Normal/Abnormal Gait Analysis based on the Statistical Registration and Modeling of the Frontal View Gait Data

Kosuke Okusa and Toshinari Kamakura

Abstract—**We study the problem of analyzing and classifying frontal view gait video data. In this study, we suppose that frontal view gait data as a mixing of scale changing, human movements and speed changing parameters. We estimate these parameters using the statistical registration and modeling on a video data. Our gait model is based on human gait structure and temporal-spatial relations between camera and subject. To demonstrate the effectiveness of our method, we conducted two sets of experiments, assessing the proposed method in gait analysis for young/elderly person and abnormal gait detection. In abnormal gait detection experiment, we apply K-nearest-neighbor classifier, using the estimated parameters, to perform normal/abnormal gait detect, and present results from an experiment involving 120 subjects (young person), and 60 subjects (elderly person). As a result, our method shows high detection rate.**

Index Terms—**Gait Analysis, Human Gait Modeling, Statistical Registration, Abnormal Gait Detection**

I. Introduction

We study the problem of analyzing and classifying frontal view gait video data. A study on the human gait analysis is very important in the field of the sports/health managements. For instance, gait analysis is one of the important method to detect the Alzheimer disease, infantile paralysis, and any other diseases [1], [2], [3].

Gait analysis is mainly based on motion capture system and video data. The motion capture system can give the precise measurements of trajectories of moving objects, but it requires the laboratory environments and we cannot be used this system in the field study. On the other hand, the video camera is handy to observe the gait motion in the field study.

Gage [4] had been proposed brain paralysis gait analysis using gait video data. Kadaba *et al.* [5] had been discussed importance of lower limb in the human gait using gait video data too. Many gait analysis have recently analyzing using video analysis software (e.g. Dartfish, Contemplas, Silicon Coach). For example, Borel *et al.* [6] and Grunt *et al.* [7] had been proposed infantile paralysis gait analysis using lateral view gait video data.

On the other hand, from the standpoint of statistics, Olshen *et al.* [8] proposed the bootstrap estimation for confidence intervals of the functional data with application to the gait cycle data observed by the motion capture system. Soriano *et al.* [9] proposed the human recognition method based on the data matching techniques using the dynamic time warping of human silhouettes.

However, most studies have not focused on frontal view gait analysis, because such data has many restrictions on analysis based on the filming conditions. The video data filmed from the frontal view is difficult to analyze, because the subject getting close in to the camera, and data includes the scale-changing parameters [10], [11]. To cope with this, Okusa *et al.* [12] and Okusa & Kamakura [13] proposed a registration for scales of moving object using the method of nonlinear least squares.

Okusa *et al.* [12] and Okusa & Kamakura [13] did not focus on the human leg swing. In this study, we focus on the gait analysis using arm and leg swing model estimated parameters. We suppose that frontal view gait data as a mixing of scale changing, human movements and speed changing parameter. We estimate these parameters using the statistical registration and modeling on a video data. Our gait model is based on the human gait structure and temporal-spatial relations between camera and human.

To demonstrate the effectiveness of our method, we conducted two sets of experiments, assessing the proposed method in gait analysis for young/elderly person and abnormal gait detection.

In young/elderly gait analysis experiment, we analyze young 120 subjects and elderly 60 subjects (normal gait: 46, abnormal gait: 14). We discuss the normal and abnormal gait features using the estimated parameters from proposed model. In abnormal gait detection experiment, we apply a K-nearest-neighbor (K-NN) classifier, using the estimated parameters, to perform abnormal gait detection, and present results from an experiment involving 180 subjects from young/elderly gait analysis. As a result, our method shows high detection rate.

II. Frontal View Gait Data

In this section, we describe an overview of frontal view gait data. Many of gait analysis using lateral view gait data, because lateral view gait is easy to detect the human gait features. However, in a corridor like structure, the subject is approaching a camera. Such case is difficult observe lateral view gait.

In a lateral view gait, at least two cycles or four steps are needed. For more robust estimation of the period of walking, about 8m is recommended. To capture this movement, the camera distance required is about 9m. Practically, having such a wide space is difficult. On the other hand, frontal view gait video is easy to observe 8m (or more) gait steps [11].

