

Augmented Observer for Fault Detection and Isolation (FDI) in Rotor Systems

Zhentao Wang, Rudolf Sebastian Schittenhelm and Stephan Rinderknecht

Abstract—Disturbances and model uncertainties often have large impact on model based fault detection and isolation (FDI) processes and result in false alarm or reduced fault detection rate. In rotor systems excited by unbalances, the predominant disturbances (e.g. unbalance forces) and model uncertainties caused by gyroscopic effect appear in a sinusoidal form with rotor rotary frequency. If the influences of disturbances and model uncertainties are to be represented using unknown inputs, the signals of unknown inputs should also be sinusoidal. Augmented observers that account for sinusoidal unknown inputs are designed in this paper to take advantage of this characteristic of rotor systems. Different configurations of the augmented observer are investigated to estimate the distribution matrix of unknown inputs and for the residual generation under the consideration of unknown inputs. In case that the faults (e.g. rotor disc break) acting on the rotor are also in sinusoidal form the augmented observer can be used for fault isolation and identification.

Index Terms— fault detection, fault diagnosis, observers, rotors

I. INTRODUCTION

Model based fault detection and isolation (FDI) methods are widely used in technical processes due to good performance and accuracy. However, disturbances and model inaccuracies often have great impact on the FDI results. In rotor systems the FDI processes often face two major problems: the disturbances acting on the shaft e.g. unbalance forces are never known to full extent and the gyroscopic effect results in rotary frequency dependent system behavior.

In the last decades FDI methods dealing with disturbances and model uncertainties have been widely investigated [1], [2]. Most of the analytical methods decouple unknown inputs or attenuate the influence of disturbances / model uncertainties and enhance the influence of faults on the residuals by means of optimization.

Wantanabe and Himmelblau introduced the idea of

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unknown inputs observer (UIO) in [12]. Since then, different authors further developed the method and methods to describe disturbances and model uncertainties as unknown inputs are investigated. The UIO is able to estimate the system states and outputs accurately despite the presence of unknown inputs. The generated residuals are thus decoupled from unknown inputs. Besides the UIO approach, other methods such as eigenstructure assignment [6], [7] and null space based methods [9], [10] are developed for FDI with the aim to decouple unknown inputs in residual generation processes.

The methods introduced above are designed for the case that no information about the unknown inputs is available. In rotor systems excited by unbalances, the predominant disturbances (e.g. unbalances) are known to be sinusoidal with rotor rotary frequency. The influence of gyroscopic effect, which results in a rotary frequency dependent system behavior, also acts on the rotor system in a sinusoidal way. Instead of describing the gyroscopic effect as a linear time variant (LTV) term in the model, it can be represented by sinusoidal disturbance moments acting on the system [11]. Often the rotor rotary frequency can be measured, such that the frequency of the above mentioned disturbances is known at all times. Although the amplitude and phase of the disturbances are not known, the FDI processes can utilize the knowledge about the signal type of the disturbances.

Based on the normal Luenberger observer, different types of augmented observers, which account for sinusoidal unknown inputs or disturbances, are designed in this paper. Different configurations of the augmented observers to estimate unknown inputs distribution matrix and for residual generation under the consideration of unknown inputs are introduced. Isolation and identification of sinusoidal faults using augmented observers are presented.

II. MODELING

A. Modeling of test rig

The applicability of the observer configurations proposed in this paper is investigated by means of simulations. The model under consideration represents a test rig at the Institute for Mechatronic Systems in Mechanical Engineering at the Technische Universität Darmstadt. It consists of a low pressure shaft of a jet engine with a large disc eccentrically mounted on it, which replicates the turbine of the engine in its mechanical attributes.

The rotor is supported by a passive bearing and an active double bearing, where four piezoelectric actuators are installed for vibration control purposes. There are eight displacement sensors installed at the rig in four planes A, B,

C and D, see Fig 1.

