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Short-term Traffic Forecasting Based on Grey Neural Network with Particle Swarm Optimization

Yuanyuan Pan, Yongdong Shi

***Abstract*—An accurate and stable short-term traffic forecasting model is very important for intelligent transportation systems (ITS). The forecasting results can be used to relieve traffic congestion and improve the mobility of transportation. This paper proposes a new hybrid model of grey system theory and neural networks with particle swarm optimization, namely, GNN-PSO. The proposed hybrid model can exploit sufficiently the characteristics of grey system model requiring less data, the non-linear map of neural networks and the quick-speed convergence of PSO, and has simpler structure. The GNN-PSO model is applied to predict the average speed of vehicle on Barbosa road in Macao. The experiment results show that the proposed model has better performance than grey forecasting model GM(1,1), back-propagation neural network model BPNN, and the combined model of them, i.e., grey neural network model (GNN), on short-term traffic forecasting.**

***Index Terms*—Short-term traffic forecasting, Grey system, Neural networks, Grey neural network, Particle Swarm Optimization (PSO).**

I. Introduction

SHORT-TERM traffic forecasting is a vital component of intelligent transportation systems (ITS), which is a process of estimating directly the anticipated traffic conditions at a future time (usually time span less than 15 minutes) given continuous short-term feedback of traffic information. The most commonly used variables in traffic forecasting are three fundamental macroscopic traffic parameters: flow, occupancy and speed [1]. The forecasting results for these parameters can be used to assist traffic control center in ITS to reduce traffic congestion and improve the mobility of transportation [2]. Therefore, it is very significant for the development of ITS to predicate effectively and accurately short-term traffic information.

In the last two decades, many scholars have devoted themselves to study this issue. Generally, the short-term traffic forecasting approaches can be divided into three categories: statistical, artificial intelligence and hybrid methods. In the conventional statistical methods, Kalman filtering [3], autoregressive integrated moving average (ARIMA) [4] and seasonal ARIMA [5] have been applied to forecast short-term traffic flow based on past data. Although these statistical techniques can achieve reasonable prediction accuracy, they may not capture the dynamics and nonlinearities existed in traffic flow. To address this issue, the artificial intelligence methods, namely, neural networks (NNs) [6] [7] [8] [9], are widely used to the predication of short-term traffic flow due to its advantages of non-linear mapping relations [10]. However, NNs require a great deal of training data and relatively long training period for robust generalization [11] [12]. To enhance the generalization capability of NNs, several previous studies have proposed hybrid NNs models by incorporating genetic algorithms [13], fuzzy logical [14] [15], Kalman filters [16], the ARIMA model [17], and so on. These hybrid models have been demonstrated to possess better performance than pure NNs in short-time traffic forecasting, but they use more parameters, have more complex structure and require greater computational power [18].

To address these issues, this paper proposes a new hybrid model of grey system theory and neural networks with particle swarm optimization, namely, GNN-PSO. Grey system theory was first introduced in early 1980s by Deng. Since then, the theory has been widely used to forecast in various fields, such as agriculture, industry and environment, because it is simple and requires only a limited amount of data to estimate the behavior of uncertainty system. Neural networks, as above mentioned, has been a primary non-linear forecasting method because of its ability of self-learning, non-linear map and parallel distributed manipulation. Particle swarm optimization (PSO), as a global optimization technique, is used to solve the commonly existing problems in NNs, such as the slow speed of convergence and the local minimum [19], by estimating the parameters of NNs. As a result, the proposed hybrid model has the characteristics of grey system theory requiring less data, the strong non-linear map of neural networks and the quick-speed convergence of particle swarm optimization. Furthermore, the structure of GNN-PSO is relatively simpler since it only executes an accumulated generating operation (AGO) and an inversely accumulated generation operation (IAGO), which are derived from grey system theory, before and after neural network, respectively. To evaluate the performance of the GNN-PSO model, it is used to predict short-term traffic speed on Babosa road in Macao. Results show that the proposed hybrid model is more accurate and stable than the single grey model, the single neural network model and the combined model of them, i.e., grey neural network model (GNN).