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Fig. 1. Frontal view gait data

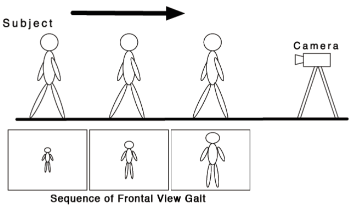

Fig. 2. Filming situation of frontal view gait data

Figure 1 is an example of frontal view gait data recorded by Figure 2 situation. Figure 1 illustrates difficulty of frontal view gait analysis. Even if subject do the same motion with the same timing, frontal view gait data is included scale changing components. Figure 3 shows subject width time-series behavior of frontal view gait data. This figure illustrates frontal view gait data contains many time-series components.

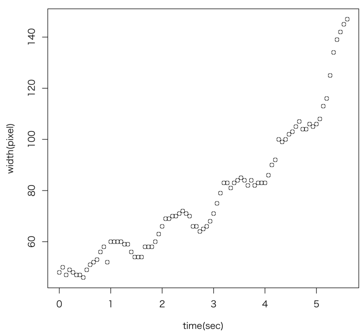

Fig. 3. Time-series behavior of frontal view subject width

III. Modeling of Frontal View Gait Data

A. Preprocessing

The raw video data is difficult to observe subject width and height time-series behavior because data contain background. We separate subject from background using inter-frame subtraction method (Eq. 1).

$$\Delta^{(T)} = |I^{(T+1)} - I^{(T)}|, T = 1, ..., (n-1),$$
$$\Delta^{(T)}(p,q) = \begin{cases} 1 & (\Delta^{(T)}(p,q) > 0) \\ 0 & (\text{Otherwise}). \end{cases} \quad (1)$$

Here, $\Delta^{(T)}$ is an inter-frame subtraction image, $I^{(T)}$ is grey scaled video data image at frame T, (p, q) is the pixel coordinate.

a) Subject Width/Height Calculation: Inter-frame subtraction method can separate the subject and background. However, it is difficult to measure the time-series behavior of the subject width and height. In this section, we describe the subject width and height calculation method using inter-frame subtraction data.

Let us suppose that inter-frame subtraction image is binary matrix. We can measure the subject height and width by integration calculation of row and column at each frame. In this study, we focus on the human gait arm and leg swing of the frontal view gait. We assume that subject width and height time-series behavior consist of the arm and leg swing behavior.

B. Relationship between camera and subject

Figure 4 shows a relationship between camera and subject. Width and height modeling has same structure. In this section, we describe the subject width modeling. We can assume simple camera structure. We consider the virtual screen exists between observation point and subject, and we define subject width on the virtual screen x_i at i-th frame($i = 1, ..., n$).

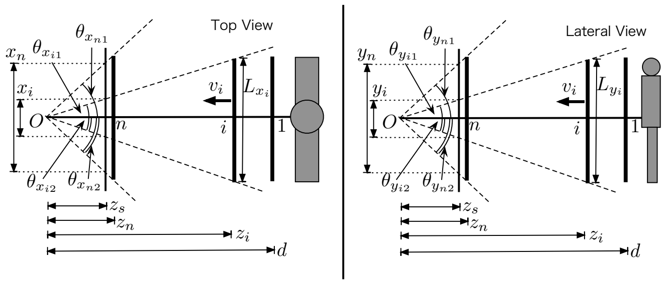

Fig. 4. Relationship between camera and subject

Here we define z_i, z_j as distance between observation point and subject at i-th, j-th frame, z_s as distance between observation point and virtual screen, $\theta_{x_{i1}}$, $\theta_{x_{i2}}$ as subject angle of view from observation point at i-th frame, d as distance between observation point and 1st frame, v_i as subject speed at i-th frame. Okusa *et al.* [12] defined the subject length L was constant. We assume that L has the time-series behavior and we define L_i is the subject length at i-th frame.

x_i at i-th frame depends on $\theta_{x_{i1}}$, $\theta_{x_{i2}}$ as shown in Figure 4.