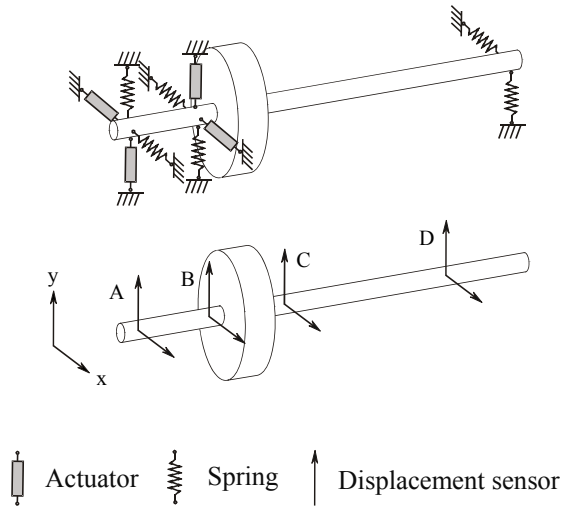


Fig 1. Structure of test rig

Due to the large disc mounted on the rotor, there are gyroscopic moments present. As a result, the system dynamics are dependent on the rotary frequency Ω . The governing differential equation in state space form reads:

$$\begin{aligned}\dot{x} &= A(\Omega)x + Bu + \tilde{E}\tilde{d} \\ y &= Cx,\end{aligned}\quad (1)$$

where x denotes the system states, u the control input, y the sensor signals and \tilde{d} the disturbances, i.e. unbalance forces. Viscous damping is introduced to the model by means of a uniform damping ratio of 0.8 % for all modes. The model (1) is reduced for the simulations in this article to an order of 16. The frequency dependent matrix $A(\Omega)$ can be approximated by

$$A(\Omega) \approx A + \Omega A_\Omega.\quad (2)$$

The frequency dependent part of $A(\Omega)$ is caused by the gyroscopic moments acting on the shaft. If the rotor runs at constant frequency, gyroscopic moments and unbalance forces work on the plant in a sinusoidal form. These moments and forces are considered as disturbances in this article. Thus the system simplifies to a linear time invariant system with extra disturbances:

$$\begin{aligned}\dot{x} &= Ax + Bu + Ed \\ y &= Cx,\end{aligned}\quad (3)$$

where $Ed = \tilde{E}\tilde{d} + \Omega A_\Omega x$ represents the effect of the disturbances and the gyroscopic moments on the system states.

B. FDI Model

For the FDI process the physical model (1) is supposed to be unavailable. Some limitations are considered in the FDI process according to the knowledge of the rotor system:

- The unbalance is never known to full extent, thus the term $\tilde{E}\tilde{d}$ in equation (1) is assumed to be unknown.
- If a physical model for the rotor is available, the gyroscopic effect can be modeled as in equation

(1). If the model is to be identified, the gyroscopic effect is hard to identify because of its LTV characteristic. Without loss of generality, it is assumed that gyroscopic effect is not modeled and only models at specific rotor rotary frequencies are assumed to be available.

The FDI process is based on the knowledge of the non-rotating rotor (i.e. $\Omega = 0$):

$$\begin{aligned}\dot{x} &= Ax + Bu + Ff \\ y &= Cx,\end{aligned}\quad (4)$$

where f is the faults to be detected and F is its input matrix. The major focus of this work is to detect sinusoidal input faults (e.g. rotor disk break), thus only input faults are considered in the model. In rotor system unbalances and gyroscopic effect can be presented as unknown inputs with an estimated distribution matrix [11]. Thus the model for the FDI process can be extended as:

$$\begin{aligned}\dot{x} &= Ax + Bu + Ed + Ff \\ y &= Cx,\end{aligned}\quad (5)$$

where d is the unknown input, which represents the influences of unbalances and gyroscopic effect and E is its distribution matrix. While the knowledge about the unknown input d is not necessary, its distribution matrix E has to be estimated first for the FDI process.

