Sloshing in Vertical Circular Tanks and Earthquakes Perturbations

Frank Otremba and José A. Romero Navarrete

***Abstract*—A simplified formulation is proposed in this paper to assess the proximity of the earthquake-related perturbation frequencies to the natural sloshing frequencies of the liquid contained in vertical cylindrical tanks. The methodology is based upon an existing gravity-waves approach, which was developed for rectangular cross-section reservoirs, and is extended in this paper to analyze circular cross-section tanks. The experimental outputs of this paper show that the existing methodology correlates at 100% with experimental data in the case of rectangular containers; while the corresponding average error in the case of a conical container and a cylindrical container, is 7% and 9.1 %, respectively. The full diameter of the cross section was considered. The use of so-validated methodology to full scale tanks, suggests that cylindrical vertical tanks with a capacity lower than 700 m^3, could be exposed to a resonance excitation when subjected to earthquake motions, regardless of the fill level.**

***Index Terms*—Sloshing, vertical circular tanks, resonance, earthquakes, mathematical modelling, experimental approaches**

I. INTRODUCTION

THE motion of the liquid within its container can be the result of different perturbations, as a function of the use given to such container, whether it is an stationary container or a transporting container. While the sloshing condition in transport containers are mainly associated to perturbations derived from the maneuvers performed by the carrying vehicle, as a function of the mode of transport, in the case of the stationary containers, the source for the sloshing phenomena is associated to environmental perturbations, whether linked to other equipment´s vibration, or to natural perturbations, due to earthquakes. The phenomenon of sloshing has been the aim of several research efforts, to disclose the potential effects of low frequency excitations on the integrity of vertical axis containers [1, 2, 3]. The perturbations on structures, derived from soil displacements associated to earthquakes are characterized by their acceleration and frequency [4, 5], amongst other factors, including the maximum acceleration and duration of the earthquake; the frequency of the prevailing waves; the amplitude and frequency relationship between horizontal and vertical movement; and the distance from the epicenter [6].

A. Perturbation frequencies

There is a wide range of vibration frequencies associated to soil movements, from a fraction of Hertz to a few dozens of Hertz. However, the predominant frequencies are always on the long period vibrations. Figures 1 and 2 illustrate some data regarding the spectral content from 20 far-field earthquake ground motion records, converted to frequency domains from the vibration period spectral data reported in [7]. While Figure 1 illustrates the percentiles, Figure 2 depicts the corresponding average spectrums. According to these data, the dominant frequencies for such seismic motions are around 1 to 2 Hz, however, there is significant activity in the range of very low frequencies.

It is the purpose of this paper, to disclose horizontal, earthquake-related potential resonant perturbations to vertical circular tanks, whether cylindrical or conical. An existing rectangular container gravity-waves formulation is used in this study, whose scope is extended for analyzing circular-cross sections vertical tanks.

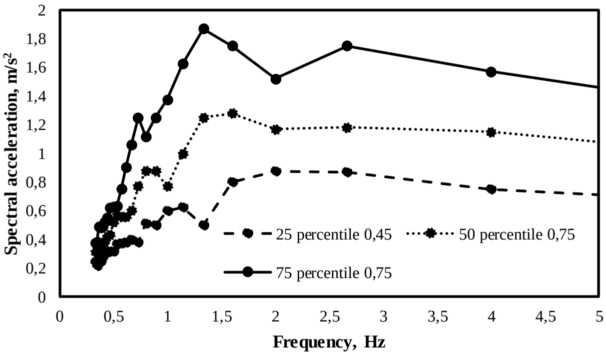

Fig. 1. Measured spectral data from earthquake

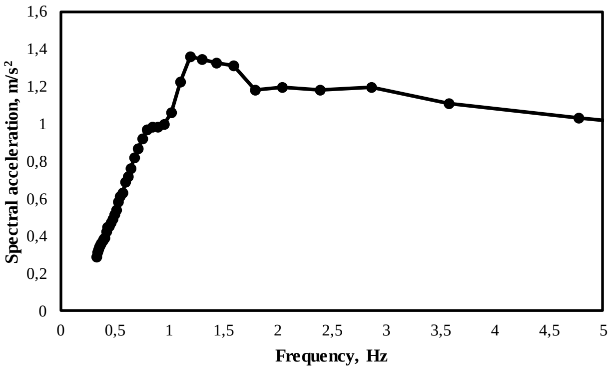

Fig. 2. Measured spectral data from earthquake

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II. EXPERIMENTAL DATA – VALIDATION