$$x_i = z_s(\tan\theta_{x_{i1}} + \tan\theta_{x_{i2}}). \quad (2)$$

Similarly, the subject length at i-th frame is

$$L_{x_i} = z_i(\tan\theta_{x_{i1}} + \tan\theta_{x_{i2}}). \quad (3)$$

From Eq.(2), Eq.(3), ratio between x_n and x_i is

$$\frac{x_n}{x_i} = \frac{L_{x_n} z_i}{L_{x_i} z_n} \quad (4)$$

Frame interval is equally-spaced (15 fps). Okusa *et al.* [12] assumed the average speed is constant. We can assume that average speed from i-th frame is $(n-i) = (z_i - z_n)/\bar{v}$, therefore z_i is $z_i = z_n - \bar{v}(n-i)$. We substitute z_i to Eq.(4)

$$x_i = \frac{M_{x_i}\gamma}{\gamma + (n-i)} x_n + \epsilon_i, \quad (5)$$

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where γ is $z_n/\bar{v}$, M_{x_i} is L_{x_i}/L_{x_n}, ϵ_i is noise. From Eq.(5), predicted value $\hat{x}_i^{(n)}$ is registration from i-th frame's scale to n-th frame's scale

$$\hat{x}_i^{(n)} = \frac{\gamma + (n-i)}{M_{x_i}\gamma} x_i. \tag{6}$$

Similarly, we can define subject height as

$$y_i = \frac{M_{y_i}\gamma}{\gamma + (n-i)} y_n + \epsilon_i, \tag{7}$$

where M_{y_i} is L_{y_i}/L_{y_n}.

Next, we discuss the scale changing, human movement, and speed changing parameter estimation model.

C. Scale changing parameter estimation

From Eq.(5), scale parameter is γ. Solve Eq.(5) for γ shows

$$\gamma = \frac{x_i(n-i)}{x_i - M_{x_i} x_n}. \tag{8}$$

Here γ is the ungaugeable parameter, and we estimate it using nonlinear least squares method

$$S(\gamma, M_{x_i}) = \sum_{i=1}^{n} \left\{ x_i - \frac{M_{x_i}\gamma}{\gamma + (n-i)} x_n \right\}^2. \tag{9}$$

D. Human movement parameter estimation

M_{x_i} and M_{y_i} are movement model of the subject. If the subject is the rigid body, movement model M_{x_i} and M_{y_i} are constant. Meanwhile, human gait is not a constant. M_{x_i} and M_{y_i} needs the movement model because the subject body is moving wildly.

b) Human gait modeling: arm swing: Collins *et al.* [14] has reported that arm swing is an very important role in the gait motion based on the simple gait model. We consider the human gait modeling based on Collins *et al.* [14] model (see Figure 5).

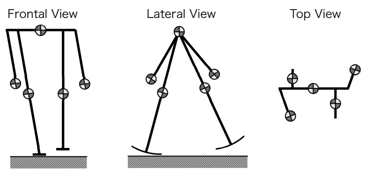

Fig. 5. Gait model

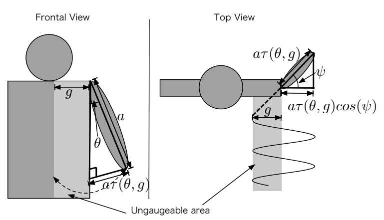

Fig. 6. Arm swing model

It seems reasonable to think that arm is single pendulum. Collins *et al.* [14] model assumed the arm swing is move to anteroposterior direction. Our model, on the other hand, can assume that arm swing move to an oblique direction (Figure 6).

Figure 6's model has an ungaugeable area. Our method's width/height calculation is based on integration calculation of row and column at each frame. If the arm move to inside body area, arm length is ungaugeable. Arm swing model is

$$\begin{aligned}
x_i &= \frac{\left(\frac{W(P_1,P_2,Q_1,Q_2,g_1,g_2,f,i)}{W(P_1,P_2,Q_1,Q_2,g_1,g_2,f,n)} + s\right)\gamma}{\gamma + (n-i)} x_n + \epsilon_i \\
&W(P_1,P_2,Q_1,Q_2,g_1,g_2,f,i) = \\
&P_1\tau(fi+Q_1,g_1) + P_2\tau(fi+Q_2,g_2) \\
&\tau(\theta,g) = \begin{cases} \sin(\theta)+g & (\sin(\theta)+g>0) \\ 0 & (\text{Otherwise}) \end{cases}
\end{aligned} \tag{10}$$

where $P_1 = a_1\cos(\psi)$ and $P_2 = a_2\cos(\psi)$. $P_1\tau(fi + Q_1, g_1)$ and $P_2\tau(fi + Q_2, g_2)$ are right and left arm model respectively. From Eq.(10), we estimate each gait parameter using nonlinear least squares method.

