Periodic Oscillations on Angular Velocity due to the ABS Operation under Specific Work Regimes

I. Vazquez-Alvarez, *Member, IAENG*, J.J. Ocampo-Hidalgo, and A. Ferreyra-Ramirez

***Abstract*—The ABS operation implies a lot of effects on the vehicle's dynamics, the oscillatory behavior represents an important study area, since in can lead to significant advances in ABS performance. In this paper we show that the ABS operation while the longitudinal contact force applied in a pneumatic system is near to the maximum value, produces an oscillatory effect on the angular velocity of the vehicle´s wheel, and that for the time intervals that the system operates the oscillation can be considered periodic.**

***Index Terms*—Antilock brake system, contact force, deviation equations, periodic oscillations, stability region.**

I. Introduction

THERE is a lot of research work on anti-lock braking systems (ABS) in transport vehicles that discuss the problem of high frequency vibrations appearance in the angular velocity of the wheel's rotation [1]-[5]. Modeling and research of forced oscillations in a deformable wheel as a result of ABS activity has been discussed in papers [1], [2]. In works [3], [4] the processes of vibration's appearance during the pressure's relief phase in the brake cylinder of the ABS are analyzed, as well as the algorithms to suppress such vibrations. In [5] the possibility of longitudinal vibrations in the chassis of an airplane during the active phase of ABS is discussed.

The modern ABS systems very often use sliding modes control [6]-[11] with switching of ABS valves. Simultaneously the nonlinear character of ABS dynamics can lead to specific periodic regimes of angular velocity change for this kind of control algorithms that make programmed switch of the valve with a given time period and duty cycle. The condition of existence of periodic changes in the angular velocity of the wheel's rotation due to the presence of specific ABS regimes is discussed in this paper.

The model of a pneumatic brake system is under consideration. The specific configuration of this system includes the next: brake disks, which hold the wheels, as a result of the increment of the air pressure in the brake cylinder (fig. 1). The entrance of the air trough the pipes from the central reservoir and the expulsion from the brake cylinder to the atmosphere is regulated by a common valve. This valve allows only one pipe to be open, when 1 is open 2 is closed and vice versa. The time response of the valve is considered small, compared with the time constant of the pneumatic systems.

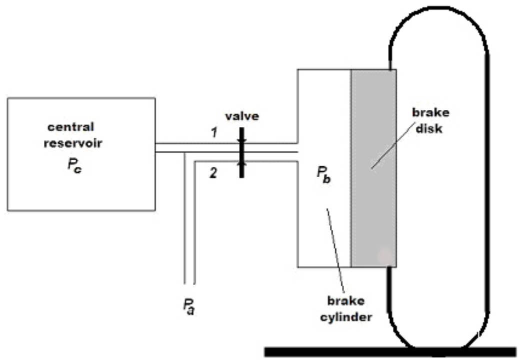

Fig. 1 Pneumatic brake model considered

We study the case of wheel's rotation control, such that the longitudinal force, due to the contact of the wheel with the road, is near from the maximum value in the period of time valid for the model. This effect is reached as a result of the ABS valve's throttling.

II. Mathematical model

A. Wheel Motion Equations

To describe the wheel's motion we use a partial mathematical model of the dynamic system (fig. 2) [3], [12]. Let's write the equation of the angular momentum change relative to the rotation axis:

$$I_y \frac{d\Omega_y}{dT} = FR + L, \tag{1}$$

where I_y - wheel's inertia moment, Ω_y - wheel's angular velocity, R - wheel's radius, F - contact force, L - brake torque.

The expression for longitudinal component of the contact force in the motion's plane according to experimental results [13] is equal

$$F = -\nu N \varphi(s). \tag{2}$$

ν is the friction coefficient between the wheel and the road, N - normal reaction.

Manuscript received July 23, 2011; revised August 07, 2011..

