Identification of Linear Periodically Time-Varying Systems Based on Hartley Series

Isidro I. Lázaro, Alberto Álvarez, Juan Anzurez

***Abstract*—The identification problem of periodically varying systems is considered in this paper. It is demonstrated that the systems with periodically varying parameters can be estimated using orthogonal functions; in this case the model can be approximated by Hartley series in the continuous range. The main advantage of using orthogonal functions is that the integro-differential equations are transformed into algebraic equations, this facilitates the identification process. The method presented in this paper was used to estimate parameters in the main rotor model of a helicopter, with satisfactory results.**

***Index Terms*— System identification, periodically varying parameters, Hartley series, basis function, operational matrix.**

I. Introduction

In a wide variety of practical applications, the characteristics of the plant dynamics are changing through time due to aging of components or the variation of the reference. When the dynamics of the system is changing at a slow pace, the least squares algorithm (with and without forgetting factor) is quite effective when it is used to follow this type of behavior. Nevertheless, the efficiency of the algorithm diminishes when the dynamics is changing rapidly. In the past years, systems with fast changing dynamics are being used more often in modern engineering applications; in particular, linear periodic time varying systems (LPTVS) have appeared in different areas such as Control, Power Electronics, Aeronautics, Communications and Signal Processing [1]. For example, in aeronautics, the equations that describe the flight dynamics in aircrafts have coefficients that are functions of time. Recently, the high speeds and accelerations that modern aircrafts can handle, have contributed to show that the parameters of the system that depend on speed will change in the same fashion. For instance, the dynamic of a main rotor in a helicopter, which can be modeled by a linear periodic system, is important from the control perspective due to the fact that a precise control of certain variables in the system can help to attenuate vibrations in the rotor components, that eventually may cause high levels of stress and fatigue [2]. Thus, there are important reasons to develop parametric identification techniques for this type of systems, some of these methodologies can be found in [3]-[5]. Also, the use of basis functions and operational matrices allows the approximation of system parameters of a previously known model; therefore, the LPTVS can be estimated through an approximation using orthogonal series [6]-[10].

The use of orthogonal series expansions is well known as an alternative for approximation and representation of functions. During the last two decades, algebraic methods have been established for the solution of problems described by differential equations, such as analysis of linear time invariant and time varying systems, model reduction, optimal control and system identification. The problem of parameter identification using orthogonal series includes linear time varying/invariant systems, lumped and distributed systems, also the nonlinear case.

The main purpose of this paper is to provide an alternative method for parametric identification. It is based on the transformation of the integro-differential equations that are used to describe the system into an algebraic set of equations. Some other advantages of this methodology are: easy implementation of the algorithm in computational packages (Matlab, Mathematica, C code, etc.) and the easy control of the identification procedure and its algorithm.

The structure of this paper is as follows: Section 2 is dedicated to the general background on approximation using Hartley series expansions and operational matrices. In section 3, the algorithm for the parametric identification for linear periodically time varying systems is presented. The application of the presented method and the associated results can be found in Section 4. The last section is devoted to the conclusions of this work.

II. Orthogonal Series Expansions and Operational Propierties via Hartley Series

In this section a brief review of the Hartley series is given. It can be stated that the kernel function of the familiar Fourier transform and Fourier series is complex exponential, $e^{-j\omega t}$.

The Hartley transform and series uses a similar frequency based kernel, the function $\cos(\omega t) + \sin(\omega t)$, also known as the cosine and sine function or $cas(\cdot)$ [11].

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A. Hartley Series

An integrable function *f(t)* defined over the interval $(0,T)$ can be approximated in terms of Hartley series as

$$f(t) = F\Phi^{t}(t) \tag{1}$$

Where

$$F = \begin{bmatrix} F_{-n} & \cdots & F_{-1} & F_0 & F_1 & \cdots & F_n \end{bmatrix}_{(1,2n+1)} \tag{2}$$

$$\Phi(t) = \begin{bmatrix} \phi_{-n} & \cdots & \phi_{-1} & \phi_0 & \phi_1 & \cdots & \phi_n \end{bmatrix}^{t} \tag{3}$$

The basis functions for the Hartley series can be defined as $\phi_n = cas(n\omega t) = \cos(n\omega t) + \sin(n\omega t)$.

