Energy Shaping Nonlinear Acceleration Control for a Mobile Inverted Pendulum with a Slider Mechanism Utilizing Instability

K. Yokoyama and M. Takahashi

***Abstract*— A nonlinear controller for accelerating a mobile inverted pendulum (MIP) with a slider mechanism is proposed. The controller shapes the total energy of the system and utilizes instability of the MIP for acceleration. The body angle and the displacement are controlled to keep states where the MIP is statically unstable, which leads to translational acceleration due to instability of the system. The total energy of the system is shaped to have the minimum at given desired states and the system is controlled to converge to them. The proposed controller can achieve various properties through the energy shaping procedure. Especially an energy function that will lead to safe operation of the MIP is proposed. The function ensures that motion of the MIP is restricted within predefined regions and converges to the desired states. The controller also returns the system back to the desired states with state-dependent gains that become large if the system comes close to fall over. Effectiveness of the proposed controller and utilization of instability for the MIP with the slider mechanism are verified through simulations.**

***Index Terms*—Energy Shaping, Mobile Inverted Pendulum, Instability, Slider Mechanism**

I. INTRODUCTION

A mobile inverted pendulum (MIP) has a small footprint and can turn in a small radius. Its mobility is energy-efficient due to it being lighter than ordinary cars. It is applied to personal mobility vehicles that are expected to be used near human living space. There are two types of MIPs. The one is the standing-type, for example, Segway. The other is the sitting-type, for example, MOBIRO and en-V, which have a slider mechanism. A diagram of the system is shown in Fig. 1. The slider shifts the center of mass (COM) of the MIP while a rider is sitting still. This mechanism enables the MIP to accelerate without the rider shifting his/her COM back and forth. Therefore various people including the elderly can use the MIP with the slider easily. In this study, this type of the MIP is focused on, and a safe controller to accelerate the mobility is proposed. We consider the safety in terms of the body angle and the slider displacement. The controller utilizes instability of the MIP. The body angle and the slider displacement are controlled to keep statically unstable but dynamically stable states while theoretically restricting motion of the MIP in a predefined range. The intentional destabilization leads to indirect control of translational acceleration.

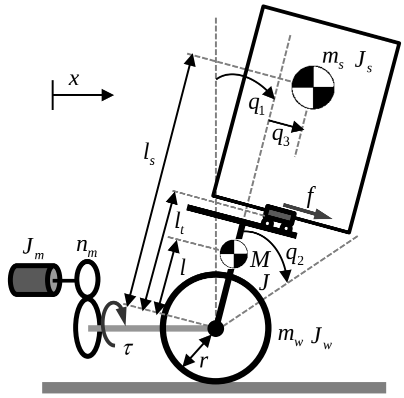

Fig. 1. A diagram of a mobile inverted pendulum with a slider

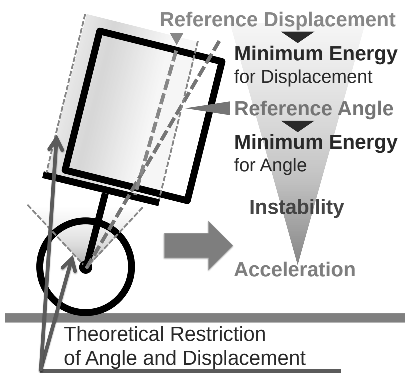

Fig. 2. The concept of the proposed controller

Typical controllers for the MIP with the slider in previous studies are linear ones based on linearly approximated equations of motion[1]-[3]. They control the system to converge to statically stable states. The reference body angle is set in the vertical direction, and the slider displacement at the position where the MIP can stand still. However, the stability is only guaranteed in the neighborhood of the equilibrium. Thus, if the MIP is largely inclined due to disturbance or the way a rider operates it, it may fall over. Driving control is based on tracking a given reference translational velocity. Although the MIP must be destabilized transiently because of the dynamics when it accelerates or

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Table I
PARAMETERS OF THE MOBILE INVERTED PENDULUM

Parameter	Unit	Value
M	kg	5
m_w	kg	1
m_s	kg	3
J	$\mathrm{kg\cdot m^2}$	6×10^{-2}
J_w	$\mathrm{kg\cdot m^2}$	3×10^{-3}
J_s	$\mathrm{kg\cdot m^2}$	2×10^{-2}
J_m	$\mathrm{kg\cdot m^2}$	7×10^{-6}
l	m	0.1
l_s	m	0.50
l_t	m	0.15
r	m	0.075
n_m	-	30

decelerates, motion of the body is not directly considered in the previous controllers. Therefore, the MIP may fall over because of an inappropriate reference velocity.