$$\begin{aligned}
&S(\gamma,P_1,P_2,Q_1,Q_2,g_1,g_2,f,s) = \\
&\sum_{i=1}^{n}\left\{x_i - \frac{\left(\frac{W(P_1,P_2,Q_1,Q_2,g_1,g_2,f,i)}{W(P_1,P_2,Q_1,Q_2,g_1,g_2,f,n)} + s\right)\gamma}{\gamma + (n-i)} x_n\right\}^2
\end{aligned} \tag{11}$$

Here, f is gait cycle frequency, s is adjustment parameter, P_1, P_2 are arm swing amplitude parameters, Q_1, Q_2 are arm phase parameters, and g_1, g_2 are ungaugeable area parameters.

c) Human gait modeling: leg swing: The leg swing modeling is simpler than arm swing model because the leg model does not have a ungaugeable area. Okusa *et al.* [12] and Okusa & Kamakura [13] does not consider the leg swing. It seems reasonable to think like arm swing that leg swing is single pendulum (Figure 7).

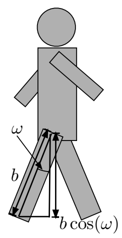

Fig. 7. Leg swing model

Leg swing model is

$$\begin{aligned}
y_i &= \frac{\left(\frac{H(b_1,Q_3,f,i)}{H(b_1,Q_3,f,n)} + s\right)\gamma}{\gamma + (n-i)} y_n + \epsilon_i \\
H(b_1,Q_3,f,i) &= b_1\cos(fi + Q_3).
\end{aligned} \tag{12}$$

Here b_1 is leg swing amplitude parameter, and Q_3 is leg phase parameter.

E. Speed changing parameter estimation

Frontal view video data is difficult to see the subject's speed. If our gait model is correct, observed value x_i and y_i is same as the fitted value of gait model at point ℓ_i. Previous model's ℓ_i assumes equally spaced ($\ell_i = i = 1, ..., n$). We estimate ℓ_{x_i} and ℓ_{y_i} value for minimize the observed value and model fitted value at ℓ_i. We can define estimated value ℓ_{x_i} and ℓ_{y_i} as a virtual space coordinate at i-th frame (Figure 8).

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Eq.5, Eq.7 with the coordinate estimation shows

$$x_i = \frac{M_{x_i}\gamma}{\gamma + (n - \ell_{x_i})} x_n + \epsilon_i$$
$$y_i = \frac{M_{y_i}\gamma}{\gamma + (n - \ell_{y_i})} y_n + \epsilon_i. \tag{13}$$

Here, $\ell_{x_i}, ..., \ell_{x_n}$ and $\ell_{y_i}, ..., \ell_{y_n}$ are virtual space coordinate parameters of width and height respectively. From Eq.13, arm swing and leg swing model with the coordinate estimation shows Eq.14, Eq.15.

$$x_i = \frac{\left(\frac{W(P_1,P_2,Q_1,Q_2,g_1,g_2,f,\ell_{x_i})}{W(P_1,P_2,Q_1,Q_2,g_1,g_2,f,\ell_{x_n})} + s\right)\gamma}{\gamma + (n - \ell_{x_i})} x_n + \epsilon_i$$
$$W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, \ell_{x_i}) =$$
$$P_1 \tau(f\ell_{x_i} + Q_1, g_1) + P_2 \tau(f\ell_{x_i} + Q_2, g_2)$$
$$\tau(\theta, g) = \begin{cases} \sin(\theta) + g & (\sin(\theta) + g > 0) \\ 0 & (\text{Otherwise}). \end{cases} \tag{14}$$

$$y_i = \frac{\left(\frac{H(b_1,Q_3,f,\ell_{y_i})}{H(b_1,Q_3,f,\ell_{y_n})} + s\right)\gamma}{\gamma + (n - \ell_{y_i})} y_n + \epsilon_i$$
$$H(b_1, Q_3, f, \ell_{y_i}) = b_1 \cos(f\ell_{y_i} + Q_3). \tag{15}$$