Each coefficient of the vector *F*, is calculated as follows

$$F_n = k\int_0^t f(t)\Phi(t)dt \tag{4}$$

Therefore, every function that complies with the Dirichlet conditions in the interval $(0,T)$ can be approximated by means of a matrix product, as indicated by (1). The number of terms in the series is related to the error level; in this case, more terms mean a better approximation and a reduction in the error. The operational properties of the orthogonal series can be written in terms of an integration matrix, where the main idea is the fact that the integral of an orthogonal series can be expressed also as an orthogonal series; in general, this property can be defined as

$$\int_0^t \underbrace{\cdots}_{n} \int_0^t T(\tau)d\tau^n = P^n\Phi(t) \tag{5}$$

In the case of the Hartley series, the structure of the integration matrix *P* is presented in [8] and it is shown in the following equation

$$P = \frac{1}{\pi}\begin{bmatrix} & & & & -\frac{1}{n} & & & & \frac{1}{n} \\ & & & & \vdots & & & \iddots & \\ & & & & -\frac{1}{2} & & \frac{1}{2} & & \\ & & & & -1 & 1 & & & \\ \frac{1}{n} & \cdots & \frac{1}{2} & 1 & \pi & -1 & -\frac{1}{2} & \cdots & \frac{1}{n} \\ & & & -1 & 1 & & & & \\ & & -\frac{1}{2} & & \frac{1}{2} & & & & \\ & \iddots & & & \vdots & & & & \\ -\frac{1}{n} & & & & \frac{1}{n} & & & & \end{bmatrix}_{(2n+1)\times(2n+1)} \tag{6}$$

For the analysis of LPTV Systems, two additional matrices must be defined, the product matrix and the coefficient matrix. The product of the vector of basis functions and its transpose gives the product matrix, this is

$$\Pi(t) = \Phi^{t}(t)\Phi(t) \tag{7}$$

The coefficient matrix is defined in terms of the product matrix and the vector of basis functions. This matrix satisfies the following relation

$$[C]\Phi(t) = \Pi(t)c \tag{8}$$

The matrix *C*, of order *2n+1* is the coefficient matrix when a vector *c* is given.

$$[C] = \frac{1}{2}\begin{bmatrix} 2c_0 & c_{-1}+c_1 & c_{-2}+c_2 & 2c_{-3} & c_{-2}-c_2 & c_{-1}+c_1 & 0 \\ c_{-1}+c_1 & 2c_0 & \begin{matrix}c_{-1}+c_1\\ +c_{-3}-c_3+\end{matrix} & 2c_{-2} & \begin{matrix}c_{-1}-c_1\\ c_{-3}\quad c_3\end{matrix} & 0 & c_1-c_{-1} \\ c_{-2}+c_2 & \begin{matrix}c_{-1}+c_1\\ +c_{-3}-c_3\end{matrix} & 2c_0 & 2c_{-1} & c_{-2}+c_2 & \begin{matrix}c_{-1}-c_1\\ +c_3+c_{-3}\end{matrix} & c_2\quad c_{-2} \\ 2c_{-3} & 2c_{-2} & 2c_{-1} & 2c_0 & 2c_1 & 2c_2 & 2c_3 \\ c_{-2}-c_2 & \begin{matrix}c_{-1}-c_1\\ +c_{-3}+c_3\end{matrix} & c_{-2}+c_2 & 2c_1 & \begin{matrix}2c_0\\ +c_2-c_{-2}\end{matrix} & \begin{matrix}c_{-1}+c_1\\ +c_2-c_{-2}\end{matrix} & c_2+c_{-2} \\ c_{-1}+c_1 & 0 & \begin{matrix}c_1-c_{-1}\\ +c_3+c_{-3}\end{matrix} & 2c_2 & \begin{matrix}c_1+c_{-1}\\ +c_3-c_{-3}\end{matrix} & 2c_0 & c_{-1}+c_1 \\ 0 & c_1-c_{-1} & c_{-2}+c_2 & 2c_3 & c_{-2}+c_2 & c_{-1}+c_1 & 2c_0 \end{bmatrix} \tag{9}$$

