. Even though there are only three parameters, PID parameter tuning is a difficult process because it must satisfying complex criteria within the limitations of system actuators. Also, the traditional PID controller only works for lower-order systems and lacks robustness against large system parameter uncertainties. This is due to the insufficient number of parameters to deal with the independent specifications of time-domain response such as settling time and overshooting [12].

In practice, an UAV autopilot uses a combination of PID feedback and feedforward controllers, such as in case of Kestrel Autopilot [13], to generate the control efforts of conventional control surfaces and engine throttle. There are about fifteen flight modes of an UAV such as manual, homing, altitude, and targeting modes. The control strategy is based upon the use of cascading controllers for which multiple PID controllers are incorporated into one input/output loop. Cascading controllers that link the output from one PID unit as the input to another PID unit are useful for more complex missions and maneuvers. As for an autonomous flight the autopilot of an UAV should be correctly configured and tuned for each flight mode, this control strategy yields a great number of parameters and requires more efficient tuning procedure.

Manual trail-and-error or analyze-and-iterate methods require a large amount of time and manpower for repetitive adjustments through computer simulations and flight tests to achieve the desired performances. Although PID parameter optimization techniques provide an optimal controller and efficiently shape the desired system dynamics, these techniques require expansive computation cost. Hence, designing control algorithms capable to assure high performance and robustness with minimum cost is highly recommended. For this purpose, we propose, in this paper, a mathematical-based framework for designing robust PID controllers for a class of higher-order systems. The

proposed strategy combines the deadbeat step response, the multi-step design system proposed in [12], and model reduction techniques in one procedure design. The resulting control system improves time-response dynamic properties, ensures robustness against large parameter uncertainties, and presents an efficient tuning tool with reduced number of tuning parameters.

In the following sections, we formalize the proposed framework mathematically, extend the design system proposed in [12] to cover arbitrary Linear Time Invariant (LTI) systems, and show the efficiency of the proposed control scheme for the design and implementation of robust PID controllers. The control loop used for illustration is the longitudinal flight control system of an UAV.

II. ROBUST DEADBEAT CONTROLLER DESIGN AND IMPLEMENTATION

The deadbeat response is defined as having the following time domain performances

1. Zero Steady-State Error (Zero-SSE).
2. Controllable settling time T_s .
3. Minimum rise time Tr_{90} (0-90% of the step height).
4. Percent Overshoot (P.O) and Percent Undershoot (P.U) less than 2%.

To design a control system that assures these performances we consider the following LTI system

$$D_p(s)y(t) = N_p(s)u(t) \quad (1)$$

where

$$N_p(s) = \sum_{i=m}^0 b_i s^i \quad (2)$$

$$D_p(s) = \sum_{j=n}^0 a_j s^j \quad (3)$$

where n is the degree of the system dynamics, $y(t)$ is the output of the system, $u(t)$ is the control input, and a_j ($j = 1, \dots, n$) and b_i ($i = 1, \dots, m$) are the system and control parameters, respectively.

Assumption 1: The orders n and m are known and $n > m$.

Assumption 2: $D_p(s)$ is a Hurwitz polynomial, and $N_p(s)$ and $D_p(s)$ are coprime.

Assumption 3: the relative degree of the system (1) with respect to its output $y(t)$ is given as

$$\rho = n - m \geq 1 \quad (4)$$

A. Deadbeat Controller Design

For an n^{th} -order LTI system such as (1), the deadbeat robust controller, as proposed in [12], is applied through the following two cell closed-loop control

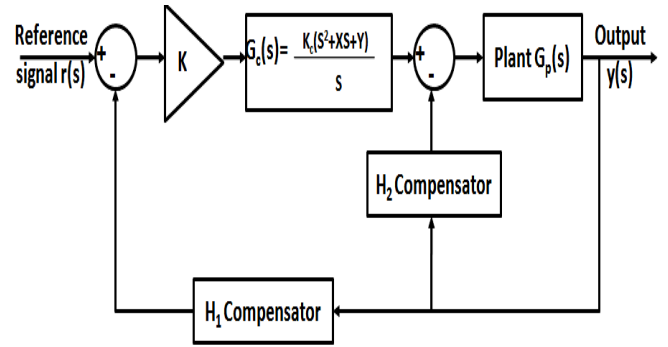


Fig. 1. Basic structure of the robust deadbeat control loop

In this design, the system (1) is actuated by the following control law

$$u(s) = u_c(s) + u_o(s) \quad (5)$$

where $u_c(s)$ is the portion of the control signal generated by a cascade PID controller as follows