The testing rig consists of a wheeled vehicle that runs on tracks and supports the testing tanks. To provide the motion to this vehicle, a pulling mechanism was designed, consisting of a falling mass attached to the vehicle chassis through a string. The instrumentation in the equipment to measure the wheel forces and to obtain the respective signals to analyze the sloshing frequency of the contained fluid, was based upon strain gages, adhered to the internal surface of grooves machined in the tracks.

Figure 3 illustrates a schematic representation of the testing rig. The impact bearing against the vehicle´s wheels impacted to get the horizontal perturbation into the vehicle, consisted of a plate, attached to a retraining wall. Four strain gages were considered to obtain the wheel forces signal. The position of such strain gages is illustrated in Figure 4, corresponding to the front and rear sides of the vehicle.

Figure 5 shows a photograph of the testing rig, includthe screen showing the data. In this respect, Figure 6 presents a screen shot of the data acquisition system, which is based on the *CatmanEasy*.

The sample rate for the data acquisition was 300 Hz, in order to have a good resolution of the data. In every testing case, the working fluid was tap water.

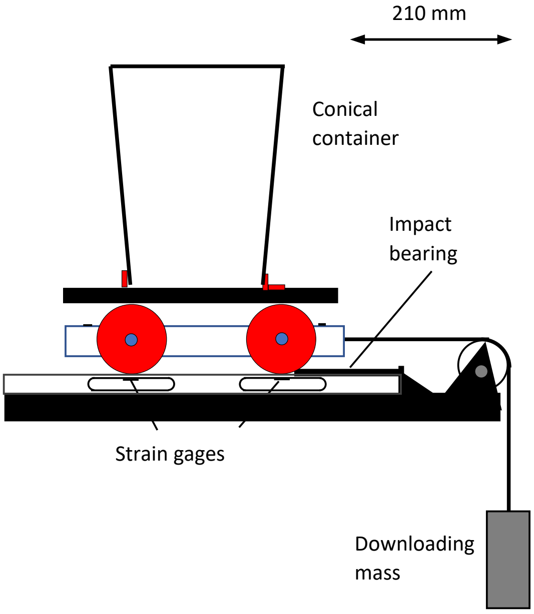

Fig. 3. Schematic representation of testing installation.

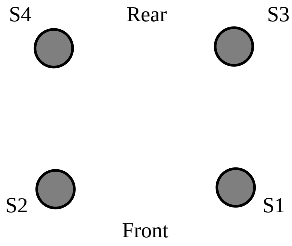

Fig. 4. Four strain sensors location.

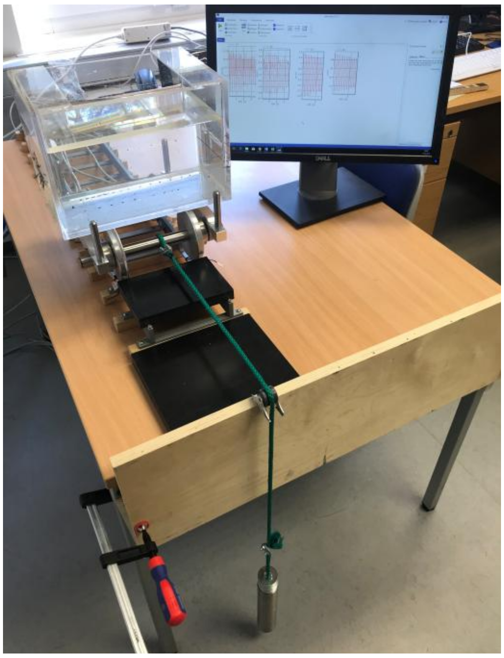

Fig. 5. Testing rig. (shown for the rectangular container)