III. Parameter Estimation for Linear Periodically Time-Varying Systems

Any given differential equation of degree *n* can be expressed as a set of state-space equations. This same set can be transformed into an algebraic set of equations using operational matrices. In this section, a methodology to solve the identification problem and estimate the unknown parameters using the input-output data is presented [12].

A. Identification Algorithm

Linear periodically time-varying systems can be described in general sense by the following set of state-space equations

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{B}(t)\boldsymbol{u}(t) \tag{10}$$

where $x(t) \in \Re^{n\times 1}$ and $u(t) \in \Re^{q\times 1}$ are the state and input vectors respectively, and $A(t) \in \Re^{n\times n}$ and $B(t) \in \Re^{n\times q}$ are the corresponding periodic coefficients with the following properties

$$\boldsymbol{A}(t) = \boldsymbol{A}(t+T) \;\; for\; 0 \le t \le T \qquad \boldsymbol{B}(t) = \boldsymbol{B}(t+T) \;\; for\; 0 \le t \le T \tag{11}$$

Using function approximations via orthogonal series, the expressions for the elements $a_{ij}(t)$ and $b_{ij}(t)$ of matrices *A(t)* and *B(t)* that satisfy the Dirichlet conditions in the interval $(0,T)$, can be expressed as

$$a_{ij}(t) \cong A_{ij}^{'}\Phi(t) \qquad b_{ij}(t) \cong B_{ij}^{'}\Phi(t) \tag{12}$$

Where

$$A_{ij}^{'} = \begin{bmatrix} A_{-n} & \cdots & A_0 & \cdots & A_n \end{bmatrix} \quad B_{ij}^{'} = \begin{bmatrix} B_{-n} & \cdots & B_0 & \cdots & B_n \end{bmatrix} \tag{13}$$

Similarly, the elements of vector $x(t) \in \Re^{nx1}$ and $u(t) \in \Re^{qx1}$ can be approximated as,

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$$x_i(t) \cong X_i^{'}\Phi(t) \qquad u_i(t) \cong U_i^{'}\Phi(t) \tag{14}$$

With this in mind, the orthogonal series of the product $\mathbf{A}(t)\mathbf{x}(t)$ is

$$\boldsymbol{A}(t)\boldsymbol{x}(t) = \begin{bmatrix} A_{11}^{'}\Phi(t)X_1^{'}\Phi(t) + \cdots + A_{1n}^{'}\Phi(t)X_n^{'}\Phi(t) \\ A_{21}^{'}\Phi(t)X_1^{'}\Phi(t) + \cdots + A_{2n}^{'}\Phi(t)X_n^{'}\Phi(t) \\ \vdots \\ A_{n1}^{'}\Phi(t)X_1^{'}\Phi(t) + \cdots + A_{nn}^{'}\Phi(t)X_n^{'}\Phi(t) \end{bmatrix} \tag{15}$$

Where every factor $A_{ij}^{\prime}T(t)X_j^{\prime}T(t)$ may be rearranged using the product and coefficient matrix as follows

$$A_{ij}^{'}\Phi(t)X_j^{'}(t)\Phi(t) = A_{ij}^{'}\Phi(t)\Phi^{'}(t)X_j = A_{ij}\left[X_j\right]\Phi(t) \tag{16}$$