Thus, the authors have proposed controllers for the MIP without the slider that restrict motion of the body in a predefined range[4][5]. Especially, the acceleration controller utilizes instability of the system[5]. A reference body angle is directly given to the system, and the body is controlled to incline it intentionally, which leads to the indirect control of translational acceleration. The method is safe because the motion of the body is directly considered and easier to predict than that of previous controllers.

Based on the background above, this study extends our acceleration controller for the MIP with the slider that utilizes instability. The body angle and the slider displacement are controlled to keep statically unstable but dynamically stable states. The translational acceleration is controlled indirectly through the direct control of the two states. The proposed control method provides the riders of a sitting MIP with a feeling of maneuvering while inclining back and forth, as can experienced with the standing MIP. In our previous study, there was a one-one relationship between the body angle and the translational acceleration at steady states. When a large acceleration needs to be achieved, the MIP has to be inclined largely. The large body angle can be suppressed using the new degrees of freedom of the slider. The combination of the two states can modify motion of the MIP, leading to safe operation. To achieve this concept, a nonlinear controller is applied called an interconnection and damping assignment passivity-based control (IDA-PBC)[6][7]. The method shapes the total energy of systems to stabilize them. The controller can achieve various properties that are difficult to achieve with a linear controller through the energy shaping procedure. In this study, an energy function is proposed that ensures that the body angle and the slider displacement are restricted within predefined regions and converge to reference states. The controller also helps to return the MIP to the desired states using state-dependent gains that become large if the MIP comes close to falling over. Effectiveness of the controller is verified in simulations. The conceptual diagram of the proposed controller is shown in Fig. 2.

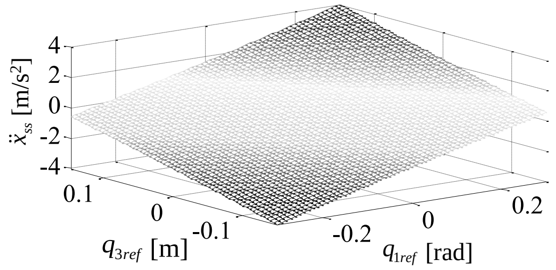


Fig. 3. Relationship between instability and translational acceleration

II. EQUATIONS OF MOTION AND ANALYSIS OF DYNAMICS

A. Equations of Motion

A diagram of the MIP is shown in Fig. 1. In this study, the shifting body means a part of the body on the slider and base body is one under it. M, m_w, and m_s represent masses of the base body, wheel, and shifting body respectively. J, J_w, J_s, and J_m are inertia of the base body, the wheel, the shifting body, and the rotor of the motor respectively. l, l_s, and l_t represent minimum distances between the center of the wheel and the COM of the base body, shifting body, and the slider respectively. r is wheel radius and n_m is reduction ratio of the actuator. Physical parameters of the system are shown in Table I. The values are selected based on an experimental setup of the MIP which is being developed in our laboratory. q_1 is the body angle with respect to the vertical direction, q_2 is the wheel angle relative to the body, and q_3 is the displacement of the slider. Its origin is set at the position where the MIP can keep the upright posture statically when $q_1 = 0$ rad. $x = r(q_1 + q_2)$ is the travel distance. $\mathbf{q} = [q_1 \quad q_2 \quad q_3]^T$ is the generalized position vector and g is the acceleration of gravity. The equations of motion of the MIP are described as follows:

$$\mathbf{M}_S\ddot{\mathbf{q}} + \mathbf{C}_S + \mathbf{G}_S = \mathbf{G}_I\mathbf{u} \tag{1}$$

$$\begin{aligned}\mathbf{M}_{S11} = {} & 2r\left(m_s l_s \cos q_1 + Ml\cos q_1 - m_s q_3 \sin q_1\right) + m_s q_3^2 \\ & + \left(M + m_s + m_w\right)r^2 + Ml^2 + m_s l_s^2 + J + J_s + J_w\end{aligned} \tag{2}$$

$$\begin{aligned}\mathbf{M}_{S12} = {} & r\left(m_s l_s \cos q_1 + Ml\cos q_1 - m_s q_3 \sin q_1\right) \\ & + \left(M + m_s + m_w\right)r^2 + J_w\end{aligned} \tag{3}$$