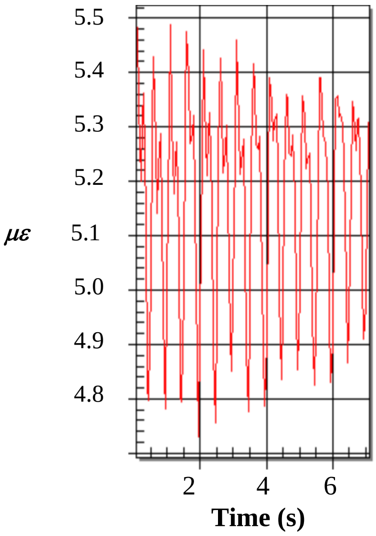

Fig. 6. Data acquisition system output, One of four channels.

B. *Theoretical approach*

The formulation considered to estimate the sloshing frequency of fluids in containers, is based upon the principles of gravity waves, which were developed in the case of a rectangular prismatic vessel having a certain fill level and a free surface. The reported basic equation for the natural sloshing frequency is described as follows [8]:

$$f = \frac{1}{2L}\left(\frac{g}{\kappa}\tanh \kappa H\right)^{1/2} \qquad (1)$$

where H is the depth of the fluid in the vessel, L is the free surface length of the fluid, κ is the wave number, equal to $2\ \pi\ /\lambda$, where $\lambda = 2\ L$. Figure 7 illustrates the different factors involved in the above equation.

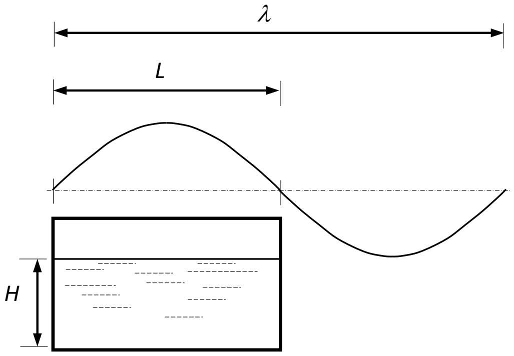

Fig. 7. Illustration for the gravity waves principles.

C. Rectangular container

The different parts of Figure 8 illustrate the time histories of the different channels, in the case of a rectangular – 10 liters – container filled at a 142 mm height. It should be noted that in these time histories, it is included the impact moment and the residual sloshing condition.

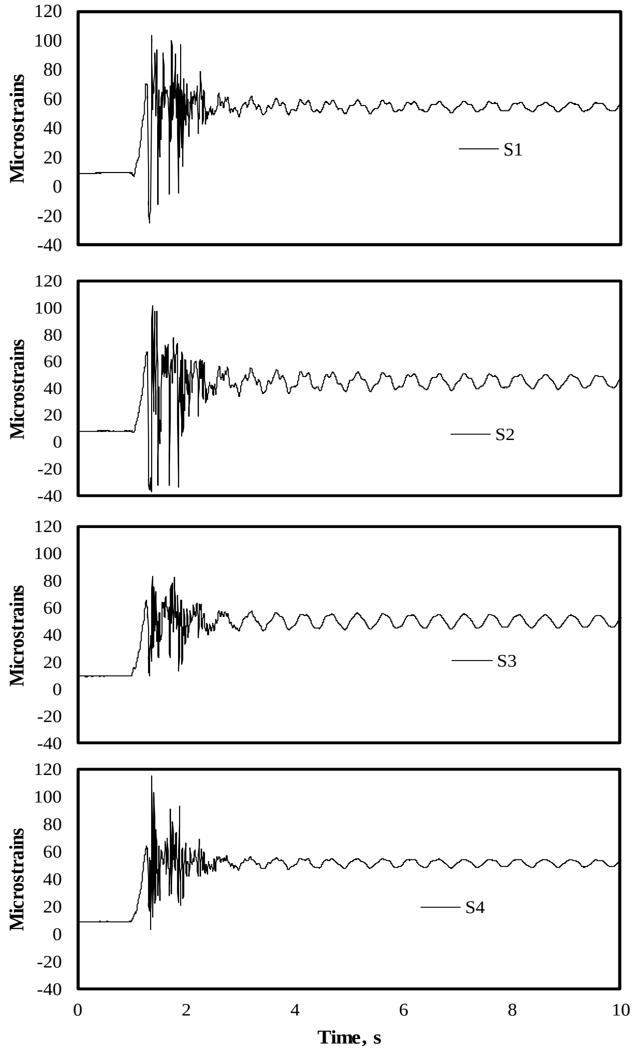

Fig. 8. Time histories of the strain at the four sensors. Rectangular container, maximum fill level (142 mm).

