A New Power System Oscillation Type Identification Method Based on Empirical Mode Decomposition and Hilbert Transform

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***Abstract*—A new method based on empirical mode decomposition and Hilbert transform to identify power oscillation types is proposed. The method utilizes and amplifies the discriminative difference of the instantaneous amplitude changing rules of the oscillation. By repeating EMD and square calculation alternately n times we can find the damp factor has been amplified by a factor of nth power of 2. So the distinction degree of different oscillation types becomes more obviously. We obtain the instantaneous amplitude by Hilbert transform, approximate it by two different expressions and analyze the correlation between the approximate results and the initial data. Finally, we determine the oscillation types according to the goodness of fit and correlation coefficients. The method is validated in actual oscillation incidents in China Southern Power Grid.**

***Index Terms*—Empirical mode decomposition, Hilbert transform, forced oscillation, weak damping oscillation, oscillation type identification**

I. INTRODUCTION

POWER oscillation has become a major problem threatening the security of large-scale interconnected power systems [1]. Power oscillation is divided into two different types according to different intrinsic inducements: free oscillation (FRO) and forced oscillation (FOO). Free oscillation is further divided into positive, zero and negative damp free oscillation. Zero and near-zero negative damp free oscillation are called weak damping oscillation (WDO) in this paper. Forced oscillation is further divided into positive, zero and negative damp forced oscillation. Practically, positive damping forced oscillation is typical forced oscillation, so the forced oscillation in this paper specifically refers to positive damping forced oscillation. Weak damping oscillation is induced by some poorly or negatively damping generators [2]-[4]. Forced oscillation is excited by external periodic disturbances [5]-[7]. A method based on hybrid dynamic simulation for disturbance source location is proposed in [8]. Energy-based methods have been proposed to locate the sources of weak damping oscillation and forced oscillation using wide area measurement system (WAMS) data when oscillation occurs [9]-[11]. After the oscillation sources being located, the key problem is to take effective emergency control actions to suppress the oscillation. For the weak damping oscillation the common control action is reducing the located generators' output powers and for the forced oscillation that is tripping the located generators. Obviously, the control actions are different for the different oscillation types. However what the oscillation type is should be identified firstly. Literature [12] presents a second order differential method to identify the power oscillation properties based on the initial period of wave. Taking the differences in response components and their oscillatory characteristics as criteria, a method for discrimination of free oscillation and forced oscillation is proposed in [13]. The methods proposed in [12] and [13] both adopt the damp factor of oscillation mode as the key discriminative information to identify the oscillation types. However in some cases the damping factors of the oscillation are so near to zero that it is hard to identify the oscillation types accurately or right. The methods proposed in [12] and [13] identifies the power oscillation types based on the initial period of wave, but sometimes initial period of wave may be unavailable due to some reasons and thus the methods are unfeasible. Because one weak (near zero) damping oscillation mode and one positive damping oscillation mode are often excited and exist in the initial period of wave at the same time, the hypothesis in [13] that free oscillation have single oscillation mode and forced oscillation have one positive damping oscillation mode and one weak (near zero) damping oscillation mode in the initial period of wave is not reasonable, and thus the method in [13] would mistakenly identify oscillation types in conditions that weak and positive damping oscillation mode are excited. Because of the damping ratio identification error and unreasonable threshold selecting in [13], the method in [13] may be invalid too. To deal with these problems, this paper proposes a method which adopts many times of the damp factors as key discriminative information to improve discrimination and doesn't not use the initial period of wave and any threshold to identify the oscillation types.

The paper is organized as follows. Section II introduces the mathematical tools used in the proposed method to identify power oscillation types in this paper. These tools includes Empirical Mode Decomposition (EMD), Hilbert Transform (HT) and Correlation Analysis (CA). Section III proposes a method to identify power oscillation type in power systems using WAMS data. The method is further developed to a systemic implementation process for practical applicability. Section V presents test results in a simple test to

Manuscript received June 28, 2014; revised July 21, 2014. This work is supported by the National Natural Science Foundation of China (51177079, 51321005).

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illustrate the distribution of the method and test results in actual oscillation incidents to demonstrate the validity of the method. Section VI is the conclusions.