$$\mathbf{M}_{S13} = m_s\left(l_s + r\cos q_1\right) \tag{4}$$

$$\mathbf{M}_{S22} = \left(M + m_s + m_w\right)r^2 + n_m^2 J_m + J_w \tag{5}$$

$$\mathbf{M}_{S23} = m_s r \cos q_1 \tag{6}$$

$$\mathbf{M}_{S33} = m_h \tag{7}$$

$$\begin{aligned}\mathbf{C}_{S1} = {} & -2m_s r\dot{q}_1\dot{q}_3 \sin q_1 + 2m_s q_3\dot{q}_1\dot{q}_3 - Mlr\dot{q}_1^2 \sin q_1 \\ & - m_s r q_3 \dot{q}_1^2 \cos q_1 - m_s l_s r\dot{q}_1^2 \sin q_1\end{aligned} \tag{8}$$

$$\begin{aligned}\mathbf{C}_{S2} = {} & -2m_s r\dot{q}_1\dot{q}_3 \sin q_1 - Mlr\dot{q}_1^2 \sin q_1 \\ & - m_s r q_3 \dot{q}_1^2 \cos q_1 - m_s l_s r\dot{q}_1^2 \sin q_1\end{aligned} \tag{9}$$

$$\mathbf{C}_{S3} = -m_s q_3 \dot{q}_1^2 \tag{10}$$

$$\mathbf{G}_{S1} = -g\left(m_s q_3 \cos q_1 + m_s l_s \sin q_1 + Ml \sin q_1\right) \tag{11}$$

$$\mathbf{G}_{S2} = 0 \tag{12}$$

$$\mathbf{G}_{S3} = -m_s g \sin q_1 \tag{13}$$

$$\mathbf{G}_I = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{14}$$

$$\mathbf{u} = \begin{bmatrix} \tau & f \end{bmatrix}^T. \tag{15}$$

$\mathbf{M}_{Sij}$ denotes the i - j term of $\mathbf{M}_S$, and $\mathbf{C}_{Si}$ and $\mathbf{G}_{Si}$ denote the i -th element of $\mathbf{C}_S$ and $\mathbf{G}_S$ respectively. The potential energy of the system are

$$V = \left(M + m_s + m_w\right) gr + \left(Ml + m_s l_s\right) g \cos q_1 - m_s g q_3 \sin q_1 . \tag{16}$$

B. Relationship between Instability and Acceleration

Consider the case in which the body angle and the displacement of the shifting body converge to reference angle $q_1 = q_{1ref}$ and displacement $q_3 = q_{3ref}$, respectively. The following relations are obtained.

$$\ddot{q}_{2ss} = \frac{g}{\mathbf{M}_{S12}} \left\{ \left(Ml + m_s l_s\right) \sin q_{1ref} + m_s q_{3ref} \cos q_{1ref} \right\} \tag{17}$$

$$\tau_{ss} = \mathbf{M}_{S22} \ddot{q}_{2ss} \tag{18}$$

$$f_{ss} = \mathbf{M}_{S23} \ddot{q}_{2ss} - m_s g \sin q_{1ref} \tag{19}$$

where $\ddot{q}_2 = \ddot{q}_{2ss}$, $\tau = \tau_{ss}$, and $f = f_{ss}$ are constants. The relationship is graphically represented in Fig. 3. The steady translational acceleration $\ddot{x}_{ss} = r\ddot{q}_{2ss}$ can be controlled by applying the reference states q_{1ref} and q_{3ref} to the system. In our previous study[5], the slider mechanism was not introduced to the MIP and the body angle and the translational acceleration had a one-one relationship. When a large translational acceleration needs to be achieved, the body angle must be large, which is undesirable in terms of safe operation of the MIP. The problem can be solved by introducing the slider mechanism. A desired translational acceleration can be achieved with small q_{1ref} combined with the degrees of freedom of q_{3ref} .

III. Derivation of Port-Hamiltonian System

To achieve the concept, a nonlinear control method called IDA-PBC[6][7] is applied. The method shapes the total energy preserving port-Hamiltonian (PH) structure[8] of the system. Stabilization is achieved using passivity of the PH system. Passivity is an essential energetic property of physical systems. In general, control methods that utilize passivity are expected to be robust[9][10]. In addition, the IDA-PBC has been shown to have powerful stabilizing performance[6]. We show that the MIP with the slider can be appropriately described as a PH system and derive the IDA-PBC controller. Feedback linearization[11][12] which will be fragile to modeling errors, and linear approximation of the equations motion[13][14] which leads to guaranteeing stability at least in the neighborhood of an equilibrium are not used. Therefore, robustness and large domain of attraction are expected in our controller. The following equations can be derived by eliminating $\ddot{q}_2$ from the equations of motion (1).