[illegible] of the theoretical approach in the case of the rectangular container. In this table there is a column regarding the outputs from the gravity waves formulation described in Eq. (1).

Figure 9 illustrates a zooming of the time output data, which was used to measure the period of the sloshing vibration. It should be noted that while there was a clear dominant frequency, some other frequencies can be identified. Figure 10 illustrates the comparison between the outputs of the experimental and the theoretical approaches, as a function of the fill level, or height of the free surface. It can be observed in these results that the theoretical approach renders a very close reproduction of the experimental data, with an average error of only 0.4 %.

TABLE I
VALIDATION OF GRAVITY WAVES FORMULATION

Test	Fluid height, mm	Theoretical Sloshing Period, s
1	142	0.4906
2	117	0.497
3	97	0.505
4	77	0.523
5	60	0.554

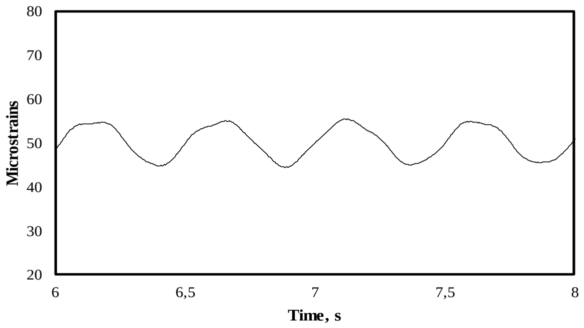

Fig. 9. Time history to estimate signal period (rectangular container)

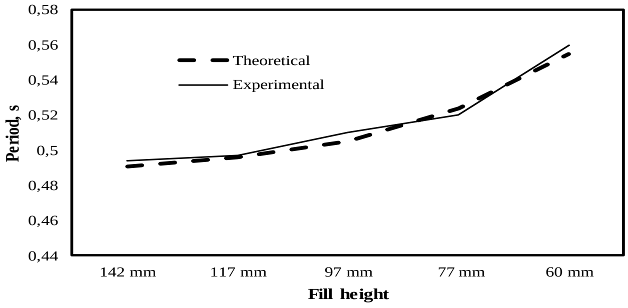

Fig. 10. Comparison of theoretical and experimental data of sloshing period, Rectangular container.

D. Cylindrical container

A cylindrical 106.6 mm – diameter container was considered to assess the applicability of the gravity waves formulation to such container shape. Figure 11 illustrates the outputs from the theoretical and experimental approaches. The corresponding errors between both approaches are presented in Figure 12. According to these data, there is a maximum discrepancy between both sets of data at 95 mm, which slightly diminishes with the lower fill levels. Figure 13 further illustrates these outputs in terms of the height to length ratios, which suggest that lower values for such ratios generate the minimum errors, for this tank shape.

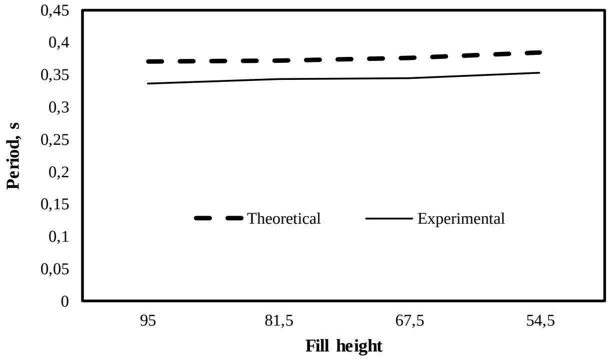

Fig. 11. Theoretical and experimental values for a cylindrical container.