This rest of this paper is organized as follows. Section II describes some fundamental concept about grey system theory, neural networks and PSO. Section III illustrates the mechanism of the GNN-PSO model. Section IV discusses the detailed experimental results. Section V finally concludes this paper.

II. Fundamental concepts

A. Grey model

In grey system theory, the most commonly used forecasting model is GM(1,1) which is called as "Grey Model First Order One Variable". This model can only be used

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in positive data sequences [12]. Since all the data of traffic speed are positive, GM(1,1) model can be used to forecast the future values of short-term traffic speed. The process of GM(1,1) can be divided into three stages. In the first stage, to smooth the randomness, the primitive data obtained from the system to form the GM(1,1) is subjected to an operator, named Accumulating Generation Operator (AGO) [20]. The differential equation of GM(1,1) is then solved to obtain the n-step ahead predicted value of the system in the second stage. Finally, using the predicted value, the Inverse Accumulating Generation Operator (IAGO) is applied to find the predicted values of original data. More details are as follows.

Stage 1: Let the original sequence be denoted by $X^{(0)}$.

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)\}, n \geq 4. \quad (1)$$

On the basis of $X^{(0)}$, a new sequence $X^{(1)}$ is generated by AGO as

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n)\} \quad (2)$$

where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, 3, ..., n$.

Stage 2: First-order differential equation of GM(1,1) model is given as

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = u \quad (3)$$

The solution of Eq. (3) can be obtained by using the least square method. That is

$$\widehat{x}^{(1)}(k+1) = [x^{(0)}(1) - \frac{u}{a}]e^{-ak} + \frac{u}{a} \quad (4)$$

where

$$[a, u]^T = (B^T B)^{-1} B^T Y \quad (5)$$

and

$$B = \begin{bmatrix} -\frac{1}{2}(x^{(1)}(1) + x^{(1)}(2)) & 1 \\ -\frac{1}{2}(x^{(1)}(2) + x^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(x^{(1)}(n-1) + x^{(1)}(n)) & 1 \end{bmatrix} \quad (6)$$

$$Y = [x^{(0)}(2), x^{(0)}(3), ..., x^{(0)}(n)]^T. \quad (7)$$

Stage 3: IAGO is applied to obtain the predicted value of the original data at time $k+1$, $\widehat{x}^{(0)}(k+1)$, since the grey forecasting model is formulated by using the data of AGO rather than original data.

$$\widehat{x}^{(0)}(k+1) = [x^{(0)}(1) - \frac{u}{a}](1 - e^{a})e^{-ak} \quad (8)$$

