Sensitivity Analysis of Linear Programming Optimization of a Manufacturing Business

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Abstract— **Approximations are usually made when Mathematical model is used to describe reality. A major challenge encountered by most business enterprises/ companies is how value change effect can be determined on optimal decisions. Linearity assumptions usually are significant approximations. Another important approximation comes because one cannot be sure of the data that one puts into the model. Knowledge of the relevant technology may be imprecise, forcing one to approximate values; moreover, information may change. In this paper the sensitivity analysis of the optimal solution of linear programming model of a business enterprise is investigated. Computer software – LINDO, was used to solve the resulting Linear programming model formed from a real-life business situation. The sensitivity analysis carried out on the optimal solutions obtained, show that an addition of a new type of plastic in the production line improved the optimal profit. A reduction in the time resources used in producing type B plastic reduced the optimal profit.**

Index Terms— **Hebron drinks, Simplex Method, Operations Research, Optimization, Linear Programming, Production Processes.**

I. Introduction

A linear programming (LP) problem might be characterized as the problem of optimizing a linear function subject to some limitations which are also linear [1,2]. The limitations might be equality or inequalities. It is a mathematical modelling used to find the most ideal way of assigning limited resources to accomplish optimum profit or least cost. It is required to satisfy a system of linear constraint [2,3].

The subject may more properly be called linear optimization. The graphical solution, usually for LP with just two variables, is the essential of sensitivity analysis. These essentials will then be reached out to the general LP problem utilizing the simplex tableau results.

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Nonetheless, for LP including more than two variables or problems including a substantial number of limitations, it is smarter to utilize methods of solution that are versatile to software. [3,4]. A strategy called the simplex technique might be utilized to locate the optimal solution for multivariable problems [4,5]. The simplex method is really an algorithm with which we look at corner point in a methodical manner until the best solution is arrived at—most noteworthy profit or least cost. The initial step of the simplex method requires that every inequality limitation in a LP formulation is changed into an equation. Less-than-or-equal-to limitations (≤) can be changed over to equations by including slack factors, which speak to the measure of an unused resource [6,7,8,9]

Sensitivity Analysis (SA) is the examination of these potential changes and mistakes and their effects on conclusions to be drawn from the model [10,11]. It is conceivably the most valuable and most generally utilized procedure accessible to modelers who wish to help decision maker. It is a deliberate investigation of how sensitive changes in the data are. It can likewise be alluded to as a technique used to decide how different values of an independent variable affect a dependent variable under a given set of assumptions. The target of sensitivity analysis is to discover new optimal solution for a LP and to perceive how sensitive these changes are to the optimal solution. In LP, the parameters of the model can change inside specific limits without making the optimal solution change. This is alluded to as sensitivity analysis. It is likewise alluded to as what-if or simulation analysis is a way to foresee the result of a decision given a specific range of variables. By making a given set of variables, the analyst or the modeler can decide how changes in a single variable effect the result. In LP models, the parameters are generally not exact. With sensitivity analysis, we can discover the effect of the uncertainty on the nature of the optimal solution [11]. Considering two cases, first, the Sensitivity of the optimal solution for change in the accessibility of resources (right-hand side of the limitations). Second, the Sensitivity of the optimal solution for changes in unit profit or unit cost. In all models, parameters are pretty much uncertain. It can be conveyed by making few changes in the model and perceive how they influence the outcomes. These changes may incorporate [12,13]:

- Changing the Objective Function coefficient.
- Changing the RHS of a limitation
- Changing the column of a decision variable
- Adding a new variable or activity
- Adding a new constraint

The aim of this paper is to help in decisions making and recommendations of the best course of action by adopting sensitivity analysis on the linear programming model outcomes.

II PROBLEM FORMULATION

A. *General linear programming model*

Max z = $\sum_{j=1}^{n} cjxj$ (Objective function)

Subject to

$\sum_{j=1}^{n} aijxj \leq bi, \quad i = 1,2, \ldots, m$ (Constraints)

$xj \geq 0, \; j = 1,2, \ldots, n$ (non negativity)

Where:
Z = value of overall performance measure
xj = level of activity j (j = 1,2,…,n)
cj = performance measure coefficient for activity j
bi = amount of resource i available (i=1,2,…,m)
aij = amount of resource i consumed by each unit of activity j

B. *Linear Programming in Standard Form*

Every linear program can be converted into standard form:

Max c1x1+c2x2+…+cnxn
Subject to
a11x1+a12x2+…+a1nxn = b1

…. …. ….

am1x1 + am2x2 +…+ amnxn = bm

x1 ≥ 0, … xn ≥ 0

C. *Business Problem Example*

Considering a manufacturer who produces three types of products. The question is how many dozens of each type of products should be produced to obtain a maximum profit?