I. V. Author is with the Universidad Autonoma Metropolitana, Azcapotzalco, 02200, Mexico D.F. Mexico (52 5553189550; e-mail: iva@correo.azc.uam.mx).

J. J. O. Author is with the Universidad Autonoma Metropolitana, Azcapotzalco, 02200, Mexico D.F. Mexico (52 5553189550; e-mail: jjoh@correo.azc.uam.mx).

A. F. Author is with the Universidad Autonoma Metropolitana, Azcapotzalco, 02200, Mexico D.F. Mexico (52 5553189550; e-mail: fra@correo.azc.uam.mx).

ISBN: 978-988-19251-7-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$s=\frac{V_x+\Omega_y R+\frac{d}{dT}\breve{\xi}}{V_x} \qquad (3)$$

s - slip rate, V_x - longitudinal velocity of the wheel mass center, $\breve{\xi}$ – longitudinal deformation of the tire's contact area element. The function $\varphi(s)$ is defined experimentally, and it looks like fig. 3.

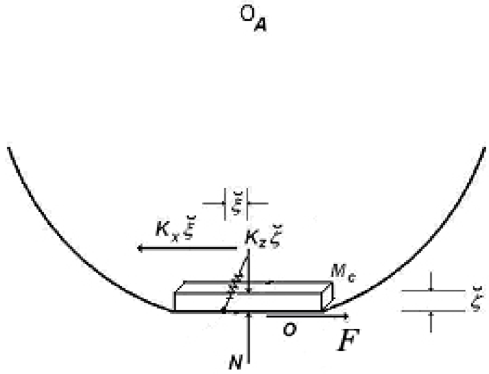

Fig. 2 Model for the contact element of the tire.

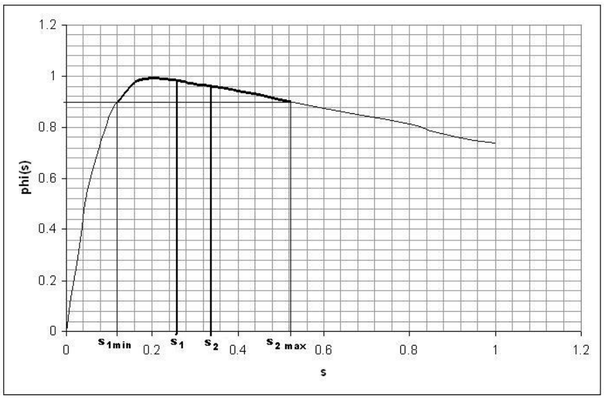

Fig. 3 Characteristic function φ(s)

The motion equation of the contact element with mass M_c is described by the tire longitudinal deformation. The interaction between this element and the rigid part of the wheel can be described with a viscoelastic forces model. The movement equation for the contact element is the next

$$M_c\frac{d}{dT}\left(V_x+\Omega_y R+\frac{d}{dT}\breve{\xi}\right)=F-C_x\frac{d}{dT}\breve{\xi}-K_x\breve{\xi} \quad (4)$$

Here C_x and K_x are longitudinal constants of viscous and elastic behavior of tire's model. The model to be used is the similar to description of first waveform in model [2].

The equations (1)-(4) characterize wheel motion. This system is closed if we assume longitudinal velocity V_x and normal reaction N as constants. This approximation is correct for time lag about seconds if longitudinal velocity and normal reaction changes slowly and their variations are small [14].

Model proposed was previously used to describe the wheel's vibration for small values of slip ratio $s<0.1$ when dependence $\varphi(s)$ is approximately linear $\varphi(s)=K_0 s$ [3]. Under these conditions, it is possible to consider that natural period of contact element vibrations in (4) is much smaller than the characteristic time of change of angular velocity and break torque. The fractional analysis method [14] can be used to reduce equation (5) to terminal form and write approximated relation $F=K_x\xi$. The wheel motion equations in this case is equivalent to pendulum equation [2,3] with viscous friction. Natural frequency of this pendulum is

$$\omega_n=\sqrt{\frac{K_x R^2}{I_y}-\frac{K_x^2 V_x^2}{4\nu^2 N^2 K_0^2}}$$

Such as been shown [3], this result is consistent with experimental effects detected in the process of ABS control algorithm tests.