III. DESIGN OF AUGMENTED OBSERVER

The augmented observer introduced in this paper is based on the idea of disturbance observer [4] [5], which uses extra states in the system model to describe the influences and behavior of disturbances. If the disturbances can be described using a disturbance model:

$$\begin{aligned}\dot{x}_d &= A_d x_d \\ d &= C_d x_d,\end{aligned}\quad (6)$$

the augmented structure of system reads:

$$\begin{aligned}\dot{x}_B &= \begin{bmatrix} A & EC_d \\ 0 & A_d \end{bmatrix} x_B + \begin{bmatrix} B \\ 0 \end{bmatrix} u \\ y &= [C \quad 0] x_B = C_B x_B\end{aligned}\quad (7)$$

with the augmented system states vector:

$$x_B = [x^T, x_d^T]^T.\quad (8)$$

The matrices A_d and C_d are respective matrices for the disturbance model and the matrix E describes how the disturbances influence the plant. If the augmented model is built with unknown inputs, the matrix E is set to be unknown input distribution matrix.

It is assumed that the unknown inputs are a set of sinusoidal signals with different amplitude and phase angles but the same frequency i.e.:

$$d = \begin{bmatrix} \delta_1 \sin(\Omega t + \theta_1) \\ \delta_2 \sin(\Omega t + \theta_2) \\ \vdots \end{bmatrix},\quad (9)$$

where Ω is the frequency of the disturbances and $\delta_1, \delta_2 \dots$ and $\theta_1, \theta_2 \dots$ are the amplitude and phase angles of the unknown inputs. Two different structures can be used for the disturbance model:

If the disturbance states vector x_d in equation (6) is described as

$$x_d = [d^T, \hat{d}^T]^T, \quad (10)$$

a disturbance model can be built as:

$$\begin{aligned} \dot{x}_d &= \begin{bmatrix} 0 & I \\ -\Omega^2 I & 0 \end{bmatrix} x_d \\ d &= [I \quad 0] x_d. \end{aligned} \quad (11)$$

Another option for disturbance model uses a complementary vector of unknown inputs

$$\hat{d} = \begin{bmatrix} \delta_1 \cos(\Omega t + \theta_1) \\ \delta_2 \cos(\Omega t + \theta_2) \\ \vdots \end{bmatrix}. \quad (12)$$

The disturbance states vector is set as:

$$x_d = [d^T, \hat{d}^T]^T. \quad (13)$$

The disturbance model is then in the form of:

$$\begin{aligned} \dot{x}_d &= \begin{bmatrix} 0 & -\Omega I \\ \Omega I & 0 \end{bmatrix} x_d \\ d &= [I \quad 0] x_d. \end{aligned} \quad (14)$$

In control theory the disturbance observer is usually used for the disturbance attenuation and the estimated disturbances are fed back to the control input for disturbance compensation [3]. Thus the matrix E is often set equal to B to achieve disturbance compensation in input signals. For the FDI processes accurate influences of unknown inputs are to be estimated, thus the matrix E has to be determined.

A. Augmented observer for estimation of unknown input distribution matrix

In [8] a method to estimate the distribution matrix by means of augmented observer for slow changing unknown input signals (i.e. $\dot{d} \approx 0$) is presented. The method is based on an augmented system model in the form of:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{d}_1 \end{bmatrix} &= \begin{bmatrix} A & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ d_1 \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u \\ y &= [C \quad 0] \begin{bmatrix} x \\ d_1 \end{bmatrix}, \end{aligned} \quad (15)$$

where d_1 is called disturbance vector with $d_1 = Ed$ and $\dot{d}_1 \approx 0$. The estimated vectors $\hat{d}_1(k)$ for discrete time steps k are the result of a mapping of unknown inputs $d(k)$ by matrix E . The matrix E is estimated as a vector space in which all the vectors $d_1(k)$ lie. A precondition for this method is that the number of independent measurements is larger than or equal to the system order of original system i.e. $\text{rank}(C) \geq \text{rank}(A)$, otherwise d_1 is not observable. In real systems with high system order this requirement is usually not fulfilled thus this method is usually not applicable.