Table 1: Shows three types of products and time requirement

Process	Type A	Type B	Type C	Total Time Available
Molding	1	2	3/2	12,000
Trimming	2/3	2/3	1	4,600
Packaging	½	1/3	½	2,400
Profit	$11	$16	$15	—

Let Type A, Type B and Type C be represented with x1, x2, x3, respectively. The linear programming model becomes:

Max Z= $11x1 + $16 x2+ $ 15x3

Subject to:

$x_1 + 2x_2 + 3/2x_3 \leq 12{,}000$
$2/3x_1 + 2/3x_2 + x_3 \leq 4{,}600$
$1/2x_1 + 1/3x_2 + 1/2x_3 \leq 2{,}400$
$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

III MODEL SOLUTION

The method used to solve the problem is the simplex method. After which, a sensitivity analysis was used to run some what-if changes to the constraints and coefficients of the objective function.

A. *Using the Simplex Method*

Max Z = $11x_1 + 16x_2 + 15x_3 + 0S_1 + 0S_2 + 0S_3$
Subject to:

$x_1 + 2x_2 + 3/2x_3 + S_1 = 12{,}000$
$2/3x_1 + 2/3x_2 + x_3 + S_2 = 4{,}600$
$1/2x_1 + 1/3x_2 + 1/2x_3 + S_3 = 2{,}400$
$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

Table 2: Initial simplex tableau

SV	X1	X2	X3	S1	S2	S3	b
S1	1	2	3/2	1	0	0	12000
S2	2/3	2/3	1	0	1	0	4600
S3	1/2	1/3	1/2	0	0	1	2400
P	11	16	15	0	0	0	

Selecting the highest contribution in P row this is referred to as the ***pivot column* = 16**, divide each value in the solution variable (i.e. b) at the right hand side by each element in the pivot column.

2nd Tableau

SV	X1	X2	X3	S1	S2	S3	b
S1	1	2	3/2	1	0	0	12000 ÷ 2 = 6000
S2	2/3	2/3	1	0	1	0	4600 ÷ 2/3 = 6900
S3	1/2	1/3	1/2	0	0	1	2400 ÷ 1/3 = 7200
P	11	16	15	0	0	0	

3rd Tableau

SV	X1	X2	X3	S1	S2	S3	b
S1	1	2	3/2	1	0	0	12000 ÷ 2 = 6000
S2	2/3	2/3	1	0	1	0	4600 ÷ 2/3 = 6900
S3	1/2	1/3	1/2	0	0	1	2400 ÷ 1/3 = 7200
P	11	16	15	0	0	0	

4th Tableau

		X1	**X2**	**X3**	**S1**	**S2**	**S3**	**b**
R1	**S1**	1/2	1	3/4	½	0	0	6000
R2	**S2**	2/3	2/3	1	0	1	0	4600
R3	**S3**	1/2	1/3	1/2	0	0	1	2400
R4	**P**	11	16	15	0	0	0	

5th Tableau

		X1	X2	X3	S1	S2	S3	b
R1	**X1**	1/2	1	¾	½	0	0	6000
R´2	**S2**	1/3	0	-1/3	1	0	0	600
R´3	**S3**	1/3	0	¼	1/6	0	1	400
R´4	P	3	0	3	-8	0	0	-96000

6th Tableau

	X1	X2	X3	S1	S2	S3	b
X2	1/2	1	3/4	1/2	0	0	6000
S2	1/3	0	-1/3	1	0	0	600
S3	1/3	0	1/4	-1/6	0	1	400
P	3	0	3	-8	0	0	-96000

7th Tableau

	X1	**X2**	**X3**	**S1**	**S2**	**S3**	**B**
X2	1/2	1	3/4	1/2	0	0	6000 ÷ ½ = 12000
S2	1/3	0	-1/3	1	0	0	600 ÷ 1/3 = 1800
S3	1/3	0	1/4	-1/6	0	1	400 ÷ 1/3 = 1200
P	3	0	3	-8	0	0	-96000

8th Tableau

		X1	**X2**	**X3**	**S1**	**S2**	**S3**	**B**
R1	**X2**	1/2	1	¾	1/2	0	0	6000
R2	**S2**	1/3	0	-1/3	1	0	0	600
R3	**S3**	1	0	¾	-1/2	0	3	1200
R4	**P**	3	0	3	-8	0	0	96000