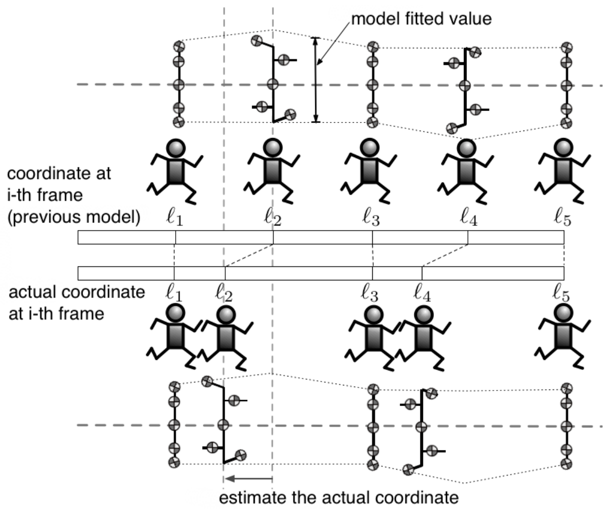

Fig. 8. Virtual space coordinate estimation

We suppose that virtual space coordinate of subject is $\hat{\ell}_i = (\hat{\ell}_{x_i} + \hat{\ell}_{y_i})/2$. Then, we can assume that subjects speed is 1st order difference of $\hat{\ell}_i$, and acceleration is 2nd order difference of $\hat{\ell}_i$.

We estimate these models parameters using Okusa & Kamakura [15] method. This method is very stable and very fast to estimate these parameters.

F. Gait parameter estimation

In this section, we discuss the gait parameter estimation.

Figure 9 is plot of the subject width(pixel) time-series behavior. Here dotted line represent fitted value of Eq.5 (scale variant estimation model), continuous line represent fitted value of Eq.10 (scale variant + arm movements estimation model), dashed line represent fitted value of Eq.14 (scale variant + arm movements + speed variant estimation model). Similarly, Figure 10 is plot of the subject height(pixel) time-series behavior. Here dotted line represent fitted value of Eq.7 (scale variant estimation model), continuous line represent fitted value of Eq.12 (scale variant + leg movements estimation model), dashed line represent fitted value of Eq.15 (scale variant + leg movements + speed variant estimation model).

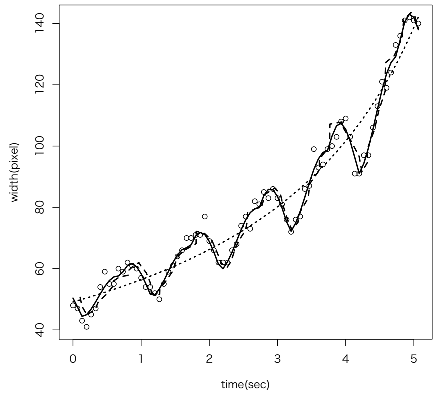

Fig. 9. Model fitting: Width

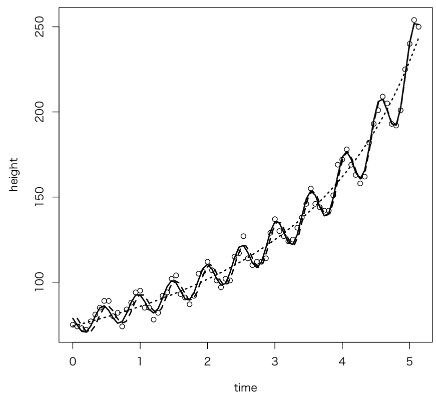

Fig. 10. Model fitting: Height

TABLE I
RSS, AIC, cAIC OF WIDTH DATA

	RSS	AIC	cAIC
1st order regression	6834.98	569.94	569.99
2nd order regression	3808.77	526.91	527.08
Eq.5 model	3077.40	508.50	510.66
Eq.10 model	425.63	372.17	374.86
Eq.14 model	189.46	463.84	-1032.56