Matrix $[X_j] \in \Re^{(m\times m)}$ is the coefficient matrix of the vector $x(t) \in \Re^{nx1}$ where $m = 2n+1$. It may be shown that with the above simplification, the product $A(t)x(t)$ may be written as

$$A(t)x(t) = \begin{bmatrix} A_{11}^{'} & A_{12}^{'} & \cdots & A_{1n}^{'} \\ A_{21}^{'} & A_{22}^{'} & \cdots & A_{2n}^{'} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1}^{'} & A_{n2}^{'} & \cdots & A_{nn}^{'} \end{bmatrix} \begin{bmatrix} \left[X_1\right] \\ \left[X_2\right] \\ \vdots \\ \left[X_n\right] \end{bmatrix} \Phi(t) \tag{17}$$

In other words,

$$A(t)x(t) = \mathrm{A}\left[\mathrm{X}\right]\Phi(t) \tag{18}$$

Where $[X_j]$ is the coefficient matrix corresponding to each coefficient, $[X]$ is the matrix formed by all the coefficient matrices $[X_j]$. $\boldsymbol{A}$ is the matrix formed for every vector $[A'_{ij}]$ and $\Phi(t)$ is the basis vector.

A similar procedure is used for $B(t)u(t)$ yielding

$$B(t)u(t) = \mathrm{B}\left[\mathrm{U}\right]\Phi(t) \tag{19}$$

Where $B(t) \in \Re^{n\times q(m)}$, $U(t) \in \Re^{q(m\times m)}$ and $\Phi(t) \in \Re^{(m\times 1)}$

Integrating in both sides (10), we have that

$$x(t) - x_0(t) = \int_0^t A(t)x(t)dt + \int_0^t B(t)u(t)dt \tag{20}$$

Using operational matrices and substituting (18) and (19) into (20) yields

$$X\Phi(t) = X_0\Phi(t) + \mathrm{A}\left[\mathrm{X}\right]P\Phi(t) + \mathrm{B}\left[\Phi\right]P \quad (t) \tag{21}$$

Where X_0 is the same size of X and contains the initial conditions of every state.

$$X_0 = \begin{bmatrix} 0 & \cdots & x_1(0) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & x_n(0) & \cdots & 0 \end{bmatrix} \tag{22}$$

Equation (21) can be rewritten as

$$X = \theta Z \tag{23}$$

Where

$$\theta = \begin{bmatrix} \mathrm{A} & \mathrm{B} & X_0 \end{bmatrix} \qquad Z = \begin{bmatrix} [\mathrm{X}]P \\ [\mathrm{U}]P \\ e \end{bmatrix} \tag{24}$$

$$e = [\ 0 \quad \cdots \quad 1 \quad \cdots \quad 0\ ] \tag{25}$$

Using the above equations and solving for the unknown parameters we obtain [9]:

$$\theta = XZ^{'}(ZZ^{'})^{-1} \tag{26}$$

From the equation (26) the parameter coefficients can be obtained provided that $(ZZ')^{-1}$ exists.

B. Analytical Redundancy

In dynamic systems, it is common to find physical redundancy, for example in the implementation of several devices of the same characteristics within the same measurement and control system. Another example consists in the use two or more sensors to anticipate every eventuality before a failure that causes system instability occurs. Another alternative, that is often more economical, is the application of analytical redundancy. This kind of redundancy involves the ability to run multiple experiments on the system to identify; the use of more than one input enables the excitation of the different operation modes in the system for every experiment. Another option is to apply the same signal at different frequencies and change the phase or/and magnitude. Subsequently, all this information is used for parameter estimation, as shown in Fig. 1. The disadvantage of this method is the off-line identification.

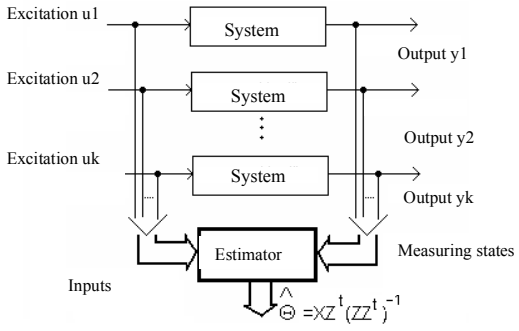

Fig. 1. Analytical redundancy scheme.