$$\begin{aligned} u_c(s) &= K G_c(s) e(s) = K \frac{N_c(s)}{D_c(s)} e(s) \\ &= K K_c \frac{(s^2 + Xs + Y)}{s} e(s) \end{aligned} \quad (6)$$

and $u_o(s)$ is an output feedback control introduced by means of the following variable feedback gain

$$u_o(s) = H_2(s)y(s) = K_a y(s) \quad (7)$$

The error $e(s)$ is defined as the difference between the reference input control $r(s)$ and the output $\hat{y}(s)$ of an additional derivative controller $H_1(s)$

$$e(s) = r(s) - \hat{y}(s) \quad (8)$$

The structure of $H_1(s)$ depends on the number of poles of the transfer function $G_c(s)G_p(s)$ as follows

$$\begin{aligned} H_1(s) &= 1 && \text{for } n_p = 2 \\ H_1(s) &= 1 + K_1 s && \text{for } n_p = 3 \text{ or } 4 \\ H_1(s) &= 1 + K_1 s + K_2 s^2 && \text{for } n_p = 5 \end{aligned} \quad (9)$$

where n_p equals the number of poles in $G_c(s)G_p(s)$ transfer function. The cascade gain K is introduced in order to deal with higher-order plants, to compensate the nonlinearities, and to be able to independently specify the desired performances such as overshoot and settling time. The different parameters and gains in the control loop shown in Fig. 1. are determined by setting the characteristic equation of the equivalent closed-loop transfer function equal to the following deadbeat equation

$$s^{n_p} + \alpha_1 \omega_n s^{n_p-1} + \alpha_2 \omega_n^2 s^{n_p-2} + \dots + \omega_n^{n_p} \quad (10)$$

where the coefficients α_i are determined from the following table depending on n_p number

TABLE I
DEADBEAT COEFFICIENTS AND RESPONSE TIMES

Order n_p	α_1	α_2	α_3	α_4	T'_{r90}	T'_s
2 nd	1.82	-----	-----	-----	3.47	4.82
3 rd	1.90	2.20	-----	-----	3.48	4.04
4 th	2.20	3.50	2.80	-----	4.16	4.81
5 th	2.70	4.90	5.40	3.40	4.84	5.43

T'_{r90} and T'_s are, respectively, the rise time and the settling time of the deadbeat step response. The normalized frequency ω_n is given as follows

$$\omega_n = \frac{T'_s}{\mu T_s} \quad (11)$$

where T_s is the desired settling time, and μ is a positive constant generally selected to be $0.8 \leq \mu \leq 0.95$.

B. Deadbeat Controller Implementation

First, the characteristic equation $p(s)$ of the closed-loop transfer function is found from

$$G(s) = \frac{q(s)}{p(s)} = \frac{KG_c(s)G_p(s)}{1 + G_p(s)H_2(s) + KG_c(s)G_p(s)H_1(s)} \quad (12)$$

which yields

$$p(s) = D_c(s)D_p(s) + D_c(s)N_p(s)H_2(s) + KN_c(s)N_p(s)H_1(s) \quad (13)$$

Selecting the feedback $H_1(s)$ as

$$H_1(s) = 1 + \sum_{i=1}^{n_p-2} K_i s^i \quad (14)$$

the development of (13) yields

$$p(s) = KN_c(s) * \sum_{l=n_p-2}^1 K_l b_l s^{l+i} + (KN_c(s) + K_a s) * \sum_{i=n_p-2}^1 b_i s^i \quad (15)$$

with

$$N_c(s) = K_c(s^2 + Xs + Y) \quad (16)$$

From (15), it is clear that to apply the deadbeat controller as proposed in [12], the orders higher than n_p should be eliminated. This results in the following conditions

$$b_i = 0 \text{ for } i > 0 \quad (17)$$

Hence, the relative degree of the system (4) should be $\rho = n$. In other words, to apply the controller (5) to an arbitrary LTI system such as (1), an order-reduction or zero-cancellation phase is needed.

