The Optimal Ordering Policy for a Perishable Inventory System

Pavee Siriruk

Abstract—**Due to inherent characteristics of products, items are subject to lose their value or usability overtime. A perishable inventory system has received increasing attentions in the past years. An optimal ordering policy for deteriorating inventory system has become more and more important. This paper develops a method to determine the optimal order quantity that minimizes the total expected cost for perishable items. In addition, the solution methodologies as well as an analysis of results are presented.**

Index Terms—**Inventory Systems; Perishable Items; Optimization; Outdating Costs.**

I. Introduction

THE ever increasing attention in determining an optimal ordering policy for perishable inventory systems has added a new level of complexity to the task of managing inventory. The traditional inventory management problem works well under deterministic demand assumption. However, when the demand is assumed to be a random variable and items are assumed to be perishable to reflect the real life situations, the classical approach leads to a poor performance and unsatisfactory management.

Nahmias [6] has done a comprehensive review of previous research on perishable inventory systems. He has classified perishable products into two categories: random lifetime and fixed lifetime. He also mentioned in his review that the first analysis of optimal policies for a fixed life perishable commodity was begun by Van Zyl. This topic was extended later by Fries [5], Namias [7, 8, 9, 10], and Chiu [2, 3]. Most of the previous studies such as those of Nahmias [10], Cohen [4], Chazan and Gal [1] have concentrated on the periodic review and multi-period lifetime problem with zero lead time.

Generally, goods having finite lifetimes are subject to the perishables. Hence, a perishable inventory, such as fashion garments, blood, and drugs, is one in which all the units of one material item in stock will be outdated if not being used before the expiration date, resulting in an additional outdating cost of perished items. Therefore, it is required that the outdating issue is taken into account to reflect the real-life situations. In this work, the focus is placed on the fixed life time and continuous review (Q, r) perishable inventory systems with a positive lead time. The objective of this research is to determine the optimal ordering quantity that minimizes the expected cost, for a perishable item over a finite horizon.

Manuscript received July 14, 2012;

Pavee Siriruk is with Department of Industrial Engineering, Suranaree University of Technology, Nakhon Ratchasima, Thailand 30000 (phone: +66-81-470-7706; e-mail: pavee@g.sut.ac.th).

The traditional model is extended to a more sophisticated model in which the outdating costs are considered. An approximate expected outdating of the current order size from [2] is used to obtain an optimal ordering policy under positive order lead time, which minimizes the total expected average cost. A solution methodology to find the optimal order quantity is presented. In addition, the behavior of the expected cost function that is composed of ordering cost, holding cost and outdating cost, is analyzed and shown that it is convex in order quantity.

The paper is organized as follows. In section two, a mathematical model for the problem is presented. The solution methodology is described in section three. In section four, the numerical example with preliminary results are shown. Finally, in section five, not only primary contributions of this work but also suggestions for the potential future research extensions are summarized.

II. Mathematical Model

A. Assumptions

The following assumptions are applied to the model:
1. One perishable item is considered. It is assumed that each unit of the item has a fixed lifetime equal to m and no loss or decrease in utility occurring before m time units.
2. Inventory levels are reviewed continuously. When the inventory level reaches the reorder point r, an order size Q, $Q > 0$ is placed.
3. All units of a replenishment order arrive in fresh condition.
4. A positive order lead time L for replenishment; L is less than the lifetime m.
5. The demand in unit time t, d_t, is a nonnegative random variable and normally distributed. It is also assumed that if $\Phi(t)$ is cumulative normal demand by time t, then $\Phi(t)$ is a stochastic process with stationary independent increments.
6. d_{m+L} is a random variable. $\Phi(m+L)$ has normal density $f_{m+L}(d_{m+L})$ and mean $(m+L)d$.
7. No shortage is allowed (all demands are met).
8. Units are always depleted according to an FIFO (First in first out) issuing policy.
9. If each unit has not been used to meet a demand before the expiration date, it must be discarded and an outdate cost equals to W is charged.