II. Mathematical Tools

A. Empirical Mode Decomposition

The empirical mode decomposition assumes that any data consists of different simple intrinsic modes of oscillation. Adopting the sifting process in [14], EMD decomposes the sample data into n-intrinsic mode functions (IMF) and a residue which can be either the mean trend or a constant. The decomposition finally obtains

$$x(t)=\sum_{i=1}^{n}c_i(t)+r(t) \tag{1}$$

Where $x(t)$ is the sample data, $c_i(t)$ is *i*th intrinsic mode function and $r(t)$ is the residue.

If the sample data includes several oscillation modes, the IMF results are physically meaningful: *i*th IMF is corresponding to the *i*th oscillation mode. Thus we can use EMD eliminates the trend and decompose the modes apart. Finally, we get a single mode

$$x(t)=A_i e^{\sigma_i t}\sin(\omega_i t+\varphi_i) \tag{2}$$

In the method illustrated in section III, we use EMD to get oscillation modes. EMD is repeatedly used to get the oscillation component of the data processed by the method in section III.

B. Hilbert Transform

Hilbert transform (HT) of $x(t)$ is defined as [14]

$$\hat{x}(t)=\frac{1}{\pi}x\int_{-\infty}^{+\infty}\frac{x(\tau)}{t-\tau}d\tau \tag{3}$$

With the Hilbert transform, the analytic signal is defined as

$$z(t)=x(t)+j\hat{x}(t)=A(t)e^{i\theta(t)} \tag{4}$$

The instantaneous amplitude of $x(t)$ is calculated by

$$A(t)=\left[x(t)^2+\hat{x}(t)^2\right]^{\frac{1}{2}} \tag{5}$$

The Hilbert transform is used to get the instantaneous amplitude of the oscillation modes and the instantaneous amplitude of the oscillation component of the data processed by the method in section III.

C. Correlation Analysis

Correlation analysis of two data series can describe their similarity. Correlation coefficient of data series x and y is defined as [15]

$$R_{xy}=\frac{\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\overline{x})^2}\sqrt{\sum_{i=1}^{n}(y_i-\overline{y})^2}} \tag{6}$$

Where

$$\overline{x}=\frac{1}{n}\sum_{i=1}^{n}x_i \tag{7}$$

$$\overline{y}=\frac{1}{n}\sum_{i=1}^{n}y_i \tag{8}$$

Where x_i is the *i*th sample point of data series x and y_i is the *i*th sample point of data series y.

The value range of R_{xy} is between -1 and +1. The more $|R_{xy}|$ is close to 1, the correlation is more strong between x and y. Plus sign and minus sign before R_{xy} indicate positive and negative correlation respectively. Zero value of R_{xy} means x and y are independent.

We utilize correlation coefficient to determine the similarity between instantaneous amplitude and its fitting equation in the proposed method to identify power system oscillation types in section III.

III. Practical Method for Oscillation Type Identification

A. Theory of the Method

If the system have weak damping mode, some small disturbance will excite the oscillation mode easily, and then system operators should take measures to improve the damping. So before the forced oscillation happens the damping is often good. So only the weak damping oscillation and positive damping forced oscillation are considered in oscillation type identification in this paper. Power system oscillation is always including multi-modes as in (9). But usually only one main mode whose damping is weakest among the modes excited dominates the oscillation. After the attenuation of positive damping oscillation modes, only the weak damping mode or the forced response called main oscillation mode remains. For free oscillation the main mode has form as (10) and for forced oscillation that has form as (11). The amplitude of (10) changes exponentially with certain directional trend. The amplitude of (11) keeps constant in ideal condition or fluctuation near a constant in ideal condition considering noises, anyway it changes without any directional trend.

$$x(t)=\sum_{i=1}^{n}A_i e^{\sigma_i t}\sin(\omega_i t+\varphi_i) \tag{9}$$

$$x(t)=Ae^{\sigma t}\sin(\omega t+\varphi) \tag{10}$$

ISBN: 978-988-19252-0-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$x(t) = A\sin(\omega t + \varphi) \tag{11}$$

We intend to choose the information whether the amplitude changes with certain directional trend as key discriminative difference to identify the oscillation types. But the difference existing in the raw oscillation data for the two type oscillation is so small that it cannot be used directly to identify the oscillation types accurately. So we propose a new method illustrated as following to amplify the difference by alternately using EMD and square calculation to process the raw oscillation data.