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9th Tableau

	X1	X2	X3	S1	S2	S3	B
X2	0	1	3/8	¾	0	-3/2	5400
S2	0	0	¼	-1/6	1	-1	200
X1	1	0	¾	-1/2	0	3	1200
P	0	0	¾	-13/2	0	-9	-99600

10th Tableau

	X1	X2	X3	S1	S2	S3	B
X2	0	1	3/8	¾	0	-3/2	5400 ÷ 3/8 = 14400
S2	0	0	¼	-1/6	1	-1	200 ÷ ¼ = 800
X1	1	0	¾	-1/2	0	3	1200 ÷ ¾ = 1600
P	0	0	¾	-13/2	0	-9	-99600

11th Tableau

		X1	X2	X3	S1	S2	S3	B
R1	**X2**	0	1	3/8	¾	0	-3/2	5400
R2	**S2**	0	0	1	-2/3	4	-4	800
R3	**X1**	1	0	3/4	-1/2	0	3	1600
R4	**P**	0	0	¾	-13/2	0	9	-99600

12th Tableau

	X1	X2	X3	S1	S2	S3	B
X2	0	1	0	1	-3/2	0	5100
X3	0	0	1	-2/3	4	-4	800
X1	1	0	0	0	-3	6	600
P	0	0	0	-6	-3	-6	-100200

From the final tableau, one can see that the maximum profit is $100,200. The optimal solution is 100200, with 600 dozen units of Type A (x1), 5,100 dozen units of Type B (x2), and 800 dozen units of Type C (x3). Substituting the decision variable values, x1=600, x2=5100, x3=800 into the objective function gives

11(600) + 16(5100) + 15(800)
= 6600 + 81600 + 12000
= 100,200

This values coincides with the results obtained using LINDO

IV. SENSITIVITY ANALYSIS

To know what happens to the solution as some parameters change in value, the following are carried out:

- Changes in the objective function
-. Changes in the resources available.
-. Changes in the constraint of the problem.

A. Changing Objective Function

Assuming the company was able to increase profit of product Type B (x_2) to $18, what would be an appropriate recommendation to the manager? Should the production plan change or should the management stick to the existing one?
The Objective function and thus the entire model changes to;

Max Z = $11x_1 + \mathbf{18x_2} + 15x_3$

Subject to:

$$x_1 + 2x_2 + 3/2x_3 \leq 12{,}000$$
$$2/3x_1 + 2/3x_2 + x_3 \leq 4{,}600$$
$$1/2x_1 + 1/3x_2 + 1/2x_3 \leq 2{,}400$$
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

Carrying out the simplex method to get the final tableau to make comparisons, the final optimal solution table for the new model is:

	X1	X2	X3	S1	S2	S3	RHS
X2	0	1	3/8	1/4	0	3/2	5400
S2	0	0	1/4	0	1	-1	200
X1	1	0	3/4	1/2	0	3	1200
P	0	0	0	-10	0	-6	-110400

The new optimal solution when Product Type B (x_2) was increased from \$16 to \$18 is 110400
Where: Type A (x_1) = 1200, and Type B (x_2) = 5400.
From the final optimal tableau, only x1 and x2 products generated an optimal profit of 110400 compared to the previous objective function where product Type B was \$16 for profit.
Increasing Type B to \$18 would enable the company to increase optimality with only two products (A and B), eliminating product C. the company should therefore change production plan to concentrate on producing Type A and Type B at profit \$11 and \$18 respectively

B. Changing the Coefficient of a Decision variable

Suppose the company has implemented some processing improvement methods and has been able to reduce the resources required for the production of Type B (x_2), does the production plan change?
What if the x_2 column is changed (resources with respect to time) from 2, 2/3, 1/3 to 1, 1 and 1 with the profit changed to \$16. The new model is as follows:

Max Z= 11 x_1 + **16** $\mathbf{x_2}$ + 15 x_3
$x_1 + \mathbf{x_2} + 3/2x_3 \leq 12{,}000$
$2/3x_1 + \mathbf{x_2} + x_3 \leq 4{,}600$
$1/2x_1 + \mathbf{x_2} + 1/2x_3 \leq 2{,}400$
$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

From the final tableau using LINDO, the optimal solution is 69575.76, when x3, and x1 are produced. Compared to the initial optimal profit of 100200, it is would be a total loss if the manager venture into the new plan.