III. REAL TIME CLOSED-LOOP CONTROL OF AN UAV

An UAV autopilot is designed to assure different levels of autonomy known as autopilot modes. These modes are classified into three categories [13]: 1-) standard modes such as manual, takeoff, land, and home modes, 2-) advanced modes such as targeting, deep stall, and PID modes, 3-) payload modes such as gimbal control and camera view modes. These autonomy capabilities allow the user to change autopilot behavior depending on the requested maneuvers. In addition, there is a pilot-in-the-loop mode which gives the user direct control of the aircraft using Radio Controlled (RC) mode for gain tuning, algorithm development, and cases where manual flight is required.

To be able to perform an autonomous flight, UAVs have to be tuned for different flight loops which are classified with respect to the relative degree as follows

Inner loops ($\rho=1$): yaw rate, roll rate, and pitch rate.

Outer loops ($\rho>1$): roll, pitch, yaw, pitch-airspeed, throttle-airspeed, throttle-altitude, and pitch-altitude.

Depending on the input-output relations, these loops are grouped into control blocks for which the control efforts are generated by cascading controllers using a combination of feedback and feedforward controls such as in Kestrel autopilot structure [13]. Assuming that all control blocks have access to the necessary sensor information, the following figure depicts the cascade structure of one of these blocks (Elevator-waypoints block control)

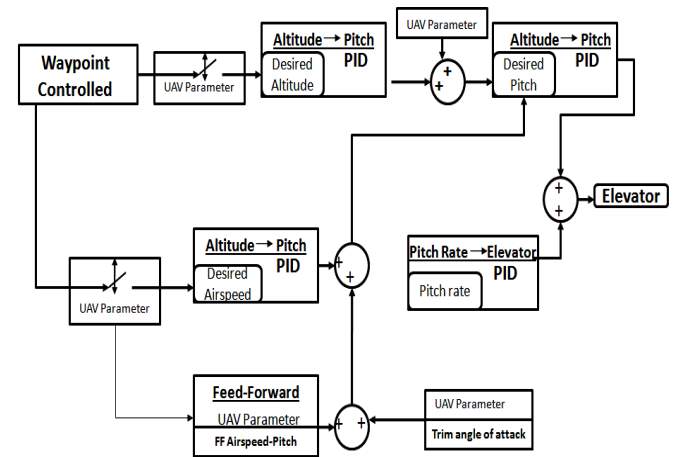


Fig. 2. Waypoint-elevator control block diagram of an UAV autopilot

From the figure above, it is clear that many PID controllers have to be tuned to successfully complete an autonomous flight. This consumes a vast amount of man power and time (three tuning parameters for each PID), and needs a great number of flight tests over the operating range. In addition, fastness and robustness are required because each control block incorporates many control loops and uses many sensor measurements (UAV parameters). In order to cope with these challenges, we present in the following section a multistep design procedure that reduces the tuning effort and ensures fastness and robustness.

IV. ROBUST DEADBEAT CONTROLLER DESIGN
FOR AN UAV AUTOPILOT

In order to perform the control system design, it is necessary to start with an intended model of the system, and then proceed to the control design. Flight dynamics of fixed-wing aircrafts are well understood and reasonably extended to model the dynamics of UAVs which are characterized by a set of coupled and strongly nonlinear differential equations. However, under special circumstances and with specific assumptions such as straight steady flight and zero cross product, the use of small perturbation theory yields linear and decoupled into lateral and longitudinal equations of motion. In the present study, the longitudinal flight control of an UAV is considered for controller design and validation.