B. Model Development

The total expected cost function consists of ordering cost, holding cost, and outdating cost as shown below.

$$EC(Q,r) = E[Ordering\ Cost + Holding\ Cost + Outdating\ Cost] \quad (1)$$

Based on the assumption that the demand is normally distributed, reorder point can be calculated by using the safety factor of normal distribution. Therefore, the following relationship can be used to express the lead time demand.

$$\mu_X = E[X] = L \times E[D] = DL \quad (2)$$
$$\sigma_X^2 = Var[X] = L \times Var[D], \text{ so that } \sigma_X = \sigma\sqrt{L} \quad (3)$$

$Pr\{X \geq r\}$ is the probability of stockout during the lead time, then choosing r such that $Pr\{X \geq r\} = q$, where q is the allowable stockout probability. Due to the normal distributed probability, the $Pr\{X \geq r\} = q$ becomes $Pr\{Z \geq k\} = q$, where $k = \frac{r-\mu_X}{\sigma_X}$ is the safety factor. Thus, $r = \mu_X + k\sigma_X$.
From (2) and (3), the reorder point using the safety factor becomes,

$$r = DL + k\sigma\sqrt{L} \quad (4)$$

After approximating the reorder point by the safety factor, equation (1) can be recognized as

$$EC(Q) = E[Ordering\ Cost + Holding\ Cost + Outdating\ Cost]. \quad (5)$$

Ordering Cost

$$Ordering\ Cost = K\ (number\ of\ cycles) = \frac{KD}{Q} \quad (6)$$

Holding Cost

The expected inventory level can be obtained by

$$E[Inventory\ Level] = 1/2(Inventory\ Level\ at\ the\ Beginning\ of\ a\ Cycle + Inventory\ Level\ at\ the\ End\ of\ a\ Cycle)$$

The inventory level at the beginning of a cycle can be computed by

$$r - E[X] + Q \quad (7)$$

The inventory level at the end of a cycle can be calculated by

$$r - E[X] \quad (8)$$

From equation (7) and (8), the expected inventory level can be written as

$$E[Inventory\ Level] = 1/2(r - E[X] + Q + r - E[X]) = \frac{Q}{2} + r - E[X]. \quad (9)$$

The holding cost function is known as
Holding Cost $= h \times E[Inventory\ Level]$

From equation (9), holding cost function can be expressed as
Holding Cost $= h\left\{\frac{Q}{2} + r - E[X]\right\}$.

From (2) and (4), the holding cost function can be rewritten as

$$\text{Holding Cost} = h\left\{\frac{Q}{2} + k\sigma\sqrt{L}\right\}. \quad (10)$$

Outdating Cost

The expected outdating approximation is borrowed from [2]. The approximate expected outdating of the current order size with a positive lead time can be obtained by

$$\text{Expected Outdating Quantity} = \int_0^{r+Q} (r + Q - d_{m+L}) f_{m+L}(d_{m+L}) \mathrm{d}d_{m+L} - \int_0^{r} (r - d_{m+L}) f_{m+L}(d_{m+L}) \mathrm{d}d_{m+L} \quad (11)$$

Hence, the expected outdating cost can be expressed as

$$\text{Expected Outdating Cost} = W \times \left[\int_0^{r+Q} (r + Q - d_{m+L}) f_{m+L}(d_{m+L}) \mathrm{d}d_{m+L} - \int_0^{r} (r - d_{m+L}) f_{m+L}(d_{m+L}) \mathrm{d}d_{m+L}\right] \quad (12)$$

The total Expected Cost Function

From equations (5), (6), (10), and (12), the total expected cost function can be written as

$$EC(Q) = \frac{KD}{Q} + h\left\{\frac{Q}{2} + k\sigma\sqrt{L}\right\} + W \times \left[\int_0^{r+Q} (r + Q - d_{m+L}) f_{m+L}(d_{m+L}) \mathrm{d}d_{m+L} - \int_0^{r} (r - d_{m+L}) f_{m+L}(d_{m+L}) \mathrm{d}d_{m+L}\right] \quad (13)$$

By using Leibniz's rule, it can be shown that the first derivatives of equation (13) is

$$\frac{\partial EC(Q)}{\partial Q} = -\frac{KD}{Q^2} + \frac{h}{2} + W\left[\int_0^{r+Q} f_{m+L}(d_{m+L}) \mathrm{d}d_{m+L}\right] \quad (14)$$

From (14), the second derivative of the total expected cost function using Leibniz's rule can be written as

$$\frac{\partial^2 EC(Q)}{\partial Q^2} = \frac{2KD}{Q^3} + W\left[\int_0^{r+Q} f_{m+L}(r + Q) \mathrm{d}(r + Q)\right] \quad (15)$$

Since equation (15) is greater than 0, $\forall Q > 0$, $EC(Q)$ is a convex function.