Further we consider the behavior of the system around the maximum value of the brake torque, it means in the region of $\varphi(s)$ maximum. The Tikhonov's theorem [14] condition used for reduction in previous paragraph is correct too, but reduced equations present singularities in $\varphi'(s)=0$. The analytic and numerical solution of the equations is difficult to obtain. Therefore it is necessary to study full system (1)-(5) properties in order to analyze periodic oscillation of the angular velocity.

We use the next approximation for $\varphi(s)$

$$\varphi(s)=\frac{a_1 s^2+a_2 s+a_3}{s^2+a_4 s+a_5} \qquad (5)$$

The parameters $a_1...a_5$ were calculated with the least squares method [15]. We use for calculation the next values:

a_1=0.8886, a_2=-0.1776, a_3=0.0155, a_4=-0.2226, a_5=0201

These values approximates top neighborhood of tire characteristics, used in [10]. .

B. Pneumatic Brake System Equations

We suppose that the brake torque L is proportional to the pressure P_m in the brake cylinder.

$$L=K_L P_m \qquad (6)$$

For the brake system we use an approximated model of pressure changes in the brake cylinder due to the opening of the valve with a first order relation. [1],[16]

$$T_e\frac{dP_m}{dT}+P_m=P_* \qquad (7)$$

Let's suppose opening and closing of valve is momentary and the parameters of the equation (7) are given by the next rules:

$a) P_*=P_c \quad T_e=T_{in}$, 1 opened and 2 closed

$b) P_*=P_a=0 \quad T_e=T_{out}$, 2 opened and 1 closed

Here P_c – pressure inside the central reservoir (constant), P_a – atmospheric pressure, that we'll consider 0. T_{in} and T_{out} - time constants of internal and external pipelines.

C. Dimensionless Equations

We desire to rewrite equations (1)-(3), (5) in a more useful form, by ignoring changes in V_x. Taking

$$\frac{d\Omega_y}{dT}$$

from (1), and writing in (5) we have:

ISBN: 978-988-19251-7-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$\begin{cases} I_y \dfrac{d\Omega_y}{dT} = \nu NR\varphi(s) - L \\ M_c \dfrac{d^2\breve{\xi}}{dT^2} + C_x \dfrac{d\breve{\xi}}{dT} + K_x\breve{\xi} = -\dfrac{M_c R}{I_y} L + \left(\dfrac{M_c R^2}{I_y} - 1\right)\nu N\varphi(s) \\ s = 1 + \Omega \dfrac{R}{V_x} + \dfrac{1}{V_x}\dfrac{d\breve{\xi}}{dT} \end{cases} \tag{8}$$

Equation (7) can be modified to following form:

$$T_e \frac{dL}{dT} - K_L P_* + L = 0 \tag{9}$$

To reduce the number of parameters we take the variables to a dimensionless form

$$l = \frac{L}{NR}, \qquad \omega = \frac{\Omega_y R}{V_x}, \qquad \xi = \frac{\breve{\xi}}{V_x T_1},$$

$$t = \frac{T}{T_1}, \qquad \text{where} \qquad T_1 = \frac{I_y V_x}{NR^2}$$

characteristic time of the angular velocity changes, according to (1).