IV. Applications

A simple theoretical second order periodic system is used as an introductory example in order to illustrate the methodology described in section III. Then, the application of the procedure is shown using the equations of the dynamics of a helicopter main rotor in forward flight.

A. Second Order Periodic System

Let´s consider a second order linear system with a periodic coefficient described by [13].

$$\dot{\boldsymbol{x}}(t)=\begin{bmatrix}0 & 1\\ -1 & -f_1\end{bmatrix}x(t)+\begin{bmatrix}0\\ 1\end{bmatrix}u(t) \tag{27}$$

Where f_1 is a system with periodically varying parameters given by

$$f_1(t)=1+\sin(5\omega t) \tag{28}$$

And excited by a periodic input $u(t)=sign\left((\sin\omega t)\right)$ and $\omega=2\pi$.

In order to perform the parametric identification of the system described by (27), equation (26) was used considering only the first 15 terms of the Hartley series and an analytical redundancy of 3. The system was excited off-line, with the purpose of having simulation data corresponding to the input-output behavior, with the following signals

$$u_1(t)=sign\left((\sin\omega t)\right)$$

$$u_2(t)=1.2\cos(3\omega t)+0.5\cos(5\omega t)$$

$$u_3(t)=1+0.2\cos(3\omega t)+0.05\cos(5\omega t) \tag{29}$$

The simulation results obtained with the help of Matlab are shown in Fig. 2. In this figure, the parametric identification of matrix *A(t)* is displayed when using only 15 terms of the Hartley series and it can be seen the difference between the estimated and real values. Fig 3. Shows the results for the constant coefficients of matrix *B(t)*

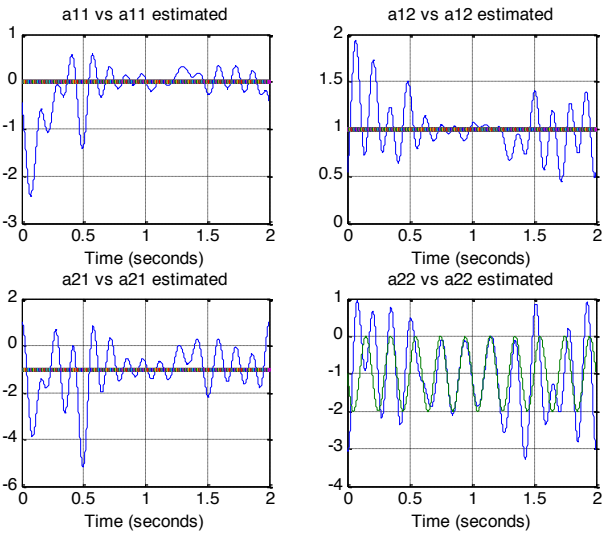

Fig. 2. Parameter estimates of *A(t)* with *n* = 15.

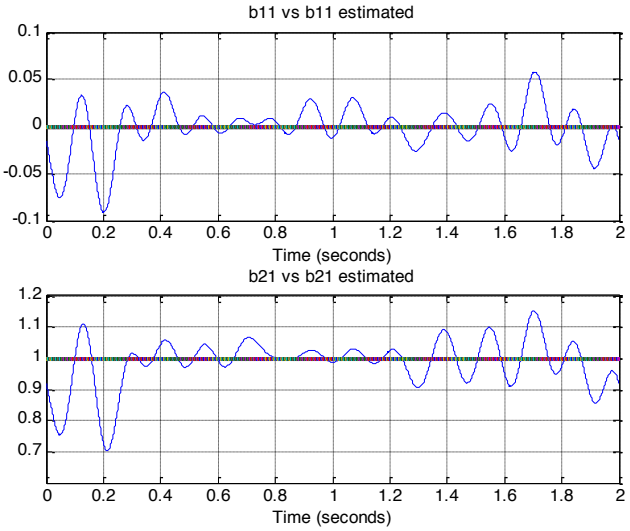

Fig. 3. Parameter estimates of *B(t)* with *n* = 15.