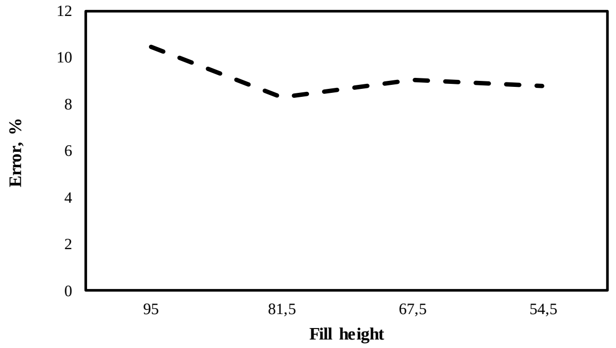

Fig. 12. Error for the cylindrical container.

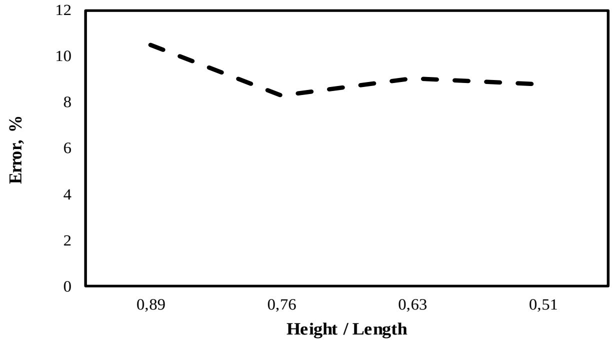

Fig. 13. Error for the cylindrical container, in terms of the height/length ratio.

E. Conical container

A conical tank is considered to assess the applicability of the theoretical formulation, to estimate the natural sloshing frequency of fluids in such tank shapes. Figure 14 illustrates the relationship between the height of the fluid surface and the corresponding diameter, or maximum free surface length. The conicity of the tank was 6.64 °. Figure 15 illustrates the comparison between the experimentally and the theoretically generated data, concerning the period of the sloshing motion, as a function of the fluid level height. It should be noted that the theoretical approach includes two outputs: one considering the diameter at the free surface, and the other one the average diameter for the whole fill height. According to the respective errors or difference between the experimental and the theoretical data, illustrated in Figure 16, the consideration of the average diameter generates a better match between the theoretical and the experimental data. Furthermore, it can be observed that the lower fill levels generate the maximum discrepancy between the experimental and the theoretical data. In this respect, the height/length ratio for which the error increases beyond 4%, is 0.4. That is, for this tank shape, the better correlations between the experimental and the theoretical data, are attained when the height of the fill level represents more than 40% of the length of the free surface. For example, for a height/length ratio of 0.78 (232 mm diameter), the error is just 0.03%, if the average diameter is considered.

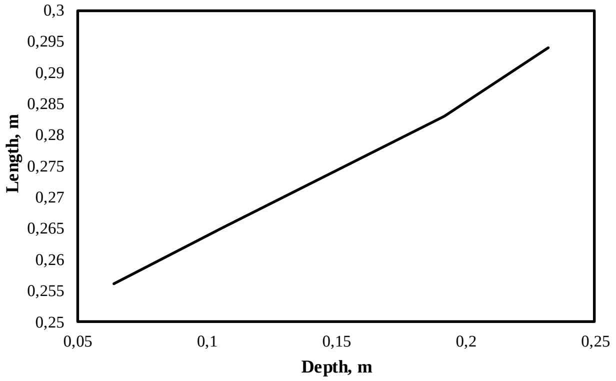

Fig. 14. Free surface length – fill depth graph for the conical container.

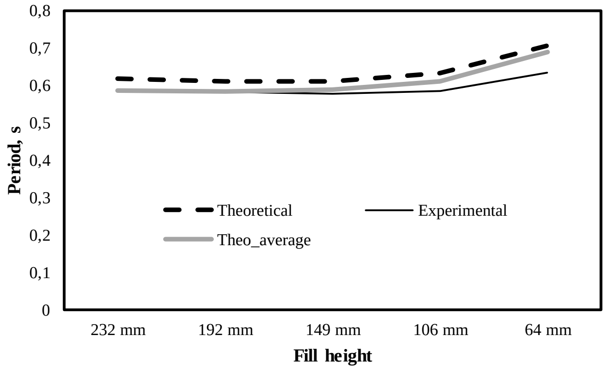

Fig. 15. Comparison of the theoretical and experimental data of the sloshing period, conical container.