A. Longitudinal Flight Dynamics of an UAV

The longitudinal flight mode, is embodied by the X-force, Z-force, and pitch moment equations as follows

$$H(s)y = \begin{bmatrix} h_x(s) \\ h_z(s) \\ h_M(s) \end{bmatrix} y = B\delta \quad (18)$$

with

$$h_x(s) = \begin{bmatrix} \left(\frac{mu}{Sq} s - C_{Xu}\right) & -C_{X\alpha} \\ -mg\cos\theta_e - C_{Xq} s \end{bmatrix} \quad (19)$$

$$h_z(s) = \begin{bmatrix} -C_{ZU} \left(\left(\frac{mu}{Sq} - \frac{c \cdot C_{Z\dot{\alpha}}}{2u}\right) s - C_{Z\alpha} \right) \\ -\left(\frac{mu}{Sq} + \frac{c \cdot C_{Zq}}{2u}\right) s - mg\sin\theta_e \end{bmatrix} \quad (20)$$

$$h_M(s) = \begin{bmatrix} -C_{Mu} & \left(-\frac{c \cdot C_{M\dot{\alpha}}}{2u} s - C_{M\alpha}\right) \\ \left(\frac{I_y}{Sq c} s^2 - \frac{c \cdot C_{Mq}}{2u} s\right) \end{bmatrix} \quad (21)$$

$$B = [b_{\delta_e} \ b_{\delta_t}] = \begin{bmatrix} C_{X\delta_e} & C_{X\delta_t} \\ C_{Z\delta_e} & C_{Z\delta_t} \\ C_{M\delta_e} & C_{M\delta_t} \end{bmatrix} \quad (22)$$

and

$$\begin{aligned} \bar{u} &= \frac{u - U_0}{U_0} \\ \bar{\alpha} &= \frac{w}{U_0} \\ \bar{\theta} &= \theta - \theta_0 \end{aligned} \quad (23)$$

The variables u and w are the change of velocity components in x and z direction, respectively, U_0 is the steady state flight velocity, θ is the actual pitch angle, and θ_0 is the trim pitch angle. The matrix $H(s)$ is the longitudinal flight matrix, and b_{δ_e} and b_{δ_t} are the aerodynamic and thrust control vector, respectively. The input and output variables are defined as follows

$$\delta = [\delta_e \ \delta_t] \quad (24)$$

$$y = [\bar{u} \ \bar{\alpha} \ \bar{\theta}]^T \quad (25)$$

where δ_e and δ_t are elevator angle and thrust angle deflection, respectively.

B. Longitudinal Deadbeat Autopilot Design

In this section, the deadbeat controller derived above is analyzed in terms of agility and robustness against parameter uncertainties. The UAV model that we use here for control design and simulation is the model given in [10-11] for which the nominal pitch-axis transfer functions with respect to the elevator deflection are given as follows

$$\begin{bmatrix} \bar{u}(s) \\ \delta_e(s) \\ \bar{\alpha}(s) \\ \delta_e(s) \\ \bar{\theta}(s) \\ \delta_e(s) \end{bmatrix} = \begin{bmatrix} h_x(s) \\ h_z(s) \\ h_M(s) \end{bmatrix}^{-1} \begin{bmatrix} C_{X\delta_e} \\ C_{Z\delta_e} \\ C_{M\delta_e} \end{bmatrix} \quad (26)$$

For a steady state flight velocity $U_0 = 12 \text{ m/sec}$ (472.44 in/sec), the nominal system transfer function $\bar{\theta}(s)/\delta_e(s)$ is given as follows

$$\begin{aligned} G_{\delta_e}^{\bar{\theta}}(s) &= \frac{\bar{\theta}(s)}{\delta_e(s)} \\ &= \frac{1.423s^2 + 0.134s + 1.839}{0.02424s^4 + 0.0683s^3 + 0.1s^2 + 0.0859s + 0.0836} \end{aligned} \quad (27)$$

The following figure depicts the open-loop time-domain response of (27) to a step unit elevator deflection

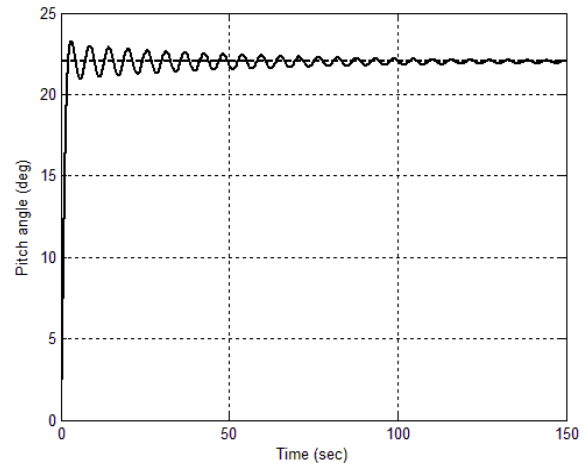


Fig. 3. Open-loop pitch angle response to a step unit elevator deflection.