TABLE II
RSS, AIC, cAIC OF HEIGHT DATA

	RSS	AIC	cAIC
1st order regression	21502.96	665.65	665.70
2nd order regression	7149.68	581.77	581.93
Eq.7 model	5248.84	555.66	557.82
Eq.12 model	631.11	406.43	409.08
Eq.15 model	401.15	519.09	-1804.91

Table I is RSS, AIC [16] and cAIC [17] value of 1st order regression, 2nd order regression, Eq.5, Eq.10 and Eq.14 models of width data. Table II is RSS, AIC and cAIC value of first order regression, second order regression, Eq.7, Eq.12 and Eq.15 models of height data.

In Table I, II, most minimal AIC model is Eq.10, Eq.12. Meanwhile, most minimal RSS and cAIC model are Eq.14, Eq.15. Burnham & Anderson [18] strongly recommend using cAIC, rather than AIC, if number of data n is small or number of parameters k is large. Since cAIC converges to AIC as n gets large, cAIC generally should be employed regardless. Therefore, we select the Eq.14, Eq.15 model. Figure 9, 10 and Table I, II illustrates our method has a good performance.

In next section, we discuss the effectiveness of our method.

IV. Experimental Details and Results

To demonstrate the effectiveness of our method, we conducted two sets of experiments, assessing the proposed method in gait analysis for young/elderly person and abnormal gait detection. We use SONY DCR-TRV70K camera. Frame rate of video data is 15 fps and resolution is 640×480.

In this paper, we focus on $\hat{\gamma}$ (speed parameter), $(\hat{P}_1 + \hat{P}_2)/2$ (width amplitude parameter), and $\hat{b}_1$ (height amplitude parameter).

A. Gait analysis: young person

In this experiment we took movie of 120 subjects walking video data from frontal view (10 steps, Male: 96 (average height: 173.24cm, sd: 5.64cm), Female: 24 (average height:156.25cm, sd: 3.96cm)) and apply to our proposed method for the gait analysis.

Figure 11 is plot of speed vs width amplitude vs height amplitude. Figure 12, 13 are width amplitude vs speed and height amplitude vs speed. The important point to note is that speed parameter $\hat{\gamma}$ is $z_n/\bar{v}$. If the subject walking fast, speed parameter $\hat{\gamma}$ is small. From Figure 11, 12, 13, width amplitude vs speed and height amplitude vs speed have a nonlinear relationship. This results means, if the subject's arm swing and leg swing moving strongly, subject's walking speed is fast.

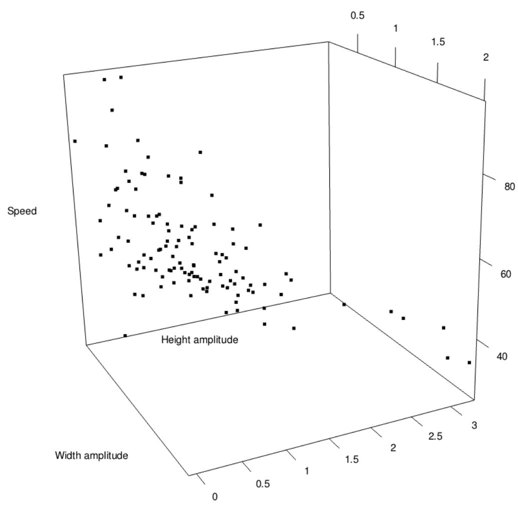

Fig. 11. Speed vs width amplitude vs height amplitude (young)

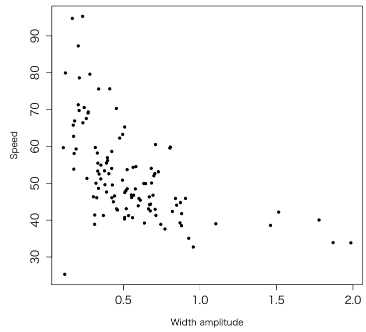

Fig. 12. Width amplitude vs speed (young)

