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The Logistic Inverse Exponential Distribution: Basic Structural Properties and Application

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***Abstract*— We extended the Inverse Exponential distribution using the Logistic family of distributions. We established some of its statistical properties and we estimated the parameters using the method of maximum likelihood estimation. Real life applications were provided and the proposed distribution was confirmed a better alternative to the Logistic distribution.**

***Index Terms*— Distributions, Family of Distributions, Generalization, Inverse Exponential, Logistic, Mathematical Statistics, Statistical Properties**

I. Introduction

Inverse Exponential (shortened as IE) distribution was proposed as an alternative to the Exponential distribution because it does not have a constant failure rate and it does not also exhibit the memoryless property. Details about the Inverse Exponential distribution are readily available in [1] and [2].

The IE distribution has been generalized in recent times resulting into Kumaraswamy Inverse Exponential distribution [3], Exponential Inverse Exponential distribution [4], Transmuted Inverse Exponential distribution [5], Exponentiated Generalized Inverse Exponential distribution [6] and Weibull Inverse Exponential distribution [7]. Developing compound distributions is a new trend in probability distribution theory and a number of these compound distributions are better than the existing standard distributions.

There are several families of distributions in the literature, a list of them are available in [8-12] and the references therein. Meanwhile in this paper, the Logistic generalized family of distribution that was developed by [13] was used to extend the Inverse Exponential distribution. The aim is to derive a compound distribution that would be more robust than the Inverse Exponential distribution yet tractable. We organize the rest of this article as follows: In section 2, the Logistic Inverse Exponential distribution is defined and its basic statistical properties are provided while in section 3, a real life application to dataset is provided.

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II. The Logistic Inverse Exponential Distribution

The densities of the Lomax family of distribution are:

$$F(x)=\left[1+\left\{-\log\left[\bar{G}(x)\right]\right\}^{-a}\right]^{-1} \tag{1}$$

and

$$f(x)=\frac{ag(x)}{\bar{G}(x)}\left\{-\log\left[\bar{G}(x)\right]\right\}^{-(a+1)}\left[1+\left\{-\log\left[\bar{G}(x)\right]\right\}^{-a}\right]^{-2} \tag{2}$$

respectively. Where $\bar{G}(x)=1-G(x)$ and $a>0$ is an extra shape parameter.

In this paper, $g(x)$ and $G(x)$ are the pdf and cdf of the Inverse Exponential distribution respectively.

$$G(x)=e^{-\left(\frac{b}{x}\right)} \tag{3}$$

and;

$$g(x)=\frac{b}{x^2}\mathrm{e}^{-\left(\frac{b}{x}\right)} \tag{4}$$

respectively.

For $x>0, b>0$

where ; b is a scale parameter

The cdf of the Lomax Inverse Exponential distribution (shortened as LIE distribution) is derived by substituting Equation (3) into Equation (1) to give:

$$F(x)=\left[1+\left\{-\log\left[1-e^{-\left(\frac{b}{x}\right)}\right]\right\}^{-a}\right]^{-1} \tag{5}$$

Its pdf is derived by making a substitution of Equations (3) and (4) into Equation (2) as:

$$f(x)=\frac{a\frac{b}{x^2}\mathrm{e}^{-\left(\frac{b}{x}\right)}}{\left[1-\mathrm{e}^{-\left(\frac{b}{x}\right)}\right]}\left\{-\log\left[1-\mathrm{e}^{-\left(\frac{b}{x}\right)}\right]\right\}^{-(a+1)}\times \tag{6}$$

$$\left[1+\left\{-\log\left[1-\mathrm{e}^{-\left(\frac{b}{x}\right)}\right]\right\}^{-a}\right]^{-2}$$

For $x > 0, a > 0, b > 0$

where a is a shape parameter which introduces skewness and b is the scale parameter.

Graphical representation of the pdf of the LIE distribution is provided in Figure 1.