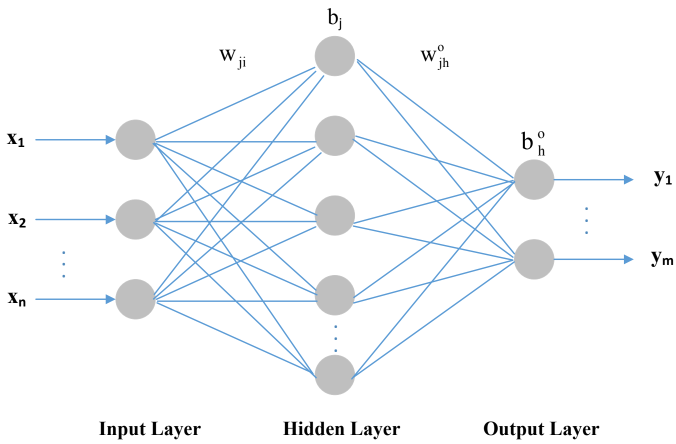

Fig. 1. The structure of BPNN

B. Neural networks

Back-propagation neural network (BPNN) is one of the most prevalent neural networks in short-term traffic forecasting. It is based on a supervised learning algorithm which minimizes the global error by using the generalized delta rule and the gradient steepest descent method. The architecture of BPNN consists of an input layer, one or more hidden layers, and an output layer. Each layer comprises several neurons connected to the neurons in neighboring layers. Since BPNN contains many interacting nonlinear neurons in multiple layers, it can capture complex phenomena [21]. The parameters of BPNN include weights and biases which are adjusted iteratively by a process of minimizing the global error or the total square error. In this paper, BPNN is a three-layer network (one hidden layer) since the previous study [7], [8] have proved that the three-layer neural network can realize non-linear map for traffic flow. The structure of BPNN can be seen in Fig. 1, where w_{ji} represents the connection weight from input node i to hidden node j, w^o_{jh} represents the connection weight from hidden node j to output node h, b_j stands for the bias of hidden node j and b^o_h stands for the bias of output node h.

C. Particle Swarm Optimization

As a population based stochastic optimization technique, Particle Swarm Optimization (PSO) was introduced by Kennedy and Eberhart in 1995. It is inspired by the social behavior of animals, such as bird flocking and fish schooling [22]. In PSO, each particle is a solution to an optimization problem. All the particles are initially distributed randomly over the search space, and then fly at certain velocity in the search space to find the global best position, namely, the global optimum, after some iterations. Suppose the dimension of search space is D and the number of particles is N. The position and velocity of particle i at iteration t are denoted by $X_i^t = \{x_{i1}^t, x_{i2}^t, ..., x_{iD}^t\}$ and $V_i^t = \{v_{i1}^t, v_{i2}^t, ..., v_{iD}^t\}$ $(i = 1, 2, ..., N)$, respectively. The best position of particle i and all particles at iteration t can be represented by $P_{best,i}^t = \{P_{i1}^t, P_{i2}^t, ..., P_{iD}^t\}$ and $G_{best}^t = \{G_1^t, G_2^t, ..., G_D^t\}$, respectively. The algorithm can be described as follow.

Step 1. Initialize randomly the positions and velocities of a group of particles.

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Step 2. Calculate the fitness value of each particle i. If its fitness value is better than the best fitness value in history, this value is set as the new best fitness value, and the current positions of this particle is set as its best positions P_{best}.

Step 3. Compare all particles' best fitness values and choose the particle with the best fitness value of them as the G_{best}.

Step 4. Update the velocity and position of each particle i according to the next two equations.

$$V_i^{t+1} = \omega V_i^t + c_1 r_1 (P_{best,i}^t - X_i^t) + c_2 r_2 (G_{Best}^t - X_i^t) \quad (9)$$

$$X_i^{t+1} = X_i^t + V_i^{t+1} \quad (10)$$

where ω is the inertia weight, c_1 and c_2 are the cognitive and social acceleration constants which are initially set to 2 [23], r_1 and r_2 are uniformly distributed random numbers in the range [0, 1]. In addition, the velocity of the particle is limited to the range $[V_{min}, V_{max}]$.

Step 5. While maximum iterations or minimum error criteria is not attained, loop back to Step 2 again.

III. The hybrid GNN-PSO model

In this paper, the proposed hybrid GNN-PSO model for short-term traffic forecasting not only combines grey model with neural networks to form Grey Neural Network (GNN), but also optimizes the GNN model with particle swarm optimization algorithm.