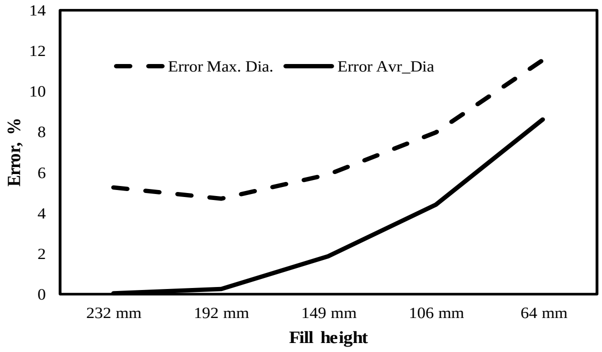

Fig. 16. Error with respect to the measured data, in the case of the conical container.

Consequently, the gravity-waves theoretical formulation is better applicable for relatively large fluid depths, to estimate the natural sloshing frequencies of the confined liquid (water).

F. *Estimation for full-scale tanks (Cylindrical)*

The formulation described above is used to estimate the natural sloshing frequencies in the case of full-scale vertical cylindrical tanks, for three different fill levels: 37.5%, 50%, 75 % and 100 %. The data used for this estimation, derives from commercial information [9]. Table II list the properties of such commercial tanks, in terms of their diameter and full height, with capacities from 100 m^3 to 50000 m^3, involving big constructions to storage a variety of liquids, such as fuels or water.

For these data, Figures 17 to 20 illustrate the estimated natural sloshing frequencies for the contained cargo for 37.5%, 50%, 75% and 100 % fill level, respectively. According to these estimations, some combinations of containers fill level and capacity, would be subjected to close to resonance situations when exposed to earthquakes.

In these figures, the range above 0.3 Hz is marked, in order to distinguish what capacity-fill level combinations would be subjected to a resonance excitation due to earthquakes, according to the measured data presented in figures 1 and 2.

It results that the combinations of tank capacity and fill level that would be prone to be subjected to resonance excitations, are tank capacities below 700 m^3, regardless of the fill level.

While the amount of resultant combinations is small when compared with the total amount of possible capacity-fill level combinations, it should be considered that the number of tanks within this range of capacity, could be the most commonly used by the different industries and commercial facilities for the storage of a variety of fluids.

On the other hand, the approach considered to perform these estimations of the natural sloshing frequencies in commercial tanks, could be considered useful and straightforward, with error estimates around 10%.

TABLE II
COMMERCIAL TANKS DATA (WITH DATA FROM [9])

Capacity, m^3	*Diameter, m*	Full height, m
100	4,73	6
200	6,63	6
300	7,58	7,5
400	8,53	7,5
500	8,45	9,25
700	10,43	9
1000	10,43	12
2000	15,18	12
3000	18,98	12
5000	22,8	12
5000	20,9	15
10000	34,2	12
10000	28,5	17,9
20000	39,9	17,8
30000	45,6	18
50000	60,7	18

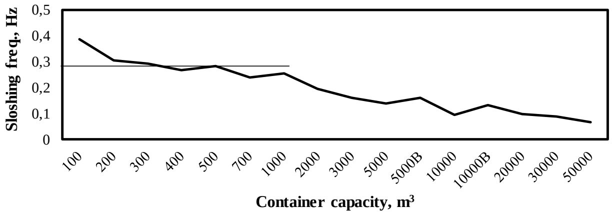

Fig. 17. Estimated sloshing frequency, commercial tanks, 37.5 % fill level.

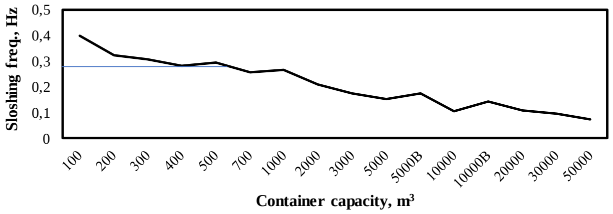

Fig. 18. Estimated sloshing frequency, commercial tanks, 50 % fill level.