With the purpose of improving the approximation between the estimated and the real parameter values, another test was performed considering 35 terms of the Hartley series. Fig. 4. Shows the improvements of the identification for the only periodic time-varying parameter in the system $a_{22}(t)$. In this case, the varying nature of the parameters is sinusoidal; therefore, the use of Hartley series for parametric identification is adequate since a low number of terms are used in the estimation process.

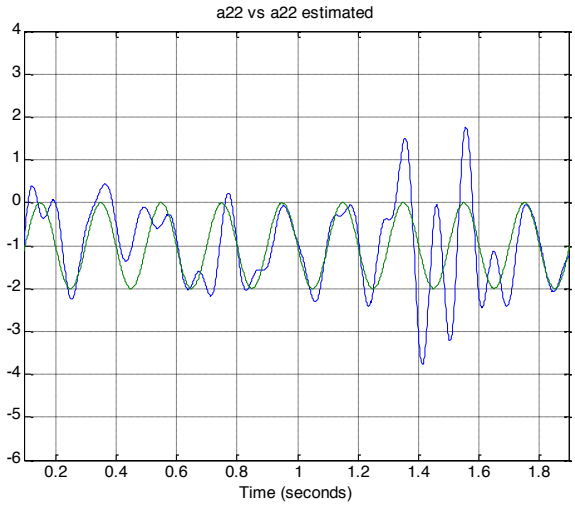

Fig. 4. Parameter estimate $a_{22}(t)$ with *n* = 35.

B. Helicopter Main Rotor Model

The equations that describe the dynamics in frontal flight of a helicopter main rotor are presented in [14]-[15]. The model for one of the blades can be written in the form of a linear periodic system, and has the following form

$$\dot{\boldsymbol{x}}(\psi)=\boldsymbol{A}(\psi)\boldsymbol{x}(\psi)+\boldsymbol{B}(\psi)\boldsymbol{u}(\psi) \tag{30}$$

Each matrix in (30) is described by

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$$A(\psi)=\begin{bmatrix} 0 & 1 \\ -\left\{1+\frac{\gamma}{8}\left(\frac{4}{3}\mu\cos\psi+\mu^2\sin 2\psi\right)\right\} & -\frac{\gamma}{8}\left(1-\frac{4}{3}\mu\sin\psi\right) \end{bmatrix} \quad (31)$$

$$B(\psi)=\begin{bmatrix} 0 \\ \frac{\gamma}{8}\left(1+\mu^2+\frac{8}{3}\mu\sin\psi \quad \mu^2\cos 2\psi\right) \end{bmatrix} \quad (32)$$

$$x(\psi)=\begin{bmatrix} \beta \\ \dot{\beta} \end{bmatrix} \quad (33)$$

Where $\mu=0.3$ represents the advance ratio of the helicopter and $\gamma=8$ is the Lock number. In this case, $\psi=\omega t$, and ω is the angular velocity. The first state variable β represents the angle of the blade with respect to the rotation plane and it is shown in Fig. 5.

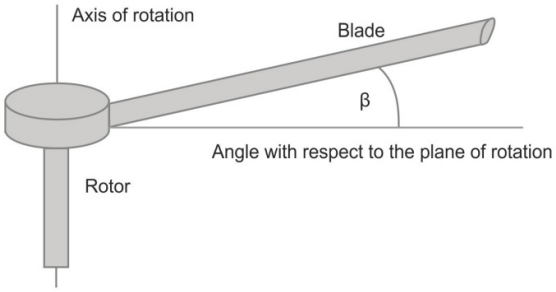

Fig. 5. Main rotor simplified diagram.

In order to illustrate the identification procedure of LPTV systems, the aforementioned rotor model is used (30). The analytical redundancy value used in the simulation was 4, and the off-line applied signals that were used to obtain input-output measurements were

$$u_1(t)=1+0.1\cos(5t)$$

$$u_2(t)=0.8\cos(2t)+0.1\cos(5t)$$

$$u_3(t)=3t+0.1\sin(3t)$$

$$u_4(t)=1.1\cos(t)+0.2\sin(3t)+0.15\cos(3t)+0.1\sin(5t) \quad (34)$$

A total of 35 terms of the Hartley Series were used to perform the identification procedure. Four different signals are used to generate the set of measurements (state $x(t)$ and input $u(t)$) when the analytical redundancy value is 4.