In order to achieve the deadbeat system implementation requirements (17), the order reduction technique presented in [14] was used and several model reduction schemes were examined. The best results were obtained with the following reduced model

$$G_{r=3}(s) = \frac{21.9976}{0.81698s^3 + 0.89665s^2 + 1.0275s + 1} \quad (28)$$

The control objective is to assure a pitch angle path tracking with overshooting/undershooting less than 2%, maximum settling time of 2 second, and robustness against large parameter uncertainties. To successfully meet these performances, the proposed multistep design procedure is applied as follows.

First step: The cascade gain K is set equal to 1 (the only tuning parameter) and the characteristic equation of the deadbeat transfer equation is constructed from table 1 with $n_p = 4$, $T_s = 2 \text{ sec}$, $\mu = 0.8$ and $\omega_n = 3.00625 \text{ rad/sec}$. Matching up this equation with (28) yields the following five nonlinear algebraic equations with six unknown design parameters

$$\begin{cases} 0.817 + K_c * 21.998 * K_2 = 1 \\ 0.897 + 21.998K_cK_1 + 21.998K_cXK_2 = 6.614 \\ 1.028 + 21.998K_c + 21.998K_cXK_1 + 21.998K_cYK_2 = 31.631 \\ 1 + 21.998ka + 21.998K_cX + 21.998K_cYK_1 = 76.074 \\ 21.998K_cY = 81.677 \end{cases}$$

Second step: The nonlinear algebraic equations above are solved for different values of K_c . Test and selection process is conducted on the reduced system (28). The best results were obtained with the following design parameters

TABLE II
 DEADBEAT DESIGN PARAMETERS FOR THE REDUCED SYSTEM

Parameter	Value
K_c	1.0000
X	1.4540
Y	3.7130
K_a	1.0390
K_1	0.2478
K_2	0.0083

Third step: The selected controller is now applied to the original system (26), and the cascade gain K is tuned until the desired performance requirements are achieved.

To measure the efficiency and robustness of the controller, a desired pitch angle $\bar{\theta} = 22 \text{ deg}$ is selected as illustration example. Computer simulations are conducted with nominal, +100% over-estimated, and 75% under-estimated system parameters. The following figures show the time-domain responses, control efforts, and tracking errors obtained

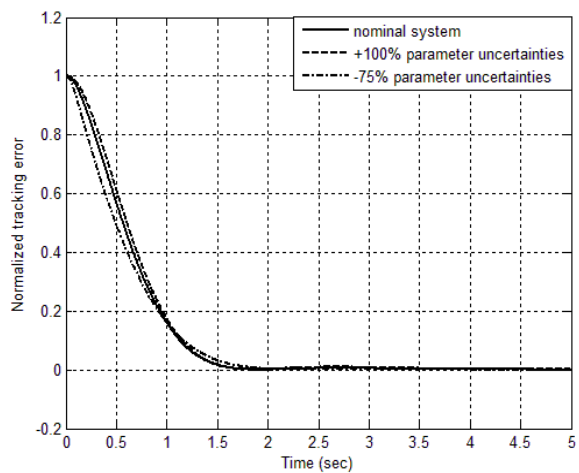
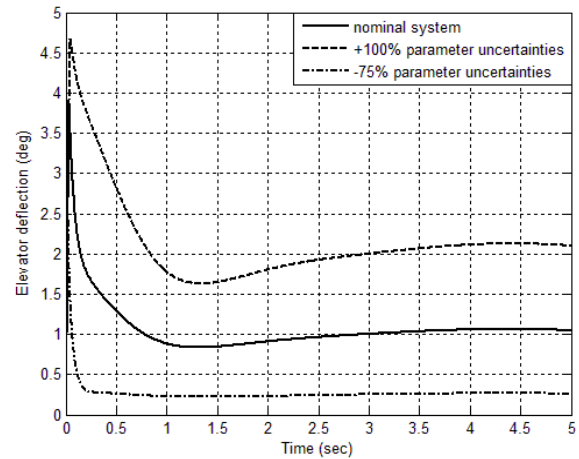
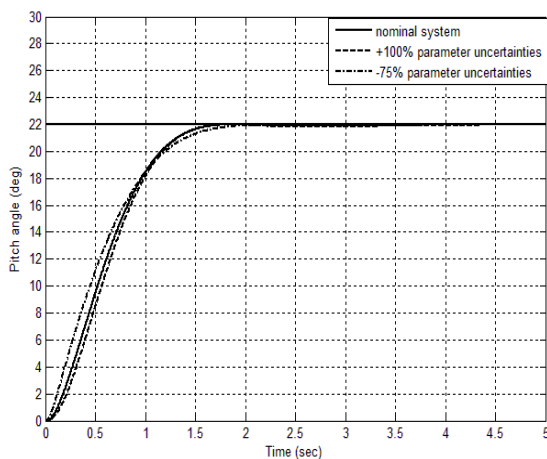