Based on the method above an augmented system model is designed for rotor system using the disturbance model for sinusoidal unknown inputs:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{x}_d \end{bmatrix} &= \underbrace{\begin{bmatrix} A & H_d \\ 0 & A_d \end{bmatrix}}_{A^*} \begin{bmatrix} x \\ x_d \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B^*} u \\ y &= \underbrace{[C \quad 0]}_{C^*} \begin{bmatrix} x \\ x_d \end{bmatrix}. \end{aligned} \quad (16)$$

The disturbance model (11) or (14) can be used for x_d . Since unknown inputs are the first half of x_d , H_d is set to:

$$H_d = [H \quad 0] \quad (17)$$

The choice of matrix H affects the observability condition of the augmented system (16). In ideal case it can be set to $H = I$ with the precondition that equation (16) is fully observable.

Lemma 1: If (A, C) is an observable pair and H has full column rank, equation (16) is observable only if

$$\text{rank}(H) \leq \text{rank}(C), \quad (18)$$

i.e. $\text{rank}(H)$ must be equal to or smaller than the number of linearly independent measurement.

Proof: Assume that $\text{rank}(A) = n$ and $\text{rank}(H) = p$. According to the observability criterion of Hautus [4] the system (16) is observable if and only if

$$\text{rank} \begin{pmatrix} \lambda_i I - A^* \\ C^* \end{pmatrix} = n + 2p \quad (19)$$

for all eigenvalues λ_i of $A^* \in \mathbb{R}^{(n+2p) \times (n+2p)}$. The system matrix A^* with

$$\det(\lambda_i I - A^*) = \det(\lambda_i I - A) \cdot \det(\lambda_i I - A_d) \quad (20)$$

has at least p pairs of eigenvalues at $\lambda_i = \pm i\Omega$ for $\det(\lambda_i I - A_d) = 0$. Thus in case of $\lambda_i = \pm i\Omega$, $\lambda_i I - A^*$ will be row rank deficient with $\text{rank}(\pm i\Omega I - A^*) \leq n + p$. Thus $\text{rank}(C)$ must be at least equal to $\text{rank}(H) = p$, in order to satisfy condition (19). \square

If there are less linearly independent measurements than the system order (i.e. $\text{rank}(C) < \text{rank}(A)$), the matrix H has to be chosen properly, so that the equation (16) is observable and the estimation of E is as accurate as possible. Considering that the frequency of unknown inputs is limited from 0 to maximal rotor rotary frequency in frequency domain and the high frequency modes normally have less influence than the low frequency ones on the system outputs. Thus influences of the modes correspondingly to the high frequencies can be approximately set to 0. Also, the modes corresponding to eigenvalues with negative imaginary parts are not excited to significant extent. As a result, they can also be neglected without introducing too much error. The matrix H can be chosen as:

$$H = [e_1, e_2, \dots, e_q], \quad (21)$$

with $q \leq \text{rank}(C)$ and e_1, e_2, \dots, e_q are eigenvectors corresponding to the eigenvalues with low positive imaginary parts.

A set of states vectors $x_d(k)$ for discrete time steps k can be observed using an observer on the basis of the augmented system model and disturbance vectors $d_1(k)$ can be calculated as

$$d_1(k) = H_d x_d(k). \quad (22)$$

Substituting disturbance model (11) or (14) into equation (7), the vector set $d_1(k)$ fulfills the relation

$$d_1(k) = Ed(k). \quad (23)$$

Note that also $E = H$ is mathematically a solution for this problem, but for the FDI purpose it is practically not applicable. Otherwise part of the faults would be represented by the unknown inputs and can thus not be detected. In order to achieve a high fault detection rate, the matrix E is to be estimated with fewer columns.