$$\mathbf{M}_L \ddot{\mathbf{q}}_L + \mathbf{C}_L + \mathbf{G}_L = \mathbf{G}_{IL} \mathbf{u} \tag{20}$$

$$\mathbf{M}_L = \begin{bmatrix} \mathbf{M}_{S11} - \dfrac{\mathbf{M}_{S12}^2}{\mathbf{M}_{S22}} & \mathbf{M}_{S13} - \dfrac{\mathbf{M}_{S12}\mathbf{M}_{S23}}{\mathbf{M}_{S22}} \\ \mathbf{M}_{S13} - \dfrac{\mathbf{M}_{S12}\mathbf{M}_{S23}}{\mathbf{M}_{S22}} & \mathbf{M}_{S33} - \dfrac{\mathbf{M}_{S23}^2}{\mathbf{M}_{S22}} \end{bmatrix} \tag{21}$$

$$\mathbf{C}_L = \begin{bmatrix} \mathbf{C}_1 - \dfrac{\mathbf{M}_{S12}}{\mathbf{M}_{S22}} \mathbf{C}_2 & \mathbf{C}_3 - \dfrac{\mathbf{M}_{S23}}{\mathbf{M}_{S22}} \mathbf{C}_2 \end{bmatrix}^T \tag{22}$$

$$\mathbf{G}_L = \begin{bmatrix} \mathbf{G}_1 - \dfrac{\mathbf{M}_{S12}}{\mathbf{M}_{S22}} \mathbf{G}_2 & \mathbf{G}_3 - \dfrac{\mathbf{M}_{S23}}{\mathbf{M}_{S22}} \mathbf{G}_2 \end{bmatrix}^T$$

$$= \begin{bmatrix} -\left(Ml + m_s l_s\right) g \sin q_1 - m_s g q_3 \cos q_1 & -m_s g \sin q_1 \end{bmatrix}^T \tag{23}$$

$$\mathbf{G}_{IL} = \begin{bmatrix} -\dfrac{\mathbf{M}_{S12}}{\mathbf{M}_{S22}} & 0 \\ -\dfrac{\mathbf{M}_{S23}}{\mathbf{M}_{S22}} & 1 \end{bmatrix} \tag{24}$$

where $\mathbf{q}_L = \begin{bmatrix} q_1 & q_3 \end{bmatrix}^T$. We assume that the potential energy of the above system that is related to $\mathbf{G}_L$ as

$$V_L = \left(Ml + m_s l_s\right) g \cos q_1 - m_s g q_3 \sin q_1 . \tag{25}$$

We also consider $\mathbf{M}_L$ as the inertia matrix of the system (20) and calculate Euler-Lagrange equations of motion

$$L_L = \frac{1}{2} \dot{\mathbf{q}}_L^T \mathbf{M}_L \dot{\mathbf{q}}_L - V_L \tag{26}$$

$$\mathbf{Q}_L = \frac{d}{dt}\left(\nabla_{\dot{\mathbf{q}}_L} L_L\right) - \nabla_{\mathbf{q}_L} L_L . \tag{27}$$

We can check $\mathbf{Q}_L$ corresponds to the left-hand side of (20). Therefore, the system can be represented as a PH system because Euler-Lagrange systems are contained in PH systems[8].

$$\begin{bmatrix} \dot{\mathbf{q}}_L \\ \dot{\mathbf{p}}_L \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_2 \\ -\mathbf{I}_2 & \mathbf{0} \end{bmatrix} \begin{bmatrix} \nabla_{\mathbf{q}_L} H_L \\ \nabla_{\mathbf{p}_L} H_L \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{G}_{IL} \end{bmatrix} \mathbf{u} \tag{28}$$

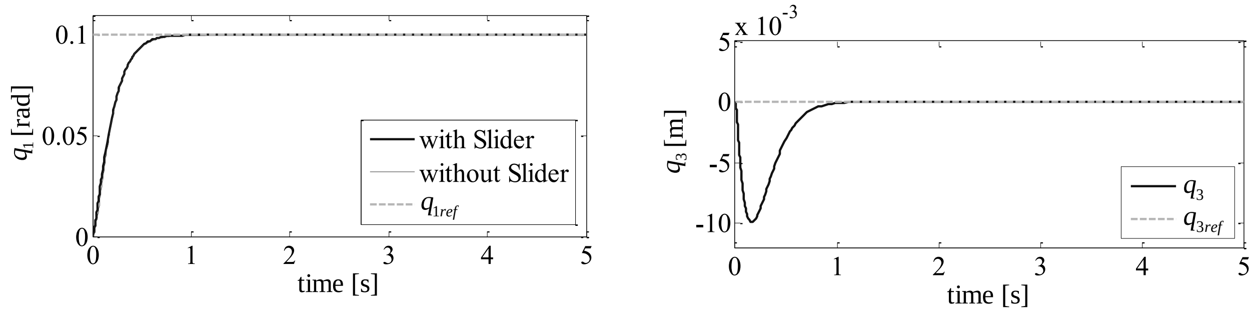
$$H_L = \frac{1}{2} \mathbf{p}_L^T \mathbf{M}_L^{-1} \mathbf{p}_L + V_L \tag{29}$$

where $\mathbf{p}_L = \mathbf{M}_L \dot{\mathbf{q}}_L$.