Since the total cost function is a convex function equating equation (14) to zero and solving yields the optimal Q^*.

$$\Phi(r + Q^*) = \frac{KD}{WQ^{*2}} - \frac{h}{2W} \quad \text{or}$$

$$\Phi(r + Q^*) - \frac{KD}{WQ^{*2}} + \frac{h}{2W} = 0 \quad (16)$$

III. Solution Methodology

The optimal order quantity that minimizes total expected cost can be calculated by using equation (16). However, equation (16) cannot be solved directly due to the computational complexity of the normal density function, which represents the demand distribution. When all fixed values are given, except for the order quantity, the only way to solve equation (16) is to apply a heuristic algorithm. For simplicity, we use a function in Microsoft Excel to find an optimal solution. We will apply a heuristic algorithm in the future. After solving equation (16), the optimal order quantity that minimizes the total expected cost is obtained.

ISBN: 978-988-19252-4-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

IV. Numerical Examples

In this section, we illustrate a numerical example. If $h = 1$, $L = 1$, $K = 10$, $W = 5$, and demand is normally distributed with mean 10 and variance 10, and the stockout probability is 0.10. Based on the safety factor, the value of k is determined as 1.28 from the normal table. After the value of k is calculated, the reorder point is obtained by using equation (4). In this paper we decided to find an optimal solution of (16) by using the "Goal Seek" function in Microsoft Excel. Solving equation (16) yields the optimal order quantity, Q^*, equal to 4.26. Table 1 demonstrates the result.

TABLE I
Results of the Numerical Example

Parameters	Values
K	10
W	5
L	1
h	1
D	10
σ^2	10
k	1.28
r	14.05
Variable	Values
Q^*	4.26

V. Conclusions

This paper has presented a solution methodology to determine the optimal ordering quantity. It is shown that the cost function to be minimized is convex in ordering quantity. Direct computation of an optimal ordering policy for a perishable product is accomplished.

Different continuous and discrete demand distributions for perishables also require further study. Furthermore, the reorder point should be considered as a variable to approximate the optimal ordering quantity more precisely. An extension of this study can be conducted in the future to formulate more accurate estimates. In addition to this, various optimal ordering policies can be compared with the proposed quantity to evaluate the performance of the solution method presented in this work. A comparison of the proposed ordering policy with simulated policies can also be carried out in order to further analyze the model. Finally, upper and lower bounds for the expected outdating can be obtained to determine a confidence interval for the outdating quantity.

References

[1] D. Chazan, S. Gal, "A Markovian Model for a Perishable Product Inventory," Management Science, Vol. 23, No. 5, 1977, pp. 512-521.
[2] H. N. Chiu, "An Approximation to the Continuous Review Inventory Model with Perishable Items and Lead Times," European Journal of Operations Research, Vol. 87, Issue 1, 1995, pp. 93-108.
[3] H. N. Chiu, "A Good Approximation of the Inventory Level in a (Q, r) Perishable Inventory System", Rairi Operations Research, Vol. 33, Issue 1, 1999, pp. 29-45.
[4] M. A. Cohen, "Analysis of Single Critical Number Ordering Policies for Perishable Inventories," Operations Research, Vol. 24, No. 4, 1976, pp. 726-741.
[5] B. E. Fries, "Optimal Ordering Policy for a Perishable Commodity with Fixed Lifetime," Operations Research, Vol. 23, No. 1, 1975, pp. 46-61.
[6] S. Nahmias, "Perishable Inventory Theory: A Review," Operations Research, Vol. 30, No. 4, 1982, pp. 680-708.
[7] S. Nahmias, "Myopic Approximations for the Perishable Inventory Problem," Management Science, Vol. 22, No. 9, 1976, pp.1002-1008.
[8] S. Nahmias, "Optimal Ordering Policies for Perishable Inventory-II," Operations Research, Vol. 23, No. 4, 1975, pp. 735-749.
[9] S. Nahmias and S. Wang, "A Heuristic Lot Size Reorder Point Model for Decaying Inventories," Management Science, Vol. 25, No. 1, 1979, pp. 90-97.
[10] S. Nahmias, "Higher Order Approximations for the Perishable Inventory Problem," Operations Research, Vol. 25, No. 4, 1977, pp. 630-640.
[11] C. P. Schmidt and S. Nahmias, "$(S-1, S)$ Policies for Perishable Inventory", Management Science, Vol. 31, No. 6, 1985, pp. 719-728.