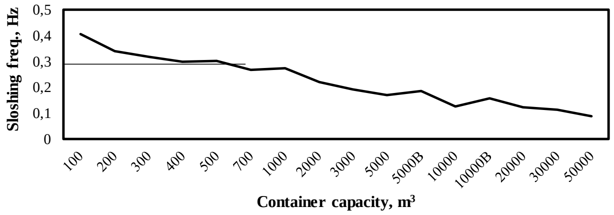

Fig. 19. Estimated sloshing frequency, commercial tanks, 75 % fill level.

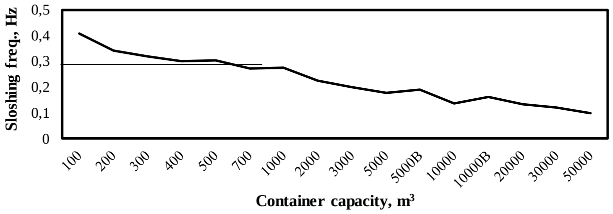

Fig. 20. Estimated sloshing frequency, commercial tanks, 100 % fill level.

III. CONCLUSIONS

A simplified methodology has been proposed in this paper, to assess the proximity of the earthquake-derived perturbation frequencies to the natural sloshing frequencies of the liquids inside vertical circular tanks. While the methodology correlates at 100% with experimental data in the case of rectangular containers, the corresponding correlation with experimental cylindrical and conical containers, produces average differences of 9.1% and 7%, respectively. The use of such so-validated methodology to analyze full scale tanks, suggests that tanks with a capacity below 700 m^3, could be exposed to a resonance excitation when subjected to earthquake motions, regardless of the fill level. Further studies should be carried out, to assess the dynamic forces associated to such resonance and close-to-resonance situations, in a context of a standard overloading situation due to sloshing forces.

REFERENCES

[1] R. Kianoush, and J. Z. Chen, “Effect of vertical acceleration on response of concrete rectangular liquid storage tanks,” *Engineering Structures*, vol. 28, no. 5, pp. 704-715, 2006.

[2] I. Gavrilyuk, I. Lukovsky, Y. Trotsenko, and A. Timokha, “Sloshing in a vertical circular cylindrical tank with an annular baffle. Part 1. Linear fundamental solutions,” *Journal of Engineering Mathematics*, vol. 54, pp. 71-88, 2006.

[3] V. Gnitko, Y. Naumenko, and E. Strelnikova, “Low frequency sloshing analysis of conical containers with flat and conical baffles,” Int. J. of Applied Mechanics and Engineering, vol. 22, no. 4., pp. 867-881, 2017.

[4] A. H. Hadjian, “Soil-structure interaction – an engineering evaluation,” *Nuclear Engineering and Design*, vol. 38, pp. 267-272, 1976.

[5] M. Kostov, “Seismic safety evaluation based on DIP,” *Nuclear Engineering and Design*, vol 269, pp. 256-261, 2014.

[6] Federal Ministry for the Environment, Nature Conservation and Nuclear Safety, Committee for pipelines (AfR), “The assessment of the endangerment of earthed pipelines by earthquakes in German earthquake areas”, *AfR Working Group on Earthquakes*, December 2011. (In German).

[7] A.H.M. Munstasir Billah, M. Shahria Alam, and M. A. Rahman Bhuiyan, “Fragility Analysis of Retrofitted Multi-Column Bridge Bent Subjected to Near Fault and Far Field Ground Motion,” *Journal of Bridge Engineering*, vol 18, no. 10, pp. 992-1004, 2013.

[8] Elmore, C.W., and Heald, M.A. *Physics of waves*, Dover Publications Inc. New York, 1989.

[9] RTW, *Vertical cylindrical steel tanks*, Catalog, Russian Tank Works, recovered 18.04.2019 from http://russiantankworks.com/catalog/vertical-storage-steel-tanks/