IV. IDA-PBC Controller Design

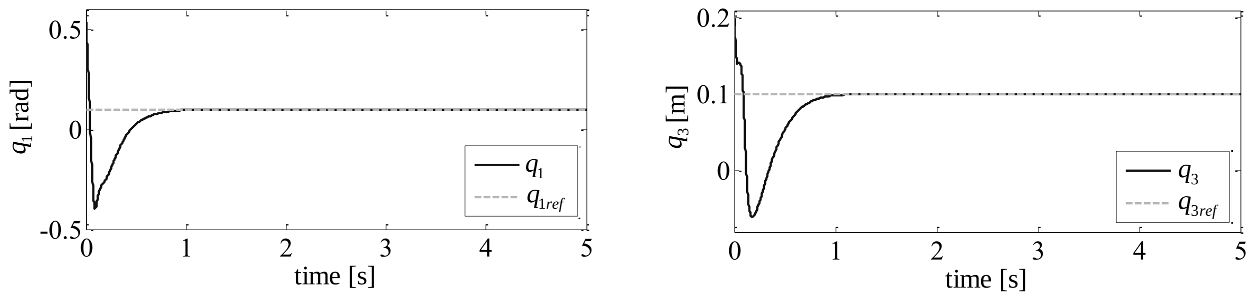
A. Derivation of Controller

In this study, the ranges of the body angle is considered in $|q_1| < \pi/2$ rad and the displacement of the slider in $|q_3| < 0.2$



(a) Body angle (b) Displacement of center of gravity

Fig. 4. Comparison of IDA-PBC controllers for systems with or without the slider mechanism



(a) Body angle (b) Displacement of center of gravity

Fig. 5. Tracking for constant reference states from the edge of the domain of attraction

m based on the experimental setup. $\mathbf{M}_L(\mathbf{q}_L) > 0$ in the above ranges can be checked numerically with the physical parameters of the MIP in Table I. We define the desired inertia matrix $\mathbf{M}_{Ld}$, the desired potential energy V_{Ld}, and the desired total energy H_{Ld}, which are going to be assigned to the PH system with the IDA-PBC controller. It can be represented as below using arbitrary H_{Ld} and a skew-symmetric matrix $\mathbf{J}_{2L}$ because the open-loop PH system is full-actuated and $\det(\mathbf{G}_{IL}) \neq 0$ [6].

$$\mathbf{u} = \mathbf{u}_{es} + \mathbf{u}_{di} \tag{30}$$

$$\mathbf{u}_{es} = \mathbf{G}_{IL}^{-1}\left(\nabla_{\mathbf{q}_L} H_L - \mathbf{M}_{Ld}\mathbf{M}_L^{-1}\nabla_{\mathbf{q}_L} H_{Ld} + \mathbf{J}_{2L}\mathbf{M}_{Ld}^{-1}\mathbf{p}_L\right) \tag{31}$$

$$\mathbf{u}_{di} = -\mathbf{K}_{Ldi}\mathbf{y}_{Lc} \tag{32}$$

where $\mathbf{K}_{di} = \mathbf{K}_{di}^T > 0$ is a parameter of the controller and $\mathbf{y}_{Lc}$ is the passive output of the closed-loop PH system. $\mathbf{u}_{es}$ is the energy shaping control input which shapes the total energy of the open-loop PH system and guarantees stability. $\mathbf{u}_{di}$ is called damping injection that is used to achieve asymptotic stability. The IDA-PBC controller (30) change the open-loop PH system into the closed-loop one.

$$\begin{bmatrix} \dot{\mathbf{q}}_L \\ \dot{\mathbf{p}}_L \end{bmatrix} = \left(\begin{bmatrix} \mathbf{0} & \mathbf{M}_L^{-1}\mathbf{M}_{Ld} \\ -\mathbf{M}_{Ld}\mathbf{M}_L^{-1} & \mathbf{J}_{2L} \end{bmatrix} - \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_{IL}\mathbf{K}_{Ldi}\mathbf{G}_{IL}^T \end{bmatrix} \right) \begin{bmatrix} \nabla_{\mathbf{q}_L} H_{Ld} \\ \nabla_{\mathbf{p}_L} H_{Ld} \end{bmatrix} \tag{33}$$

$$H_{Ld} = \frac{1}{2}\mathbf{p}_L^T\mathbf{M}_{Ld}^{-1}\mathbf{p}_L + V_{Ld} \tag{34}$$

$$\mathbf{y}_{Lc} = \mathbf{G}_{IL}\nabla_{\mathbf{p}_L} H_{Ld} = \mathbf{G}_{IL}\mathbf{M}_{Ld}^{-1}\mathbf{p}_L . \tag{35}$$