C. Adding a New Activity

This is most times seen in a production company as adding a new product to the pool of products you are already producing.
Suppose a new activity (Type D i.e. a new x_4) added to the production system and its resource consumption as well as its profit is given below, with Profit contribution of \$25 for x_4 (Type D plastic) and Resources (with respect to time for the three process; molding, trimming and packaging) = (2, 1, 2)
The new LP model is given as:

Max Z = $11x_1 + 16x_2 + 15x_3 + 25x_4$
Subject to:
$x_1 + 2x_2 + 3/2x_3 + 2x_4 \leq 12{,}000$
$2/3x_1 + 2/3x_2 + x_3 + x_2 \leq 4{,}600$
$1/2x_1 + 1/3x_2 + 1/2x_3 + 2x_3 \leq 2{,}400$
$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

The optimal solution when a new product is added to the pool of products produced is 101675, where x1 =0, x2 = 5000, x3= 960 and x4 = 310. This yields a better profit compared to the first optimal solution of 100200. The manager should adapt to the new production plan by introducing product D.

V. RESULTS AND DISCUSION

The optimal solution of the original problem is 100200, with 600 units of Type A (x1), 5,100 units of Type B (x2), and 800 units of Type C (x3). This implies that the profit is maximized when 600, 5100 and 800 units of time are used in producing the three types of plastic. Changing the objective function by altering the profit contribution of type B plastic from 16 to 18 dollars improved the optimal profit, from is \$100,200 to \$110,400. The time required to produce Type A (x_1) reduced to 1200 units while time for Type B (x_2) increased to 5400 units. When the x_2 column is changed (resources with respect to time) from 2, 2/3, 1/3 to 1, 1 and 1 with the profit contribution of \$16, the optimal profit decreased to 69575.76 dollars. This implies suppose the company has implemented some processing improvement methods and has been able to reduce the resources required to produce Type B plastic (x_2), reduces the profit. So in this situation it is advisable stick to the previous production plans. Suppose a new activity (Type D i.e. a new x_4) added to the production system with Profit contribution of \$25 and Resources, with respect to time for the three processes; molding, trimming and packaging equals 2, 1, 2 respectively. It was noticed that the optimal profit is 101675, which is an improvement over the original optimal profit of 100200. So a new type of product can be carefully incorporated into the production plan.

VI. CONCLUSION

This paper set to investigate how the optimal profit of a manufacturing outfit can improved by carrying out sensitivity analysis on her business plan modeled as a linear programming problem. Solving the new resulting models because of changes in some decision variables and parameters produced different optimal profits. When compared with the original optimal profit, there was an improvement in some cases and reduction in optimal profit in other cases. Specifically, in the example considered, the management can be advised to add another plastic type to their production line. Also the management can be advised to increase the profit contribution of type B plastic. Both advises if adhered to will lead to an improvement in the optimal profit. However, the management should advised against reducing the time resources required for the production of type B plastic, as this will reduce the optimal profit.

In conclusion, sensitivity analysis on any Linear programming optimal solution helps to proffer better optimal solution if possible compared to the previous assumed optimal solution. With this, Manufacturing companies and other related firms will have a better confidence in running their business; not based on guesses, mere assumptions or approximations but on facts.

Appendices

```
LP OPTIMUM FOUND AT STEP      0

       OBJECTIVE FUNCTION VALUE

       1)      100200.0

 VARIABLE        VALUE          REDUCED COST
       X1       600.000061           0.000000
       X2      5100.000000           0.000000
       X3       799.999756           0.000000

      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000           6.000000
       3)         0.000000           3.000000
       4)         0.000000           6.000000

NO. ITERATIONS=       0
```

```
LP OPTIMUM FOUND AT STEP      0

       OBJECTIVE FUNCTION VALUE

       1)      110400.0

 VARIABLE        VALUE          REDUCED COST
       X1      1200.000366           0.000000
       X2      5400.000000           0.000000
       X3         0.000000           0.000000

      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000           8.000000
       3)       199.999756           0.000000
       4)         0.000000           6.000000

NO. ITERATIONS=       0
```

```
LP OPTIMUM FOUND AT STEP      2

       OBJECTIVE FUNCTION VALUE

       1)      69575.76

 VARIABLE        VALUE          REDUCED COST
       X1       606.060608           0.000000
       X2         0.000000          19.757576
       X3      4193.939453           0.000000

      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)      5103.030273           0.000000
       3)         0.000000          12.121212
       4)         0.000000           5.757576

NO. ITERATIONS=       2
```

```
LP OPTIMUM FOUND AT STEP      3

       OBJECTIVE FUNCTION VALUE

       1)      101674.8

 VARIABLE        VALUE          REDUCED COST
       X1         0.000000           0.292327
       X2      4969.549316           0.000000
       X3       959.805115           0.000000
       X4       310.596832           0.000000

      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000           5.557247
       3)         0.000000           5.389769
       4)         0.000000           4.247869

NO. ITERATIONS=       3
```

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