The system (1),(8),(9) has the next dimensionless form

$$\begin{cases} \dfrac{d\omega}{dt} = l - \nu\varphi(s) & s = 1 + \omega + \dfrac{d\xi}{dt} \\ \dfrac{d^2\xi}{dt^2} + q\dfrac{d\xi}{dt} + p\xi = -l - \nu k\varphi(s) & \\ \dfrac{T_e}{T_1}\dfrac{dl}{dt} = l_s - l & a) l_s = l_c = const \quad T_e = T_{in} \\ & b) l_s = 0 \quad T_e = T_{out} \end{cases} \tag{10}$$

Where

$$q = \frac{C_x T_1}{M_c}, \qquad p = \frac{K_x T_1^2}{M_c}, \qquad k = \frac{I_y}{M_c R^2} - 1$$

$$l_c = \frac{K_L P_c}{L_*}.$$

III. Periodic Solutions Finding

The main goal of this work is the study of periodic regimes produced by programmed switching of the valve with a given period and duty cycle.

To search for periodic regimes we analyze an auxiliary task: control with a relay feedback built such that the system switches the valve when the slip ratio s reaches the arbitrary limit values s_1 and s_2. We analyze the values s_1, s_2 for which the function $\varphi(s)$ changes around the maximum value (fig. 3). In this region the numerical value of the contact force is less or equal than 10% down the maximum value, for a constant normal reaction between the wheel and the road.

To find periodic solutions $[l_p, \xi_p, \omega_p]$ we integrate numerically the equation system (10) for initial conditions that can be present in real systems [3]. As a result of this integration we have solutions for which the values a) work in the interval $\Delta_1 = \tau_1 - \tau_0$, and the values b) in the interval $\Delta_f = \tau_f - \tau_1$ (fig. 4).

We consider that a periodic regime was found if the integration if the next criteria is true

$$\max\left(l_f - l_{P_0}, \xi_f - \xi_{P_0}, \frac{d\xi_f}{dt} - \frac{d\xi_{P_0}}{dt}, \omega_f - \omega_{P_0} \right) \le 0.01$$

Here (l_p, ξ_p, ω_p) and (l_f, ξ_f, ω_f) are the variables in two successive periods at the moment of valve's opening. (l_0, ξ_0, ω_0) - are the initial conditions of computed periodic solution.

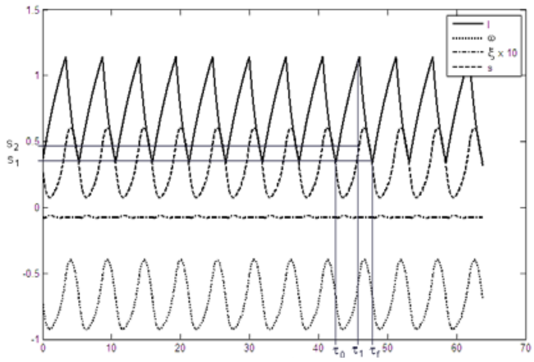

Fig. 4 Periodic Solution

All the possible values Δ_1, Δ_f and the corresponding initial conditions of the periodic solutions at the opening moment were obtained by solving the system for different pairs (s_1, s_2) within the time interval (s_{1min}, s_{2max}). The region of founded values, Δ_1, Δ_f for different friction coefficient value ν can be seen in fig. 5. The parameters for calculations are:

T_{in}=0.0043,
T_{out}=0.0085,
p=1000,
q=100,
k=10,
l_s=0.4755,
T_1=0.0848.

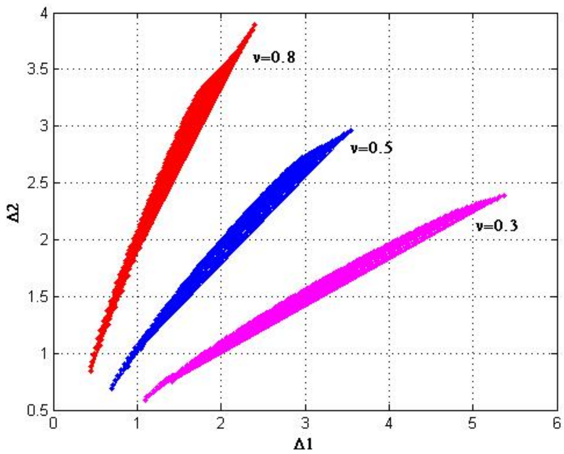

Fig. 5 Periodic Oscillation Regions for Different Friction Coefficients

ISBN: 978-988-19251-7-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

IV. Conclusion

Since the ABS operation is based on a switching process, oscillatory affects are produced, and the results can have consequences on performance, security and comfort of the vehicle, it is important to analyze the properties of such oscillations. The case of maximum longitudinal force before the wheel locks was considered. The simulation showed that the oscillations on the angular velocity of the wheel have a periodic behavior, that information can be helpful to design control algorithms.