B. *Conditions for Stability*

Conditions to guarantee stability with the Lyapunov function H_{Ld} at least in the neighborhood of $(\mathbf{q}_L, \mathbf{p}_L) = (\mathbf{q}_{Lref}, \mathbf{0})$ are as follows:

$$\mathbf{M}_{Ld}(\mathbf{q}_{Lref}) > 0 \tag{36}$$

$$\mathbf{q}_{Lref} = \arg\min V_{Ld}(\mathbf{q}_L) \tag{37}$$

where $\mathbf{q}_{Lref} = \begin{bmatrix} q_{1ref} & q_{3ref} \end{bmatrix}^T$. The condition (37) can be interpreted as

$$\nabla_{\mathbf{q}_L} V_{Ld}(\mathbf{q}_{Lref}) = \mathbf{0} \tag{38}$$

$$\nabla_{\mathbf{q}_L}^2 V_{Ld}(\mathbf{q}_{Lref}) > 0 . \tag{39}$$

Under all the conditions above, we have

$$\dot{H}_{Ld} = -\mathbf{y}_{Lc}^T\mathbf{K}_{Ldi}\mathbf{y}_{Lc} \leq 0 . \tag{40}$$

Asymptotic stability is guaranteed when zero-state detectability is satisfied. The IDA-PBC controller has degrees of freedom in designing $\mathbf{M}_{Ld}$ and V_{Ld}. Various properties can be achieved that are difficult to realize with a linear controller.

C. *Design of Energy Function*

In this study, the kinetic energy is not going to be shaped, that is, $\mathbf{M}_{Ld} = \mathbf{M}_L > 0$, to verify basic properties of the proposed controller. The desired potential energy is designed as

$$V_{Ld} = \frac{1}{q_{1l}^2 - q_1^2}\cdot\frac{K_{p1}}{2}\left(q_1 - q_{1ref}\right)^2 + \frac{1}{q_{3l}^2 - q_3^2}\cdot\frac{K_{p3}}{2}\left(q_3 - q_{3ref}\right)^2 . \tag{41}$$

q_{1l} and q_{3l} are parameters which have to be set to restrict motion of the MIP in $|q_1| < q_{1l}$ and $|q_3| < q_{3l}$. K_{p1} and K_{p3}