In the GNN-PSO model, there are three stages. The first stage initializes the original traffic speed series data by accumulating generation operator (AGO) in grey model in order to weaken the randomness of the original data, and then these data are feed into a back-propagation neural network (BPNN). The second stage focuses on the optimization of BPNN by using PSO algorithm to decide the optimal original value of parameters, such as weight and bias, in BPNN. In this stage, the structure of three-layer BPNN $N_{in} * N_{hindden} * 1$ is first determined, where N_{in} refers to the number of data input from the first stage, 1 refers to the number of output forecasting value, N_{hidden} refers to the node number of hidden layer. According to the structure, the dimension of search space in PSO D is then decided, where D is equal to the number of all weights and biases in BPNN. In other words, for each particle, say particle i, its position can be expressed as $X_i = \{w_{mn}, b_m, w_{hm}^o, b_h^o\}$ $(1 \leq n \leq N_{in},\ 1 \leq m \leq N_{hidden},\ h = 1)$. The fitness function of particle i, denoted by $f(P_i)$, is set to the error sum of squares between the predicted value $\widehat{x}^{(0)}(k+1)$ and the real value $x^{(0)}(k+1)$, which is expressed as follows.

$$f(P_i) = \frac{1}{M} \sum_{k=1}^{M} (\widehat{x}^{(0)}(k+1) - x^{(0)}(k+1))^2 \quad (11)$$

where M refers to the number of sample data. According to the steps of PSO algorithm as described above, the globe optimum G_{best} which is corresponding to the optimal weights and biases can be find. At last, BPNN that takes the weights and biases optimized by PSO as its initial parameters is trained continuously until the fixed iterations or the desired error is obtained. This aims to improve the robustness and accuracy of BPNN for short-term traffic forecasting. In the third stage, to obtain the predicted values, the output data from the optimized BPNN are processed by the inverse accumulating generation operator (IAGO) in grey model. The structure of hybrid model GNN-PSO is shown in Fig. 2.

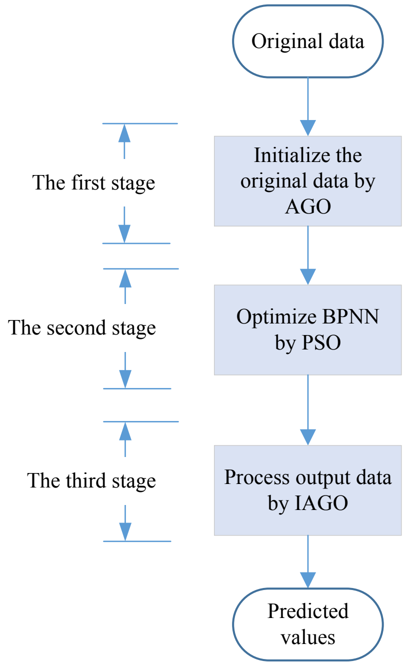

Fig. 2. The structure of hybrid model GNN-PSO

IV. Experiment results

In this section, the proposed hybrid model GNN-PSO is used to forecast the future vehicular speed on Barbosa road in Macao during the period of PM 14:00-15:00, 19th August. The forecasting results are compared with that obtained from the single models, such as GM(1,1) and BPNN, and the combined model of them, i.e., Grey Neural Network (GNN) [24].

A. Data source

The speed data used for forecasting was collected by the real-time traffic information system which has been developed by intelligent transportation systems (ITS) research laboratory of Macau University of Science and Technology. This system adopts dynamic data-collecting technology. The GPS terminals in 140 vehicles send data of the position and the speed of vehicles to the lab server in every 1 minute. The received data are saved in MySQL database. By calculating the arithmetic mean of the speed data in every last 3 minutes, we get the average real-time speed of vehicles in the road. So, we get 60 speed data totally that are calculated in every 1 minute between 14:00-15:00.