We can decompose oscillation data into *n*-intrinsic mode functions (IMF) and a residue and then extract the instantaneous amplitude of every IMF by Hilbert transform. Choose the IMF whose instantaneous amplitude is the biggest as the main oscillation mode and then normalize it by dividing it by its instantaneous amplitude at *t*=0.

For weak damping oscillation, the main oscillation mode $c_m^{'}(t)$ has form

$$c_m^{'}(t) = e^{\sigma t}\sin(\omega t + \varphi) \tag{12}$$

Square $c_m^{'}(t)$ and multiply the result by two, then we obtain

$$2(c_m^{'}(t))^2 = e^{2\sigma t}(1 - \cos(2\omega t + 2\varphi)) \tag{13}$$

Abstract oscillation component from (13) by MED and then we obtain

$$x(t,1) = -e^{2\sigma t}\cos(2\omega t + 2\varphi) \tag{14}$$

Repeating square calculation in (13) and EMD in (14) alternately *n* times, finally we obtain

$$x(t,n) = (-1)^n e^{2^n \sigma t}\cos(2^n \omega t + 2^n \varphi) \tag{15}$$

The instantaneous amplitude of oscillation data processed in (15) is

$$A(t) = e^{2^n \sigma t} \tag{16}$$

The damping factor and oscillation frequency of oscillation have been amplified by a factor of *n*th power of 2. The exponentially directional changing trend of the instantaneous amplitude of oscillation has also been amplified and has become even steeper.

For forced oscillation, the main oscillation mode $c_m^{'}(t)$ has form

$$c_m^{'}(t) = \sin(\omega t + \varphi) \tag{17}$$

Repeating square calculation and EMD alternately *n* times, finally we obtain

$$x(t,n) = (-1)^n \cos(2^n \omega t + 2^n \varphi) \tag{18}$$

The instantaneous amplitude of oscillation data processed in (18) is

$$A(t) = 1 \tag{19}$$

The oscillation frequency of oscillation has also been amplified by a factor of nth power of 2. But instantaneous amplitude of oscillation keeps constant not having any certain directional changing trend because the damping factor is zero in (17).

The above analysis results are obtained in the ideal conditions. Factually, the practical oscillation data contains disturbances or noises. Thankfully, the disturbances or noises are generally irregular and nondirective, therefor they will not impact the directional changing characteristic of the instantaneous amplitude of the oscillation. So in the actual conditions, (16) and (19) have the general forms respectively as (20) and (21)

$$A(t) = ae^{bt} + c \tag{20}$$

$$A(t) = a\sin bt + c \tag{21}$$

We can use the difference in the general form of instantaneous amplitude of the oscillation processed to determine the oscillation type.

First, extract the instantaneous amplitude of (15) for weak damping oscillation or (18) for forced oscillation and fit it by expression (20) and (21) respectively, finally we get the curve fitting result $A(t)$ and the goodness of fit: sum of squares due to error (SSE), root mean squared error (RMSE) and coefficient of determination (CD).

Second, calculate the correlation coefficient *R* between instantaneous amplitude and its fitting expressions (20) and (21).

Third, if SSE and RMSE for fitting expression (20) are smaller than that for fitting expression (21) and meanwhile CD and R for fitting expression (20) is bigger than that for fitting expression (21), we know that fitting expression (20) is more similar to instantaneous amplitude and can conclude that the oscillation type is weak damping oscillation. If SSE and RMSE for fitting expression (20) are bigger than that for fitting expression (21) and meanwhile CD and R for fitting expression (20) is smaller than that for fitting expression (21), we know that fitting expression (21) is more similar to instantaneous amplitude and can conclude that the oscillation type is forced oscillation.

When the practical oscillation incidents occur, system operators would take actions to restrain the oscillation which may change the oscillation modes or forced disturbances, and thus the oscillating curve would not have the general form as (10) having a fixed mode or (11) having continuous forced disturbance. Before the oscillating curve decaying obviously, these changes about oscillation modes or forced disturbances are so little that the existing oscillation didn't change its property and we can use the oscillating curve to identify the oscillation types.