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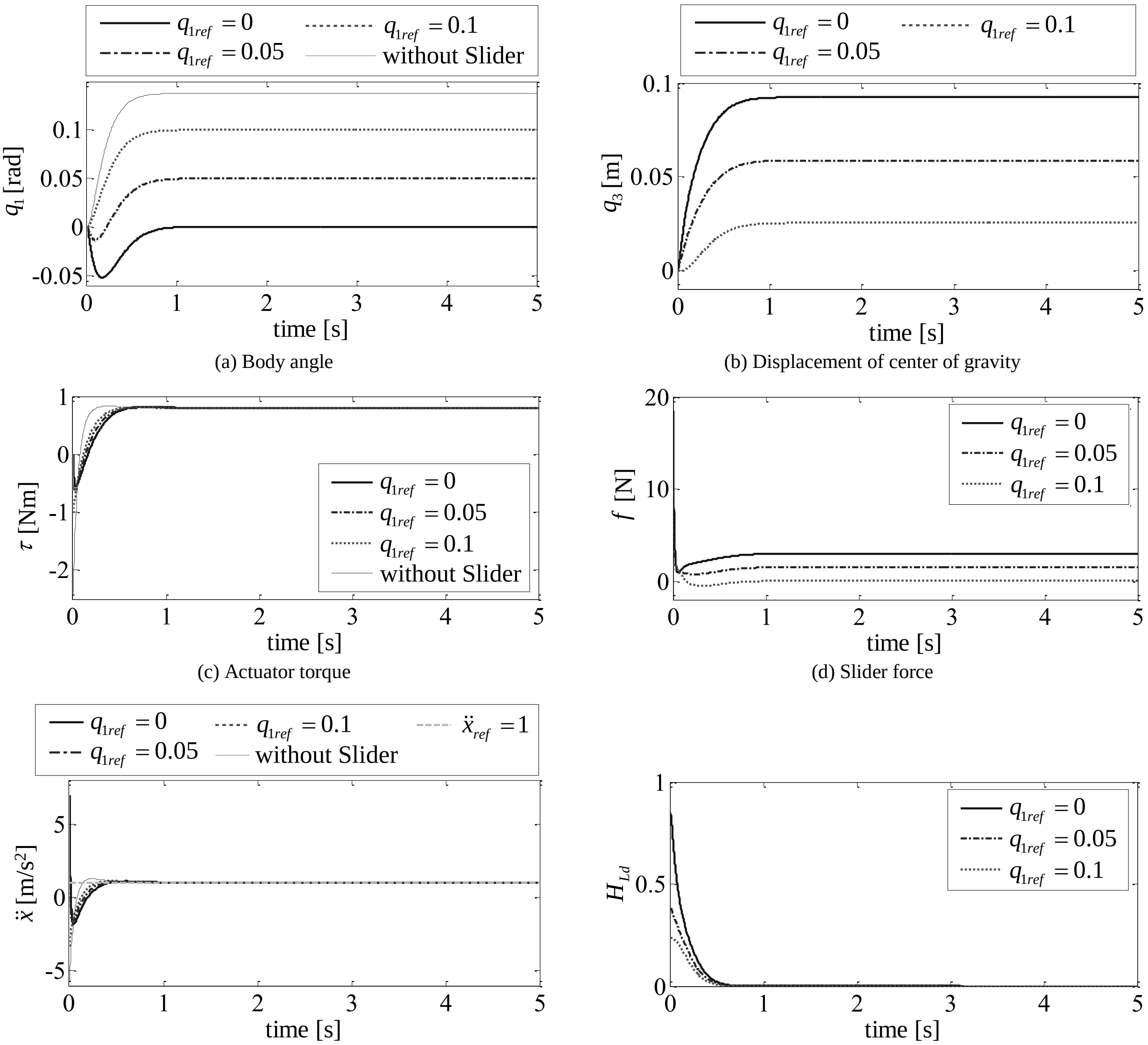

(a) Body angle (b) Displacement of center of gravity

(c) Actuator torque (d) Slider force

(e) Translational acceleration (f) Total energy of closed-loop PH system

Fig. 6. Simulation results of acceleration control

are positive constants. As long as initial conditions are set satisfying $|q_1| < q_{1l}$, $|q_3| < q_{3l}$ and reference states are also selected from the ranges, the proposed desired potential energy (41) satisfies the condition (37). V_{Ld} has the unique minimum at $(\mathbf{q}_L, \mathbf{p}_L) = (\mathbf{q}_{Lref}, \mathbf{0})$. Stability of the system is guaranteed with the Lyapunov function H_{Ld} . We can also check $\mathbf{y}_{Lc} \equiv \mathbf{0}$, $\mathbf{u}_{di} \equiv \mathbf{0} \Rightarrow \mathbf{G}_{lL} \nabla_{\mathbf{p}_L} H_{Ld} \equiv \mathbf{0} \Rightarrow \mathbf{p}_L \equiv \mathbf{0}$, and $\nabla_{\mathbf{q}_L} H_{Ld} \equiv \mathbf{0} \Rightarrow \nabla_{\mathbf{q}_L} V_{Ld} \equiv \mathbf{0} \Rightarrow \mathbf{q}_L \equiv \mathbf{q}_{Lref}$ using (33). Zero-state detectability is satisfied and asymptotic stability is guaranteed.

D. Properties of Proposed Controller

The desired potential energy V_{Ld} contributes the control input in the form of

$$\nabla_{\mathbf{q}_L} V_{Ld} = \left[K_{pv1} \left(q_1 - q_{1ref} \right) \quad K_{pv3} \left(q_3 - q_{3ref} \right) \right]^T \tag{42}$$

$$K_{pvi} = \frac{\left(q_{il}^2 - q_i q_{iref} \right)}{\left(q_{il}^2 - q_i^2 \right)^2} \cdot K_{pi} \qquad \left(i = 1,3 \right). \tag{43}$$

This is a kind of a proportional control using deviations $q_i - q_{iref}$ multiplied with the state-dependent gains $K_{pvi}(q_i)$. The gain property is designed based on our previous study[5]. If the body angle comes close to falling over beyond the reference angle, the gain increases and the controller actively returns the MIP from the dangerous states. A similar property is also achieved for the displacement of the slider. They are desirable in terms of safe operation of the MIP.