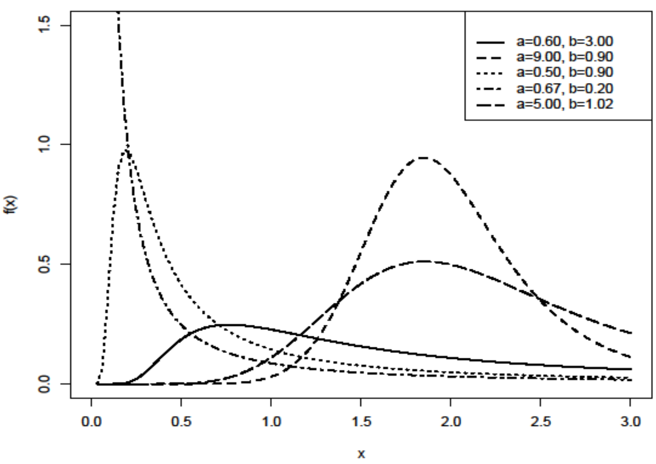

Figure 1: PDF of the LIE Distribution

The plot in Figure 1 show that the LIE distribution exhibits unimodal (inverted bathtub) and decreasing shapes.

Some Structural Properties of the LIE Distribution

We provide some basic structural properties of the LIE distribution as follows:

Reliability Analysis

The reliability (or survival function) is derived from:

$$S(x) = 1 - F(x)$$

Therefore, we obtain the reliability function for the LIE distribution as:

$$S(x) = 1 - \left[1 + \left\{-\log\left[1 - e^{-\left(\frac{b}{x}\right)}\right]\right\}^{-a}\right]^{-1} \qquad (7)$$

For $x > 0, a > 0, b > 0$

The failure rate (hazard function) is derived from:

$$h(x) = \frac{f(x)}{1 - F(x)}$$

We therefore obtain the failure rate for the LIE distribution as:

$$h(x) = \frac{\dfrac{a\frac{b}{x^2}e^{-\left(\frac{b}{x}\right)}}{\left[1-e^{-\left(\frac{b}{x}\right)}\right]}\left\{-\log\left[1-e^{-\left(\frac{b}{x}\right)}\right]\right\}^{-(a+1)}\left[1+\left\{-\log\left[1-e^{-\left(\frac{b}{x}\right)}\right]\right\}^{-a}\right]^{-2}}{1-\left[1+\left\{-\log\left[1-e^{-\left(\frac{b}{x}\right)}\right]\right\}^{-a}\right]^{-1}} \qquad (8)$$

For $x > 0, a > 0, b > 0$

A graphical representation of the failure rate of the LIE distribution is provided in Figure 2.

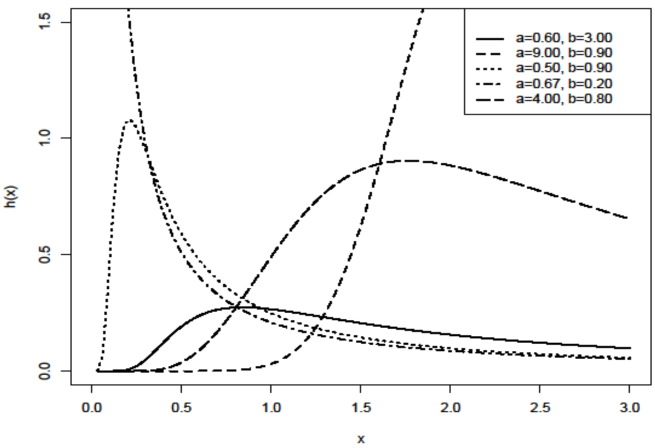

Figure 2: Failure Rate of the LIE Distibution

As displayed in Figure 2, the failure rate of the LIE distribution has increasing, decreasing and unimodal shapes.

The reversed hazard function is obtained from:

$$r(x) = \frac{f(x)}{F(x)}$$

So, we obtain the reversed hazard function of the LIE distribution as:

$$r(x) = \frac{\dfrac{a\frac{b}{x^2}e^{-\left(\frac{b}{x}\right)}}{\left[1-e^{-\left(\frac{b}{x}\right)}\right]}\left\{-\log\left[1-e^{-\left(\frac{b}{x}\right)}\right]\right\}^{-(a+1)}}{\left[1+\left\{-\log\left[1-e^{-\left(\frac{b}{x}\right)}\right]\right\}^{-a}\right]^{-1}} \qquad (9)$$

For $x > 0, a > 0, b > 0$

The odds function is mathematically expressed as:

$$O(x) = \frac{F(x)}{1 - F(x)}$$

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So, we obtain the odds function of the LIE distribution as:

$$O(x)=\frac{\left[1+\left\{-\log\left[1-e^{-\left(\frac{b}{x}\right)}\right]\right\}^{-a}\right]^{-1}}{1-\left[1+\left\{-\log\left[1-e^{-\left(\frac{b}{x}\right)}\right]\right\}^{-a}\right]^{-1}} \quad (10)$$