ISBN: 978-988-19252-0-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

B. Determination of Repeating Times n

After repeating EMD and square calculation alternately *n* times, according Shannon theorem we have

$$2^n f_o < \frac{1}{2} f_s \tag{22}$$

Where f_o is the oscillation frequency, and f_s is the sample frequency.

And then the maximum of *n* satisfies

$$n_{\max} \le \log_2 f_s - \log_2 f_o - 1 \tag{23}$$

The PMU data sampling frequency is 100 Hz and the oscillation frequency in power system is 0.1～2.5Hz. So in practical application we choose *n*=5 in section IV.

C. Systemic Implementation Process of the Method

Active power has high measurement accuracy and good observability for electromechanical oscillation. We choose active power of generators to identify oscillation types. The sample data contains noises therefor we need filtering processing. The oscillation has multi-modes therefor we need separate main oscillation mode from sample data by EMD. The aim of identifying the oscillation type is to take right action to eliminate the oscillation, so we also need locate the oscillation source in advance. The systemic implementation process of the method proposed above is as following.

1) Locate the oscillation sources using the method in [11] and choose these generators' active power as input data for oscillation type identification. The input data are written as $x(t)$.
2) Filter $x(t)$ by band-pass filter to eliminate the noises and the components beyond the frequency range of the electromechanical oscillation. Then we obtain $x^{'}(t)$.
3) Abstract all IMFs from $x^{'}(t)$ by EMD. Extract the instantaneous amplitude $A_i(t)$ of $c_i(t)$ by Hilbert transformation for i from 1 to m. Choose $c_i(t)$ whose $A_i(t)$ is the biggest as the main oscillation mode $c_m(t)$ whose instantaneous amplitude is $A_m(t)$. Normalize $c_m(t)$ to $c_m^{'}(t,j)$ by dividing $c_m(t)$ by $A_m(0)$, $j=0$,
4) Calculate 2 $(c_m^{'}(t,j))^2$ and abstract its oscillation component $c_m^{'}(t, j+1)$, $j = j+1$
5) Repeat 4) *n* times, then we get $c_m^{'}(t,n)$. Here $c_m^{'}(t,n)$ denotes simply as $y(t,n)$.
6) Extract the instantaneous amplitude $A(t)$ of $y(t,n)$ by Hilbert transformation. Fit $A(t)$ using expression as (20) and (21) and then we get fitting results $A_e(t)$ and $A_l(t)$ respectively. At the same time we obtain the goodness of fit: SSE, RMSE and CD. Calculate the correlation coefficient R between $A(t)$ and the fitting results.
7) Identify the oscillation type using the goodness of fit and the correlation coefficients.

IV. TEST RESULTS

A. Simple test

The free oscillation in (24) has one positive damping oscillation mode and one weak damping oscillation mode.

$$x_{free} = A_1 e^{\sigma_1 t}\sin(\omega_1 t) - A_2 e^{\sigma_2 t}\sin(\omega_2 t) \tag{24}$$

The forced oscillation in (25) is positive damping forced oscillation.

$$x_{forced} = A_1 e^{\sigma_1 t}\sin(\omega_1 t) - A_2 \sin(\omega_2 t) \tag{25}$$

Where A_1=A_2=1, $\sigma_1 = 0.08$, $\sigma_2 = 0.004$, $f_s = 0.1$ Hz, $\omega_1 = \omega_2 = 3.768$ rad/s.

The initial oscillation curves are showed in Fig.1. After systemic implementation process of the method the instantaneous amplitude of processed free and forced oscillation are showed in Fig.2.