Applying the concepts of operational matrices, the [X] and [U] coefficient matrices are generated and employed in the off-line identification technique.

The algorithms to identify the periodic parameters in the model were developed in Matlab and the results obtained from comparing the estimated vs. real parameters of matrix $A(t)$ can be seen in Fig. 6 and 7 when n = 15. A small error can be observed and it is even smaller for the second cycle of operation in the system. The error tends to decrease when the value of n increases.

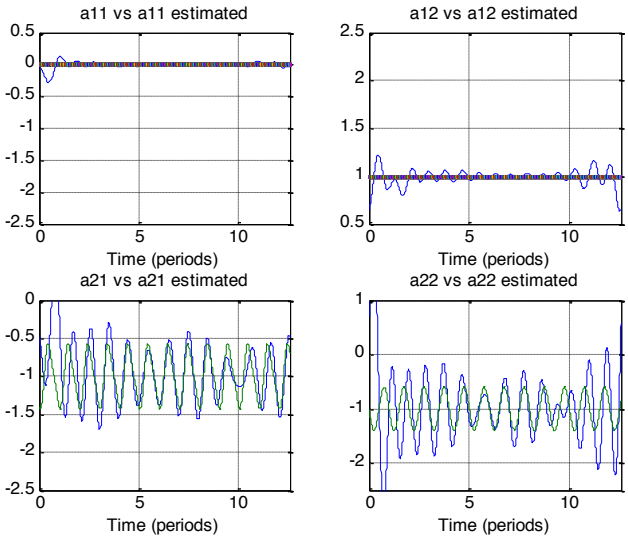

Fig. 6. Parameter estimates of $A(t)$ with n = 15.

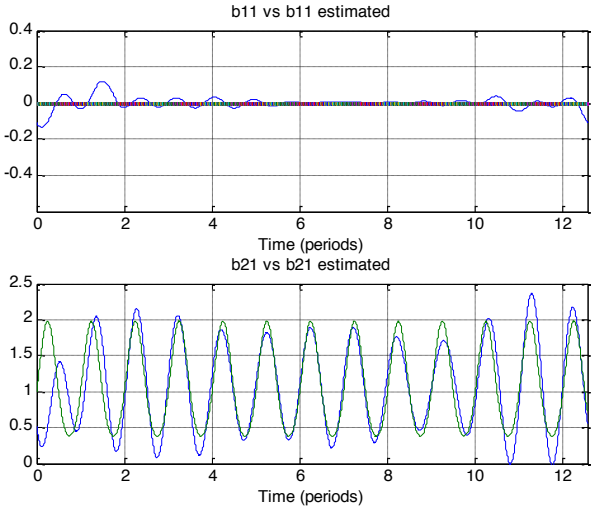

Fig. 7. Parameter estimates of $B(t)$ with n = 15.

Figures 8 and 9 show the estimated parameters for matrices $A(t)$ and $B(t)$ when n = 35. Again, the correlation between the real values and the estimated ones, for both invariant and periodically variant parameters, can be seen.

In Fig. 10, the absolute error of the periodic parameters in matrix $A(t)$ is displayed. From this graphic, it can be seen that the error between estimated and real values decreases drastically after the transitory effect of the identification test and when the number of terms used in the Hartley series increases.