For N time steps of measurement, a set M of vectors $d_1(k)$ with can be calculated:

$$M = [d_1(1), d_1(2), \dots, d_1(N)]. \quad (24)$$

Using singular value decomposition M can be decomposed as:

$$M = U[\text{diag}(\sigma_1, \sigma_2, \dots, \sigma_n), 0]V^T. \quad (25)$$

The matrices U , V are left and right singular matrices and $\sigma_1, \dots, \sigma_n$ are the singular values with $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_n$. The desired low rank approximation of E is obtained by keeping a few of the most significant singular values i.e.:

$$E = U[\text{diag}(\sigma_1, \sigma_2, \dots, \sigma_p)], \quad (26)$$

with $p \leq n$ and $\sigma_1 \gg \sigma_{p+1} \geq \sigma_n$.

B. Augmented observer for FDI

For fault detection purpose an augmented observer based on the augmented model (7) in normal Luenberger observer form can be used. Since the unknown inputs are considered in the observer, the influences of unknown inputs are included in the estimated output \hat{y} . The generated residual:

$$r = y - \hat{y}, \quad (27)$$

is thus decoupled from the unknown inputs. In rotor system some of the faults (e.g. rotor disc break) are also in sinusoidal form. For the purpose of fault isolation and fault identification of sinusoidal faults, an augmented system model can be constructed with the structure:

$$\begin{aligned} \dot{x}_B &= \begin{bmatrix} A & [E \ F]C_d \\ 0 & A_d \end{bmatrix} x_B + \begin{bmatrix} B \\ 0 \end{bmatrix} u \\ y &= [C \ 0]x_B. \end{aligned} \quad (28)$$

The respective sinusoidal faults f can be observed directly using an augmented observer based on model (28). Note that the usage of the augmented observer for fault isolation and identification is often limited by the observability condition introduced above.

IV. APPLICATION

A. Estimation of matrix E

The functionality tests of the augmented observer for the estimation of unknown input distribution matrix are done at different constant rotor rotary frequencies. It is assumed that models at these frequencies are known including the gyroscopic effect. The unknown inputs are thus only from unbalance forces \tilde{d} in model (1). The matrix E is compared

with \tilde{E} in model (1) and is thus estimated with the same column number as \tilde{E} . For the evaluation, the matrices E and \tilde{E} are considered as vector spaces and the angles between the vector spaces are calculated.

The estimations are done at 10 equal distant rotor rotary frequencies that cover whole rotary frequency range. The poles of augmented observer are set in a region that is about 10 times left the fastest pole of the test rig model.

For the feasibility test, 8 extra sensors are added to the system. The total sensor number is then 16 (equals the system order) and the matrix H is set to $H = I$ for the augmented model (16). The augmented system is observable and the matrix E can be theoretically exactly estimated in this configuration. As expected, the estimations are very accurate, the angles between E and \tilde{E} are in the order of $10^{-6}\pi$.

For the test using the configuration of the test rig with 8 sensors the influences of eigenforms corresponding to the negative eigenfrequencies are neglected so that the augmented system is observable. The matrix H is chosen in the form of (21). As result, the estimates still show good accuracy. The angles between E and \tilde{E} lie around 0.0025π .

Test with 6 sensors are also done by neglecting the influences of eigenforms corresponding to negative eigenfrequencies and the 2 highest eigenfrequencies. The accuracy of the estimates is not acceptable (about 0.3π). Additional columns for the matrix E and the use of faster poles for the augmented observer increased the accuracy. The resulted angles between estimates of E with 6 columns and \tilde{E} are from 0.007π at low rotary frequency to 0.042π at high rotary frequency.

Note that the gyroscopic effect can be represented using periodical unknown input [11] and this estimation method is also suitable for the estimation of matrix E under consideration of gyroscopic effect. The estimation results with consideration of the gyroscopic effect are hard to evaluate using simple comparison and are thus not presented here. The estimate of E under consideration of gyroscopic effect is used in the FDI process of this paper.