References

[1] C. L. Clover, and J. E. Bernard, “Longitudinal Tire Dynamics”, Vehicle System Dynamics, Vol 29, 1998, pp. 231-259.

[2] S. T. H. Jansen, P. W. A Zegelaar, and H. B. Pacejka, “The Influence of In-Plane Tyre Dynamics on ABS Braking of a Quarter Vehicle Model”, Vehicle System Dynamics, Vol 32, 1999, pp. 249-261.

[3] P. A. Kruchinin, M. K. Mogamedov, and I. V. Novozhilov, “Mathematical Model of an Automobile Wheel for antilock modes of motion” Mechanics of Solids, Vol. 36, no. 6, 2001, pp. 52-57.

[4] P. A. Kruchinin, M. K. Mogamedov, and L. M. Makarov, “Active Supression of Parasitic Oscillations During the Operation of a Pneumatic Antilock System” Moscow University Mechanics Bulletin, no. 5, 2003, pp. 25-29.

[5] V. S. Gozdek, and V. I. Goncharenko, “About verification of ACS Stability in the Brake Wheel Process”, Promislova Gidravlika I Pnevmatika, Vol. 6, no. 3, 2004, pp. 73-76.

[6] S. V. Drakunov, U. Ozguner, P. Dix, and B. Ashrafi, “ABS Control Using Optimum Search via Slidind Modes” IEEE Transactions on Control Systems Technology, Vol. 3, no. 1, March 1995, pp. 78-85.

[7] W. Ming-Chin, and S. Ming-Chang, “Simulated and Experimental Study of Hydraulic Anti-lock Braking System Using Sliding Mode PWM Control”, Mechatronics, no. 13, 2003, pp. 331-351.

[8] C. Unsal, and K. Pushkin, “Sliding Mode Measurement Feedback Control for Antilock Braking Systems” IEEE Transactions on Control Systems Technology, 1999.

[9] V. Utkin, J. Guldner, and J. Shi, “Sliding Mode Control in Electromechanical Systems”, Taylor and Francis, 1999.

[10] M. K. Mogamedov, V. V. Alexandrov, and K. A. Pupkov, “Robust Adaptative Stabilization of Moving Car Under Braking with ABS in Control Circuit”, Proceedings of the Automotive and Transportation Technology Congress and Exposition, SAE, Vol. 6, October 2001.

[11] A. El Hadri, J. C. Cadiou, and N. K. M’Sirdi, “Adaptative Sliding Mode Control for Vehicle Traction”, Proceedings of the IFAC World Congress, 2002.

[12] I. V. Novozhilov, P. A. Kruchinin, and M. K. Mogamedov, “Contact Force Relation Between the Wheel and the Contact Surface”, Collection of Scientific and Methodic Papers, Teoreticheskaya Mekhanika, MSU, Vol. 23, 2000, pp. 86-95.

[13] H. B. Pacejka, “In-plane and Out-plane Dynamics of Pneumatic Tyres”, Vehicle System Dynamics, Vol. 8, no. 4-5, 1981, pp. 221-251

[14] I. V. Novozhilov, “Fractional Analysis. Methods of Motion Decomposition”, Birkhauser, 1997.

[15] C. Lawson, and R. Henson, “Solving Least Squares Problems”, Prentice Hall, 1974.

ISBN: 978-988-19251-7-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)