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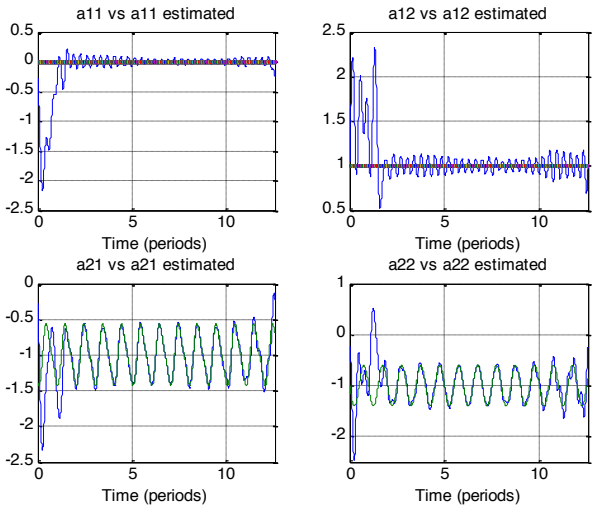

Fig. 8. Parameter estimates of $A(t)$ with $n = 35$.

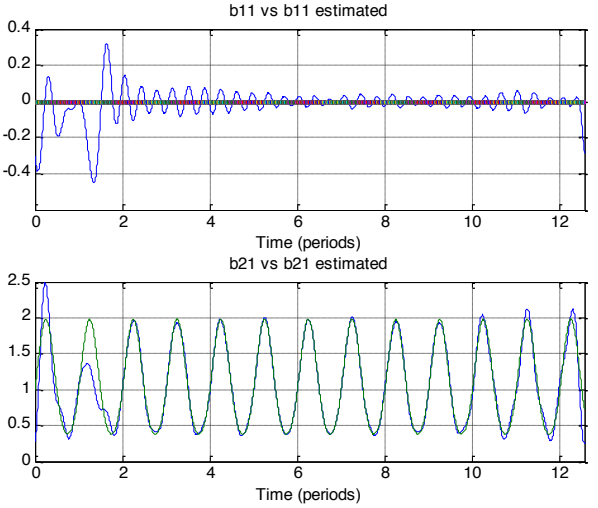

Fig. 9. Parameter estimates of $A(t)$ with $n = 35$.

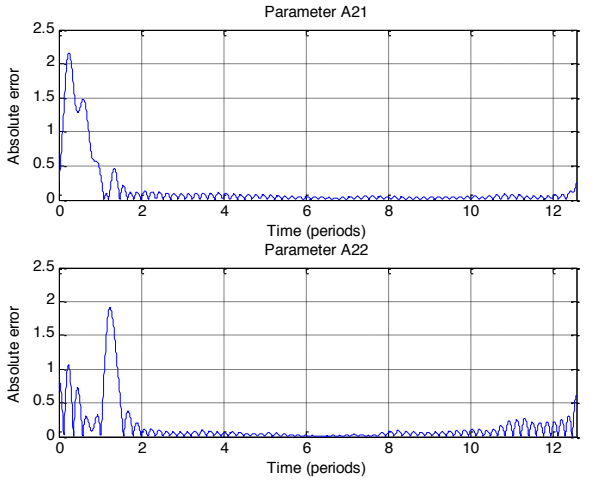

Fig. 10. Absolute error of $A_{21}(t)$ and $A_{22}(t)$ with $n = 35$.

It is worth mentioning that not necessarily all the terms in the orthogonal series that are used in the identification process have great significance. For instance, the estimated parameter $a_{22}(t)$ has only 15 terms that are significant; the values of the rest of the terms are less than 10% of the Hartley series coefficient of higher magnitude. Therefore, to estimate the parameter, only an expansion in orthogonal series of 15 out of 35 terms is needed.

V. Conclusions

A frequency domain technique for off-line parameter estimation of LPTV systems has been presented. The operational matrices tools and the integration matrix in particular, are used to yield algebraic equations that allow the parameter identification procedure to take place. These algebraic equations can be solved through the use of a pseudoinverse. The presented procedure is applied to two different cases using Hartley series as the basis function. These properties are used to estimate the parameters of a second order theoretical system with only one time-varying parameter, and also of the model that describes the dynamics of the main rotor of a helicopter in forward flight.

From the comparison made between the estimated and real parameters, for both cases, it can be seen that the average error decreases when the number of terms used in the series is increased. It is also important to notice that the number of terms to be considered in the estimation process can be reduced if only the parameters with higher magnitudes are taken into account.

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