B. Data forecasting

To evaluate the performance of the GNN-PSO model for short-term traffic forecasting, the time series data from 14:00

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TABLE I
THE COMPARISONS OF FORECASTING RESULTS

Time	Actual data	GNN-PSO		GM		BPNN		GNN	
		Forecasting value	Relative error(%)	Forecasting value	Relative error(%)	Forecasting value	Relative error(%)	Forecasting value	Relative error(%)
14:51	21.245	20.644	2.83	19.049	10.34	18.474	13.04	20.089	5.44
14:52	29.275	22.916	21.72	21.789	25.57	24.376	16.74	25.445	13.08
14:53	30.92	29.708	3.92	29.248	5.41	33.057	6.91	23.334	24.53
14:54	27.95	27.73	0.79	28.209	0.93	38.172	36.57	22.973	17.81
14:55	21.75	24.229	11.40	22.035	1.31	32.752	50.58	27.062	24.42
14:56	14.34	15.468	7.87	18.908	31.86	21.957	53.12	12.243	14.62
14:57	10.88	11.886	9.25	14.394	32.30	13.123	20.61	16.075	47.75
14:58	15.80	8.311	47.40	12.244	22.51	7.9807	49.49	11.563	26.82
14:59	10.94	11.138	1.81	14.144	29.29	10.336	5.52	11.09	1.37
15:00	10.76	12.862	19.54	11.893	10.53	11.748	9.18	8.018	25.49

to 15:00 were divided into two sub-sets. The first sub-set of the time series data, namely training data, collected from 14:00-14:50, were used for training the GNN-PSO model, the second sub-set of that, namely test data, collected from 14:51:15:00, were used to evaluate the generalization capability of this model. Whether training or evaluating, this model selects a time series data with equal dimension to predict the next data. The so-called equal dimension means, for a time series data, after predicting one traffic speed data, a new datum is added to the sequence at the end, meanwhile the oldest datum from the head of the sequence is take out, which results a new time series data with same dimension is generated to forecast the next traffic speed data. For example, using a time series data $\{x^{(0)}(k), x^{(0)}(k+1), ..., x^{(0)}(k+4)\}$, the GNN-PSO model predicts the value after one sampling times (i.e., 1 minutes) $x^{(0)}(k+5)$. In the next steps, the first data is always shifted to the second. It means that the model uses a time series data $\{x^{(0)}(k+1), x^{(0)}(k+2), ..., x^{(0)}(k+5)\}$ to forecast the value of $x^{(0)}(k+6)$. In this way, the new superseding the old, forecasting one by one, all need prediction results can be obtained. In this paper, the dimension of the time series data is set to 5. This means there are 45 and 10 pieces of speed data to train and validate the GNN-PSO model, respectively.

In the GNN-PSO model, a time series data with 5 data are first selected to generate a new sequence by accumulated generating operation (AGO). The new sequence is used as the input data of BPNN. According to the number of input and output data, the structure of BPNN is decided as $5*N_{hidden}*1$. Since the node number of hidden layer N_{hidden} is not the goal of this paper, we use one method recommended in [25], where $N_{hidden} \approx \log_2(N_{tr})$ with N_{tr} be the number of training data. As mentioned above, N_{tr} is equal to 45, so N_{hidden} is 6 in this case, i.e., $\log_2(45) \approx 6$. Based on the structure of BPNN, the dimension of search space in PSO is set to 43. The other parameters were used in PSO as follows: the population size of swarm is set to 20, the number of iterations is predefined as 200, ω is set to 0.8 and the training errors are below 10%. We use the optimum obtained by PSO as the initial parameters of BPNN, and then train again the optimized BPNN based on training data. The trained BPNN is used to forecast the last 10 data. Finally, we deal with the output 10 data of neural network by inverse accumulated generation operation (IAGO) to get the prediction value we need.

The performance of the hybrid GNN-PSO model is compared with the following models.