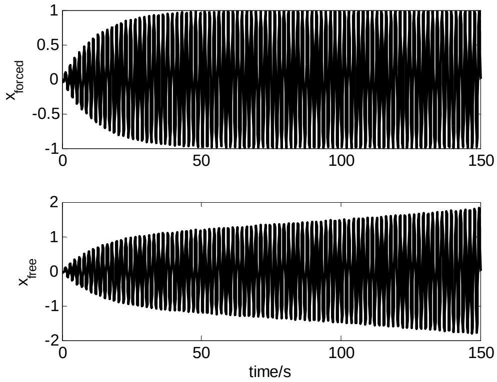

Fig. 1 Oscillating curves of free oscillation and forced oscillation

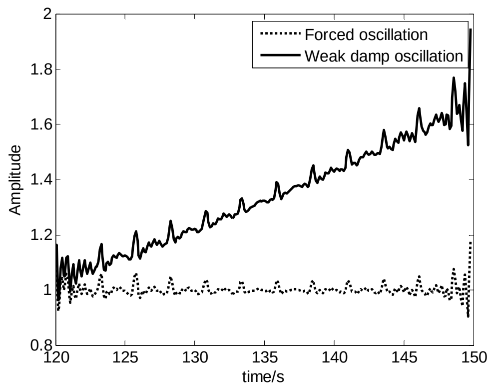

Fig. 2 Instantaneous amplitude of processed free and forced oscillation

The free oscillation in (24) would be mistakenly identified as forced oscillation by the method in [13] which assume that free oscillation has one weak damping oscillation mode and forced oscillation has one positive damping oscillation mode and one weak (near zero) damp oscillation mode in the initial period of wave. The instantaneous amplitude of processed

ISBN: 978-988-19252-0-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

free oscillation has directional changing trend, but the instantaneous amplitude of processed forced oscillation doesn't have directional changing trend. So the oscillation types of oscillation in (24) and (25) can be identified exactly by the method proposed in this paper. So the method in this paper can overcome the shortcoming of the method in [13].

B. *Actual incidents*

Consider two practical oscillation incidents in China Southern Power Grid. Oscillation in Ping Ban and Fa Er is showed in Fig.3 and Fig.5 respectively. Data between two vertical lines in Fig.1 and Fig.3 is chosen as the input data. Employing the systemic implementation process in section Ⅲ, we get fitting coefficients and goodness, correlation coefficients and oscillation type identification results showed in Table I. The curves of instantaneous amplitude and its fitting results are showed in Fig.4 and Fig.6. In Ping Ban oscillation, SSE and RMSE under (20) are smaller than that under (21), and meanwhile CD and R under (20) are bigger than that under (21), therefor we consider the oscillation as weak damping oscillation. In Fa Er oscillation, SSE and RMSE under (20) are bigger than that under (21), and meanwhile CD and R under (20) are smaller than that under (21), therefor we consider the oscillation as forced oscillation. In order to improve the identification accuracy, we choose data in different time ranges as input data and employ the systemic implementation process in section Ⅲ. The identification results for every single data range are showed in Table II. And we conduct statistical analysis for the identification results. The oscillation type whose number is more than 50% of the total time range number for every oscillation incident is the type identified at last. Oscillation types identified by the proposed method are consistent with the post-fault offline analysis results [16]-[17]. In Fa Er plant oscillation, the initial period of wave is unavailable, so the method in [12]-[13] cannot be used to identify the oscillation type. But the method proposed in this paper identified the oscillation type accurately.