For $x>0, a>0, b>0$

Parameter Estimation

Let $x_1, x_2, ..., x_n$ denote random samples from the LIE distribution as defined in Equation (6), by the method of maximum likelihood estimation (MLE), we obtain the likelihood function as:

$$f\left(x_1,x_2,..,x_n;a,b\right)=$$

$$\prod_{i=1}^{n}\left[\frac{a\frac{b}{x_i^2}\mathrm{e}^{-\left(\frac{b}{x_i}\right)}}{\left[1-\mathrm{e}^{-\left(\frac{b}{x_i}\right)}\right]}\left\{-\log\left[1-\mathrm{e}^{-\left(\frac{b}{x_i}\right)}\right]\right\}^{-(a+1)}\times\left[1+\left\{-\log\left[1-\mathrm{e}^{-\left(\frac{b}{x_i}\right)}\right]\right\}^{-a}\right]^{-2}\right]$$

Let $l=\log\left[f\left(x_1,x_2,...,x_n;a,b\right)\right]$

Then, the log-likelihood function $'l'$ is:

$$l=n\log(a)+n\log(b)-2\sum_{i=1}^{n}\log(x_i)-b\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)-$$

$$(a+1)\sum_{i=1}^{n}\log\left\{-\log\left[1-e^{-\left(\frac{b}{x_i}\right)}\right]\right\}-2\sum_{i=1}^{n}\log\left[1+\left\{-\log\left[1-e^{-\left(\frac{b}{x_i}\right)}\right]\right\}^{-a}\right] \quad (11)$$

When the system of non-linear equations of $\frac{\partial l}{\partial a}=0$ and $\frac{\partial l}{\partial b}=0$ are solved, the solution is the maximum likelihood estimates of parameters a and b respectively. The solution cannot be obtained in closed form but it can be obtained with the aid of statistical software. In this paper, R software was however adopted.

III. APPLICATION

We applied the LIE distribution to a real data set and its potentials over the Logistic distribution was assessed. As a result, we make use of the Negative log-likelihood (NLL), Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) to select the distribution with the best fit. The lower the values of these criteria the better the distribution.

The data used in this paper represents the strengths of 1.5 cm glass fibres obtained by workers at the UK National Physical Laboratory. The data has 63 observations and it has already been studied by [10], [14] and many others. The data set is as follows:

0.55, 0.74, 0.77, 0.81, 0.84, 1.24, 0.93, 1.04, 1.11, 1.13, 1.30, 1.25, 1.27, 1.28, 1.29, 1.48, 1.36, 1.39, 1.42, 1.48, 1.51, 1.49, 1.49, 1.50, 1.50, 1.55, 1.52, 1.53, 1.54, 1.55, 1.61, 1.58, 1.59, 1.60, 1.61, 1.63, 1.61, 1.61, 1.62, 1.62, 1.67, 1.64, 1.66, 1.66, 1.66, 1.70, 1.68, 1.68, 1.69, 1.70, 1.78, 1.73, 1.76, 1.76, 1.77, 1.89, 1.81, 1.82, 1.84, 1.84, 2.00, 2.01, 2.24.

We present the summary of the data in Table 1:

Table 1: Data summary on glass fibre

Mean	Median	Variance	Skewness	Kurtosis
1.507	1.590	0.1051	-0.8999	3.9237

The results of the two competing distribution are shown in Table 2:

Table 2: Table of result

Models	Estimates	NLL	AIC	BIC
LIE	$\hat{a}=8.90775$ $\hat{b}=0.72284$	89.7886	183.5773	187.8636
Logistic	$\hat{a}=1.01272$	121.1714	244.3428	246.4859

From the result in Table 2, we can see that the LIE distribution is indeed an improvement over the Logistic distribution as it has the lowest NLL, AIC and BIC values.

IV. CONCLUSION

We have successfully defined the Logistic Inverse Exponential (LIE) distribution. The shape of the LIE distribution is unimodal and decreasing; this however depends on the parameter values. We have derived and established the expressions for the reliability, hazard, reversed hazard and odds functions. The shapes of the failure rate indicate that the LIE distribution is capable of modeling data sets with decreasing, increasing and inverted bathtub failure rates. An illustration with the aid of real life dataset reveals that the LIE distribution is better than the Logistic distribution as the extra shape parameter helped in withstanding data set with strong asymmetry.

Further research would involve making comparison between the LIE distribution and some other generalized or compound distributions, and a simulation study to investigate the behavior of the parameters.

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