1) The single GM(1,1) model. Like the GNN-PSO model, the GM(1,1) model is built by using a time series data with 5 data and applying the equal dimension. But, due to no requirement of training neural network, the GM(1,1) model takes a time series data start from 14:46 to build grey model and forecast one data. This model iterates 10 times, then gives the last 10 speed forecasting results.
2) The single BPNN model. The structure and parameters of BPNN are identical to those of GNN-PSO. For instance, adopting $5*6*1$ neural network, using Tansig as the transfer function between input layer and hidden layer, and applying Pureline for output layer. However, unlike GNN-PSO, the initial neurons connection weights of the single BPNN model are set to stochastic real number belonging to [-1, 1].
3) The hybrid grey neural network (GNN) model. It has three basic parts: a grey layer, a back-propagation neural network (BPNN), and a white layer [26]. The grey layer before neural network executes accumulated generating operation (AGO) to initialize input data. These new data generated by AGO are then feed into the neural network. Finally, the white layer after neural network inverses accumulated generation to the output data of the neural network. Therefore, the prediction value we need is obtained. The parameters used in GNN are identical to those used in GNN-PSO, except that there is no optimization in GNN by PSO.

The forecasting values and relative errors of the four models are as shown in Table I.

C. Evaluation and comparison

To investigating the viability of short-term traffic forecasting models, several performance measures have been applied in the previous studies. In this paper, the mean absolute percentage error (MAPE) and the root-mean-square error (RMSE) [18], shown as Eqs. (12) and (13), are used as the measures for comparison in these forecasting models. MAPE and RMSE respectively reflect the mean prediction accuracy and stability. The smaller MAPE is, the more accurate the prediction is. Similarly, the smaller RMSE is, the more stable the prediction is.

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}|\frac{\widehat{x}(i)-x(i)}{x(i)}| \tag{12}$$

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$$RMSE = \frac{1}{n}\sqrt{\sum_{i=1}^{n}(\frac{\widehat{x}(i)-x(i)}{x(i)})^2} \tag{13}$$

where $x(i)$ is the actual value at the ith time interval, $\widehat{x}(i)$ is the forecasted value at the ith time interval, and n is the total number of the forecasted value.

Table II summarizes the results of MAPE and RMSE values obtained by the four models, i.e., GNN-PSO, GNN, GM(1,1) and BPNN. It can be seen that the MAPE value of GNN-PSO (12.65%) is the lowest, and decreases 25.6%, 51.67%, 37.17% compared to that of GNN, GM(1,1) and BPNN, respectively. This implies that the GNN-PSO model can improve the accuracy of forecasting short-term traffic speed. Furthermore, as shown in Table II, the RMSE values of GNN-PSO, GNN, GM(1,1) and BPNN are 3.33, 3.48, 6.22, 4.28, respectively. This indicates that GNN-PSO is more stable than other three model in short-term traffic forecasting. In a word, the results of MAPE and RMSE show that the proposed hybrid GNN-PSO model has the higher prediction accuracy and stability among the four models.

TABLE II
THE MAPE AND RMSE VALUES OF DIFFERENT MODELS

Criteria	GNN-PSO	GNN	GM	BPNN
MAPE (%)	12.65	17.00	26.18	20.13
RMSE	3.33	3.48	6.22	4.28

V. CONCLUSION

In this paper, a new hybrid model of grey system theory and neural networks with particle swarm optimization, namely, GNN-PSO, has been proposed to predict short-term traffic speed. It aims to address some issues existed in the previous studies, including: (i) a great deal of history data; (ii) the non-linear characteristic of traffic data; (iii) the slow speed of convergence; (iv) great parameters and complex structure. The proposed model can be divided into three stages. In the first stage, the original traffic data are initialized by AGO in grey model to generate a new data sequence which is input into a BPNN. In the second stage, the BPNN is optimized by PSO algorithm. In the third stage, the data output from the optimized BPNN are processed by IAGO in grey model, thus the predicted values are obtained. This paper uses MAPE and RMSE to measure the accuracy and stability of forecasting models. The experimental results reveal that the predicted values obtained from GNN-PSO are more accurate and stable than that from the single GM(1,1), the single BPNN, and the combined model of them GNN. Of course, like most studies, the structure of GNN-PSO, such as the dimension of the time series data input into BPNN and the number of hidden nodes, is required to be pre-defined and fixed, which can not guarantee the optimal one will be obtained. How to optimize the structure of GNN-PSO with respect to time is left for our future work.

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