B. Fault detection using augmented observer

For the FDI process the model (5) is used as basis model. To test the feasibility of this method, the matrix E is estimated with high accuracy on the basis of a 16-sensors configuration. At each of the 10 rotary frequencies a 6-column matrix E_m ($m = 1 \dots 10$) is estimated to represent both gyroscopic effect and unbalances. According to equations (24), (25) and (26) the estimates of E_m are proportional to the amplitude of unknown inputs. Since the rotor excitations (i.e. unbalance forces) used for the estimation are proportional to Ω^2 , the 10 E_m are weighted with $1/\Omega^2$ and then combined in E_M :

$$E_M = [E_1^*, E_2^*, \dots, E_{10}^*], \quad (29)$$

where matrices E_m^* are weighted matrices E_m . Using the singular value decomposition technique presented in (25) and (26), E_M is approximated by a 6-column E -matrix. This E -matrix is used for the whole rotary frequency range considered in this paper.

As an example, rotor disc break as fault is taken into consideration. The effect of this fault can be considered as an additional unbalance. Thus both unknown inputs and fault have the same frequency as rotor rotary frequency. Since faults of other frequencies as unknown inputs can be simply separated in frequency domain, it is of authors' interest to detect faults that have the same frequency as disturbances.

The FDI processes are based on the model (5) with 8 sensors. The measurements are simulated using the test rig model (1). For the evaluation of the results the frequency responses of both disturbances and fault on the residuals are considered. In observer based FDI, the effect of control inputs are compensated on the residual. According to the superposition principle the control inputs are thus not considered and set to 0 in the simulation.

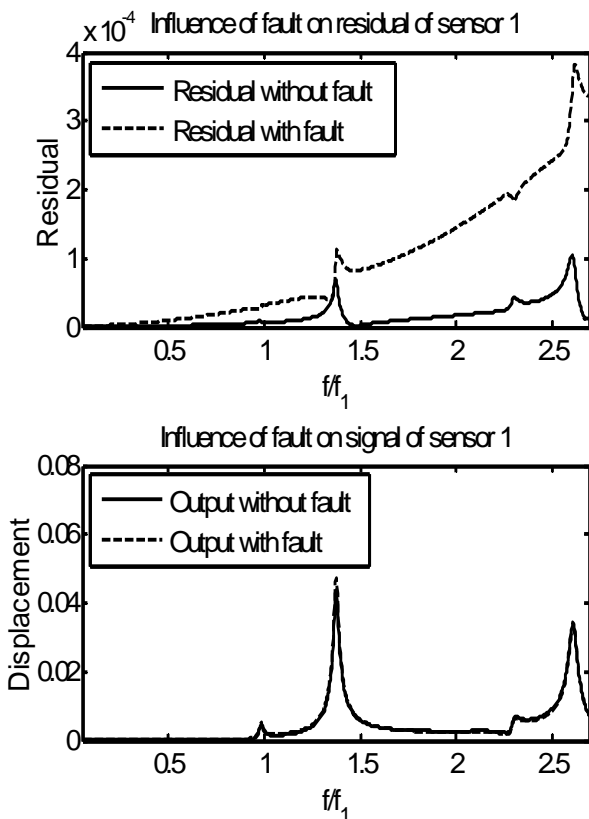


Fig 2. Influence of fault on output and residual

An augmented observer based on the augmented model (7) is built for the fault detection. A constant feedback term is designed for the augmented observer, which lead to a stable system in the whole frequency range. The frequency responses on sensor 1 for fault free case (only excited by disturbances) and in case of fault (excited by both disturbances and fault) are presented in Fig 2. Other sensors show similar results and are thus not presented. The frequency is normalized to the first eigenfrequency f_1 of standing still rotor and displacement and residual are also normalized values. The frequency response on signal of sensor 1 gives a brief overview about how strong the fault affects the system. The rotor disc break as fault is chosen in such a way, that its result is almost invisible on the output. It has to be pointed out, that without disturbance the fault results similar amplitude as disturbances on the output.