V. SIMULATION

In simulations, the proposed controller is compared with one from our previous study that deals with a MIP without the slider[5]. Effectiveness of the acceleration control using the intentional destabilization of the body angle and the displacement is verified. The control input (30) is applied to the system (1) in the following simulations. In all cases, initial values of the wheel angle and its angular velocity are zeros.

The parameters of the proposed controller are set as $q_{1l} = 0.55$ rad, $q_{3l} = 0.2$ m, $K_{p1} = 10.5$, $K_{p3} = 8$, $\mathbf{J}_{2L} = \mathbf{0}$, and $\mathbf{K}_{Ldi} = \text{diag}(0.6, 35)$. Although the IDA-PBC controller for the MIP without the slider has a structure different from that with the slider, we omit the detail in this study. Please refer to our previous study[5].

Fig. 4 compares basic performance of the two controllers.

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The initial conditions are all zeros. The reference body angle and displacement are $q_{1ref} = 0.1$ rad, $q_{3ref} = 0$ m. The controller for the MIP without the slider is applied to the system (1) with a constraint $q_3 \equiv 0$ m. Both controllers are designed to show similar responses in the body angles, as shown in Fig. 4 (a). In Fig. 4 (b), the shifting body slightly moves and converges to zero in about 1 s.

Theoretical restrictions of $|q_1| < q_{1l}$ and $|q_3| < q_{3l}$ are checked in Fig. 5. The initial conditions are set as $q_{10} = 0.54$ rad and $q_{30} = 0.19$ m, which are on the edges of the restriction ranges. To make the situation difficult, we set the initial body angular velocity $\dot{q}_{10} = \pi$ rad/s and the slider velocity $\dot{q}_{30} = 2$ m/s which are in the directions of coming closer to the edges. The reference states are $q_{1ref} = 0.1$ rad, $q_{3ref} = 0.1$ m. In Fig. 5, although the responses are precipitous, the two states converge to the references without violating the restriction.

Fig. 6 compares the two controllers with the reference translational acceleration $\ddot{x}_{ref} = \ddot{x}_{ss} = 1$ m/s^2. The initial conditions are all zeros. From (17) to (19), there are many combinations of q_{1ref} and q_{3ref} to achieve the given reference acceleration. Three cases of q_{1ref} are verified in Fig. 6. q_{3ref} is uniquely determined for each q_{1ref} . Fig. 6 (e) shows the translational acceleration converge to the reference in all cases. Fig. 6 (f) shows that H_{Ld} is monotonically non-increasing in all cases and plays the role of the Lyapunov function as is theoretically expected. In Fig. 6 (a), the case “without Slider” shows the largest maximum body angle, whereas it is suppressed in the other three cases by utilizing the slider. In Fig. 6 (b), the case “$q_{1ref} = 0$” shows the largest maximum displacement, which is about $q_3 = 0.9$ m. However, there is a margin against the restriction of $|q_3| < q_{3l} = 0.2$ m. The reference acceleration is reasonably achieved by utilizing the combination of the body angle and the displacement. The cases “$q_{1ref} = 0$” and “$q_{1ref} = 0.05$” in Fig. 6 (a) show undershoot. This will be undesirable for the rider of the MIP. Although controlling the MIP with the reference angle as the vertical direction has been very common in previous studies, the case “$q_{1ref} = 0.1$” does not show undershoot. The results indicate that utilizing instability effectively accelerates the MIP with the slider. The proposed controller can modify the motion of the body angle using the shifting mechanism appropriately. In Fig. 6 (c), the maximum input torques of the proposed controllers are more suppressed than previous one. This will prevent slipping when the MIP starts to accelerate. Fig. 6 (d) shows that the larger the reference angle, the smaller the maximum slider force. The acceleration control with the combination of inclining and shifting the body has the merit of reducing the required maximum torque and force of the actuators as well as modifying the motion of the MIP.

VI. Conclusion

An energy shaping nonlinear acceleration controller for a mobile inverted pendulum (MIP) with a slider mechanism is proposed. The controller accelerates the mobility utilizing instability of the system. The body angle and the slider displacement are controlled to keep states where the system is statically unstable, which leads to translational acceleration due to the instability of the MIP. Various controller properties can be achieved through the energy shaping procedure. Especially an energy function which will lead to safe operation of the MIP is proposed. The function ensures that the body angle and the slider displacement are restricted in predefined regions and converges to a desired state. The controller also returns the system back to the desired states with state-dependent gains that become large if that the system comes close to fall over. Effectiveness of the proposed method is verified in simulations. The results have indicated that motion of the MIP can be modified by the combination of intentional inclination and shift of the body. Required maximum input torque and force to accelerate the MIP are shown to be suppressed with the utilization of instability.

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