Fig. 4. Responses, control efforts, and tracking errors for an UAV pitch angle flight control system.

The following table summarizes the results of cascade gain tuning and time domain performances of the three systems above.

TABLE III
 TIME-DOMAIN RESPONSES FOR DESIGN METHOD APPLIED TO NOMINAL AND UNCERTAIN SYSTEMS

Parameter uncertainties	K	P.O [%]	P.U [%]	Tr_{90} [sec]	T_s [sec]
Nominal	0.77	0.00	0.55	1.134	1.472
100% increase	0.73	0.00	0.90	1.150	1.467
75% decrease	0.84	0.00	0.18	1.167	1.620

To check the ability of the designed controller to force an UAV to follow an arbitrary command, the same design parameter values shown in table 2 are used for an arbitrary pitch angle command pattern tracking. Again, the cascade gain K is tuned and the final values that correspond to the desired response are found to be as follows

TABLE IV
 TIME-DOMAIN RESPONSES FOR DESIGN METHOD APPLIED TO PATH TRACKING WITH NOMINAL AND UNCERTAIN SYSTEMS

Parameter uncertainties	K	P.O [%]	P.U [%]
Nominal	0.72	0.00	<1%
100% increase	0.70	0.00	<1%
75% decrease	0.76	0.00	<1%

The responses and needed control sequences for a pattern tracking mission are shown in the following figures

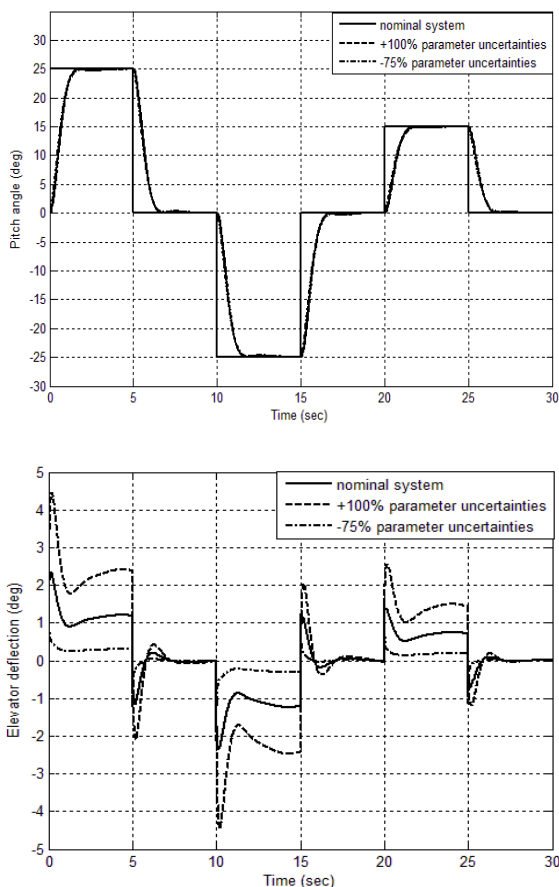


Fig. 5. Responses and control efforts for a pitch angle command sequence.