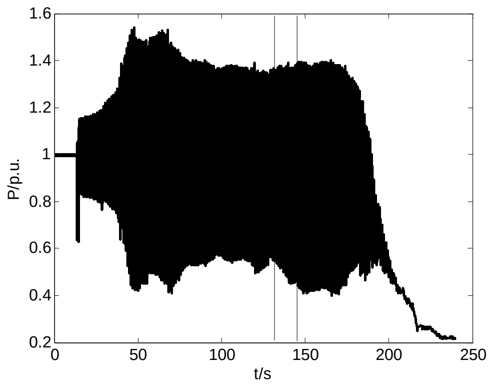

Fig. 3 Active power oscillation of G1 in Ping Ban

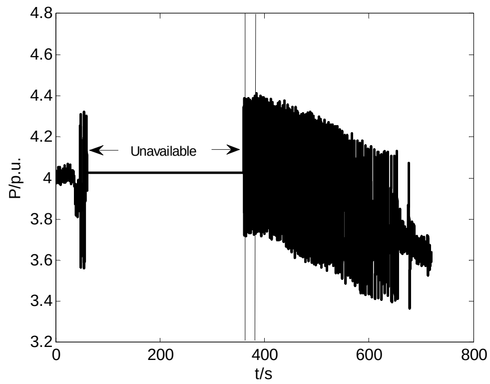

Fig. 5 Active power oscillation of G4 in Fa Er

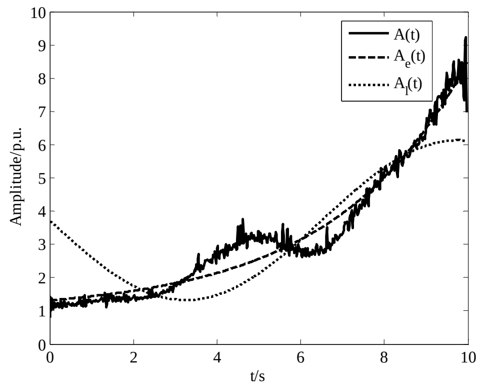

Fig. 4 Instantaneous amplitude of processed active power of G1 in Ping Ban

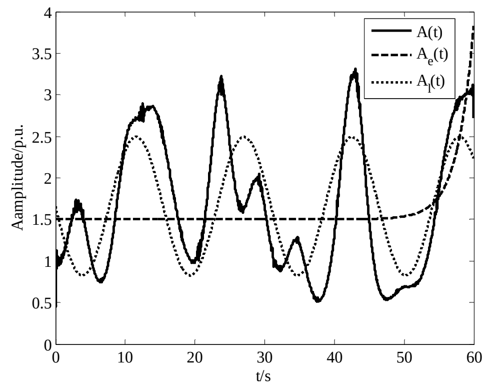

Fig. 6 Instantaneous amplitude of processed active power of G4 in Fa Er

TABLE I
FITTING COEFFICIENTS AND GOODNESS, CORRELATION COEFFICIENTS AND OSCILLATION TYPE IDENTIFICATION RESULTS

Plant	Expression	a	b	c	SSE	RMSE	CD	R	Oscillation Type Identified
Ping Ban	$ae^{bt}+c$	0.3272	0.3136	0.991	153.4	0.3925	0.9602	0.9799	Weak damping Oscillation
	$a\sin bt+c$	-2.404	0.482	3.728	1119	1.06	0.7103	0.8428	
Fa Er	$ae^{bt}+c$	-0.833	0.4064	1.66	3465	0.7602	0.1281	0.3652	Forced Oscillation
	$a\sin bt+c$	1.713e-15	0.4274	1.502	1851	0.5343	0.5556	0.7309	

ISBN: 978-988-19252-0-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

TABLE II
OSCILLATION TYPE IDENTIFICATION RESULTS IN DIFFERENT TIME RANGES

Plant	Start Time (s)	End Time (s)	Oscillation Type Identified	Statistics	Oscillation Type Identified at Last
Ping Ban	70	80	Weak damping Oscillation	88% WDO	Weak damping Oscillation
	80	90	Forced Oscillation		
	90	100	Weak damping Oscillation		
	100	110	Weak damping Oscillation		
	110	120	Weak damping Oscillation		
	120	130	Weak damping Oscillation		
	130	140	Weak damping Oscillation		
	140	150	Weak damping Oscillation		
Fa Er	360	380	Forced Oscillation	88% FOO	Forced Oscillation
	380	400	Forced Oscillation		
	400	420	Forced Oscillation		
	420	440	Forced Oscillation		
	440	460	Forced Oscillation		
	460	480	Weak damping Oscillation		
	480	500	Forced Oscillation		
	500	520	Forced Oscillation		

V. CONCLUSION

The instantaneous amplitude of weak damping oscillation has certain directional trend but that of forced oscillation does not have. We grasp and amplify this discriminative difference to identify oscillation types by a method based on EMD and HT. In this method, repeating EMD and square calculation alternately *n* times, the damp factor has been amplified by a factor of nth power of 2. And then here come two different results: directional changing trend of the amplitude of weak damping oscillation becomes more obviously but the amplitude in forced oscillation keeps constant or fluctuates around a constant not having any directional changing trend. We acquire the instantaneous amplitude of oscillation and use two different expressions to approximate it and analyze the correlation between the fitting results and it. Finally, we determine the oscillation types according to the goodness of fit and correlation coefficients. We use a simple test to illustrate the advantage of the method. The method is validated in actual oscillation incidents. The method utilizes WAMS data and has the potential for online application.

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