Because of the phase difference the resulted amplitude of disturbance and fault is similar as in fault free case.

It can be seen, that in general the fault has a much stronger influence on the residual as disturbances and can thus be simply detected. Low frequency response of the fault is observed near the rotary frequency $f = 1.4f_1$. This problem is caused by the usage of constant feedback term in observer. Since the augmented observer is dependent on rotor rotary frequency Ω . A constant feedback term can only be designed for a single rotary frequency. Using the pole placement technique, the poles at design point are set in a region that is 10 times faster than the fastest pole of the original system. For other rotary frequency Ω , the poles drift away from the designed region. At some Ω some of the poles of the augmented observer are even slower than the fastest pole of the system and are thus inefficient for fault detection. A frequency dependent feedback term might improve the performance and will be studied in the authors' future work.

C. Fault isolation and identification using augmented observer

As example for fault isolation and identification, the rotor system (1) at a constant rotary frequency is considered. It is assumed that a rotor model with gyroscopic effect (e.g. through identification) at this frequency is known. Except the fault from rotor disc (fault 1), an extra unbalance in the middle of the rotor is also considered as fault (fault 2). A 2-column matrix E is estimated to represent the influence of disturbances. Based on the augmented model (28), an augmented observer is constructed. The faults are observed in the corresponding states x_d of the augmented states vector x_B . The time domain diagnosis is presented in Fig 3.

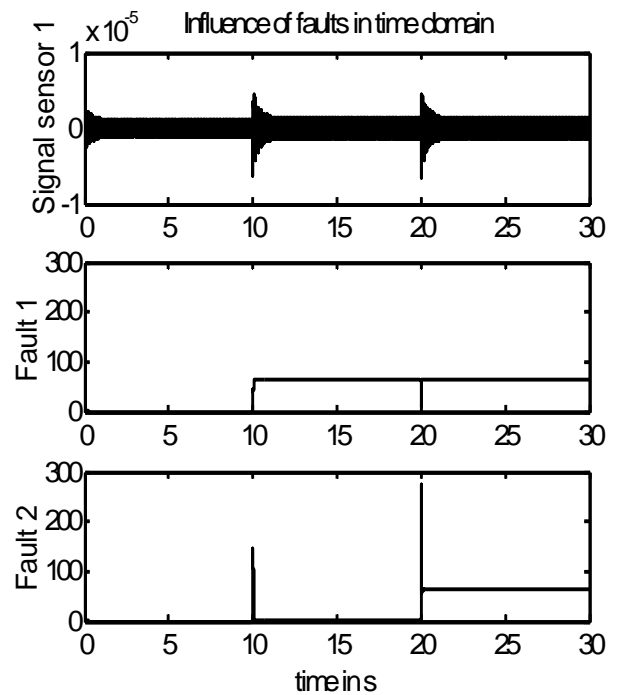


Fig 3. Observed fault amplitude

Fault 1 takes place at 10s and fault 2 takes place at 20s. The fault amplitude is to be understood as amplitude of unbalance forces. The sensor signal and fault amplitudes are

normalized values. A very good FDI result can be seen from Fig 3. The observed system is not in minimal phase. Thus large amplitudes are observed when faults take place.

V. CONCLUSION

Different methods to design augmented observers for FDI processes in rotor systems with sinusoidal disturbances are presented in this paper. The augmented observers can be used for the estimation of unknown input distribution matrix and for fault detection, isolation and also identification in rotor system. Restrictions are discussed and examples considering unbalances and gyroscopic effect as disturbances are presented.

The augmented observer is rotary frequency dependent. Although constant feedback term can be used for the observer design, the dynamics at design point cannot be held at other rotary frequencies. As further development of this method, a rotary frequency dependent feedback term can be developed to improve the performance.

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