The results above show that a high closed-loop performance response with high robustness against large parameter uncertainties can be obtained with the proposed control framework. The designed deadbeat controller is also able to successfully perform tracking tasks under complex criteria within the limitations of system actuators.

V. CONCLUSION

A new strategy to design robust PID controller for uncertain higher-order systems is presented in this paper. The formal mathematical concepts of deadbeat control approach are developed and a new feature of robust deadbeat system is successfully implemented within an effective control framework. Computer simulations show that, for LTI systems, impressive time-domain performances and robustness to modeling uncertainties can be obtained with the proposed control strategy. Since the controller parameters are computed in advance and the tuning procedure is limited only to the cascade gain, the framework also provides an efficient and practical way for real-time PID parameter tuning.

REFERENCES

- [1] O. A. A. Orqueda, X. T. Zhang, and R. Fierro, "An output feedback nonlinear decentralized controller for unmanned vehicle coordination," *International Journal Of Robust And Nonlinear Control*. Int. J. Robust Nonlinear Control 2007; 17:1106–1128.
- [2] M. Sadraey, R. Colgren, "Robust nonlinear controller design for a complete UAV mission," *AIAA Guidance, Navigation, and Control Conference and Exhibit 21 - 24 August 2006, Keystone, Colorado*.
- [3] A. Benallegue I, A. Mokhtari, and L. Fridman, "High-order sliding-mode observer for a quadrotor UAV," *International Journal Of Robust And Nonlinear Control* Int. J. Robust Nonlinear Control. Copyright © 2007 John Wiley & Sons, Ltd.
- [4] I. K. Nikolos, Kimon P. Valavanis, Nikos C. Tsoveloudis, and Anargyros N. Kostaras. "Evolutionary algorithm based offline/online path planner for UAV navigation," *IEEE Transactions On Systems, Man, And Cybernetics—Part B: Cybernetics*, Vol. 33, No. 6, December 2003.
- [5] I. K. Nikolos, N. Tsoveloudis, and K. P. Valavanis, "Evolutionary algorithm based 3-D path planner for UAV navigation," in *Proc. CD-ROM. 9th Mediterranean Conf. Control Automation*, Dubrovnik, Croatia, 2001
- [6] J. Z. Sasiadek and I. Duleba, "3-D local trajectory planner for UAV," *J. Intelligent Robotic Systems*, vol. 29, pp. 191–210, 2000.
- [7] A. C. Nearchou, "Adaptive navigation of autonomous vehicles using evolutionary algorithms," *Artif. Intell. in Eng.*, vol. 13, pp. 159–173, 1999.
- [8] S. Kurnaz , O.Cetin , O.Kaynak, "Fuzzy logic based approach to design of flight control and navigation tasks for autonomous unmanned Aerial Vehicles," *J Intell Robot System*, 54:229–244, 2009.
- [9] Zelinski S, Koo T, Sastry S. "Optimization-based formation reconfiguration planning for autonomous vehicles," *Proceedings of the IEEE International Conference on Robotics and Automation*, Taipei, Taiwan, September 2003; 3758–3763.
- [10] K. Turkoglu, U. Ozdemir, M. Nikbay, E. Jafarov, "PID parameter optimization of a UAV longitudinal Flight control system," *World Academy of Science, Engineering and Technology* 45, 2008.
- [11] K. Turkoglu, and E. M. Jafarov, "H inf. loop shaping robust control vs. classical PI(D) control: A case study on the longitudinal dynamics of hezarfen UAV," *World Academy of Science, Engineering and Technology* 45, 2008.
- [12] J. Dawes, L. Ng, R. Dorf, and C. Tam, "Design of deadbeat robust systems," *Glasgow, UK*, , pp1597-1598, 1994.
- [13] *Kestrel Autopilot System: User Guide*, Copyright © 2004 – 2008. Procerus ® Technologies.
- [14] R. Prasad, S.P. Yadav, and R. Rani, "A new computing technique for order reduction of linear time invariant systems using stability equation method," *Journal of the institution of Engineers IE(I) Journal EL*, Vol. 86, Sept. 2005, pp 133-135, 2005.