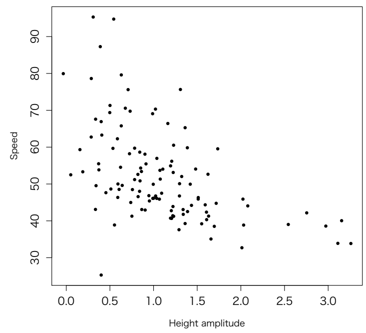

Fig. 13. Height amplitude vs speed (young)

B. Gait analysis: elderly person

In this experiment we took movie of 60 subjects walking video data from frontal view (10 steps / average age: 76.97・sd: 4.16 / abnormal gait subjects; average age: 77.56・sd: 4.35 / normal gail subjects; average age: 75.37・sd: 3.18) and apply to our proposed method for the gait analysis.

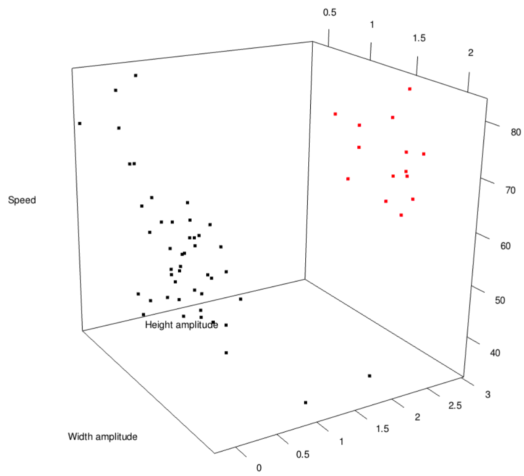

Fig. 14. Speed vs width amplitude vs height amplitude (elderly)

Figure 14 is plot of speed vs width amplitude vs height amplitude. Black and red dot means normal and abnormal gait subjects respectively.

Figure 15, 16 are width amplitude vs speed and height amplitude vs speed. Black and red dot means normal and abnormal gait subjects respectively. From Figure 14, 15, 16, normal gait subject width amplitude vs speed and height amplitude vs speed have a nonlinear relationship like a young person. However, on the other hand, abnormal gait subject estimated parameters does not have nonlinear relationship. These abnormal gait parameters clustered in different place from normal gait subjects. The result leads to our presumption that the abnormal gait subject trying to moving fast, but this effort is not effective to moving speed.

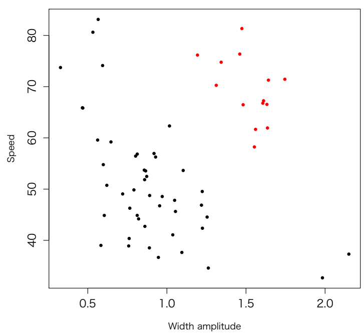

Fig. 15. Width amplitude vs speed (elderly)

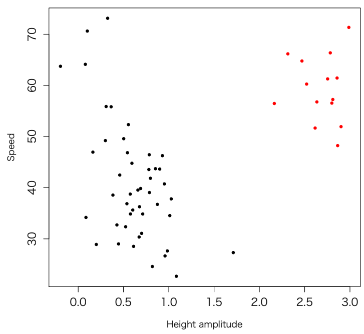

Fig. 16. Height amplitude vs speed (elderly)

C. Abnormal gait detection

In this section, we apply K-NN classifier (K=3), using the estimated parameters, to perform normal/abnormal gait detect, and present results from an experiment involving 120 subjects (young person), and 60 subjects (elderly person).

To evaluate our estimated parameters, we apply these parameters to leave-one-out cross-validation test. Table III is average detection rate of normal/abnormal gait. Table III shows our estimated parameters may be used for the normal/abnormal gait detection.

TABLE III
NORMAL/ABNORMAL GAIT AVERAGE DETECTION RATE(%)

	Normal Gait	Abnormal Gait
Normal Gait	98.2	1.8
Abnormal Gait	0	100

V. CONCLUSIONS

In this article, focusing on the human gait cycles, we consider the human gait modeling based on simple gait structure. We estimate the parameters of the human gait cycles using the method of nonlinear least squares. We also show that estimated parameters may be used for the human gait analysis and abnormal gait detection. Experimental results verify that our model can estimate the various parameters and these parameters are good feature values of the human gait. We plan to implement this scheme for the sports analysis of the long-distance runner.

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