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Electrical Resistance of Superconductors

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***Abstract*—A theoretical model has been created to explain why the electrical resistance of superconductors reaches zero. Although a description of electron behavior in solids requires some knowledge of quantum physics, understanding this electrical transport phenomenon in this model only requires knowledge of electrical circuits. Therefore, electrical circuit theory is used to help explain the electrical resistance behavior in superconducting materials.**

***Index Terms*—atomic lattice, conductivity, electron, temperature, theoretical model**

I. Introduction

Superconductivity is a phenomenon where electrons experience no electrical resistance and thus can flow through a material unimpeded [1–4]. It is well known that electrical resistance is a function of temperature that typically decreases as temperature decreases. Superconductors have no electrical resistance when their temperature decreases below a critical temperature. Below the critical temperature, conduction electrons in superconductors will not lose energy due to interactions with lattice atoms.

The core of electrical engineering involves transmitting energy or information from one point to another efficiently and reliably. Therefore, understanding superconductivity is especially important in the field of electrical engineering. Electrical engineering specializes in generating and transmitting high-power electricity as well as constructing microelectronic circuitry for various applications. So, advances in superconductivity will have a major impact on major industrial sectors such as electrical power transmission companies as well as on integrated circuit and computer technology firms [5,6].

To understand the phenomenon of superconductivity and to make progress in superconductivity research, theoretical models are needed. As a result, various microscopic or atomic theories have been developed [4]. It is understood that although not all theories are entirely correct, even incorrect or partially correct theories can help lead researchers to correct understanding of phenomena. The BCS superconductivity theory is the most successful theory because it explains certain aspects of type I superconductors through the formation of Cooper paired electrons and their interaction with lattice atoms. However, the BCS theory has difficulty in explaining why type II materials (or equivalently high temperature superconductors) exhibit superconducting properties since Cooper pairs cannot form at higher temperatures. Additionally, there isn’t a theory that clearly demonstrates why the electrical resistance of superconductors has a drastic change in resistance at their critical temperature. Therefore, a new theory is needed to explain the electrical resistance behavior.

A theoretical model has been developed using atomic theory to explain the relationship between electrical conductivity and temperature [7]. This theory produces a relationship between conductivity and temperature from basic principles and then demonstrates its accuracy by comparing it to known linear responses from platinum and nickel.

However, this model did not specifically account for superconducting effects, so it has been modified to provide a complete description of electrical conductivity. This will be accomplished by analyzing conduction electron interactions with atoms at low temperatures. The electrical resistance can then be developed and compared to the response of superconducting materials to demonstrate the accuracy of this theoretical model.

II. Theoretical Model

A theoretical model consisting of a two-dimensional lattice structure has been created and will be used to represent the atoms and conduction electrons associated with a conductor [7]. An illustration of this model is shown in Figure 1. Atoms vibrate at a rate that is a function of temperature. Higher temperatures produce higher or larger vibrations. As temperature decreases, the gap between atoms or the unimpeded space will increase. When the gap size increases, a higher proportion of electrons will exist in the gasp ant this will lead to higher electrical conductivity.

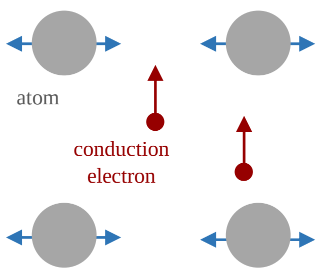

Fig. 1. The atomic model showing a conduction electron (small circle) that will be move through the material unimpeded by the atoms (large circles) and another electron that will encounter some resistance.

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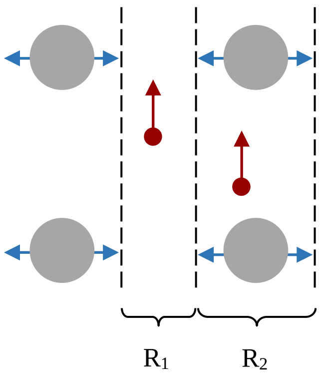

Fig. 2. The atomic model showing the corresponding electrical resistances R_1 and R_2 that conduction electrons could encounter.

The original model produces an equation that describes the relationship between conductivity (or resistivity) and temperature. However, based on the assumptions of the model, it does not attempt to address superconductivity and the behavior of the electrical resistance at sufficiently low temperatures. Therefore, this modified or enhanced model will be used to explain this phenomenon.

Based on Figures 1 and 2, those conduction electrons that are in the gap between atoms will encounter an electrical impedance that will be represented by resistance R_1. Likewise, electrons that will be directly affected or impacted by lattice atoms will encounter an electrical impedance that will be represented by resistance R_2. Since these resistors are in parallel, they can be represented by an equivalent resistance R. In equation form, the equivalent resistance is found from

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad (1)$$

Figure 3 provides an illustration of the two resistors R_1 and R_2 that are in parallel along with the equivalent resistance R. This equivalent resistance R is the same resistance that would be measured in the material. Resistance R is the results of the two paths that electrons can travel in the material.

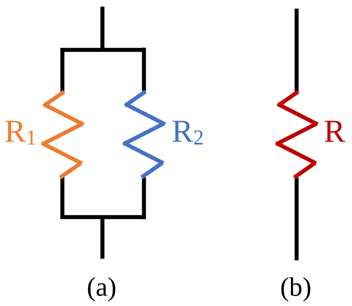

Fig. 3. Resistance of a material. (a) Resistor R_1 represents the path or corridor between lattice atoms and resistor R_2 represents lattice atoms in the material. (b) The equivalent resistance R is the parallel combination of R_1 and R_2.

For resistance R_1, when electrons are in 'gaps', they will only directly encounter other electrons. These other electrons will contribute to the electrical resistivity (which is the inverse of the elecctrical conductivity) in this region. This electrical resistivity will be termperature dependent and thus will be modeled by a linearly decreasing function of temperature.

Since the resistivity ρ associated with R_1 decreases with decreasing temperature, resistance R_1 will also decrease with temperature. Accounting for lattice defects such as grain boundaries and impurities, resistance R_1 will have a non-zero value when T~0 and will appear as shown in Figure 4. This represents the resistance of the gaps in the model regardless of whether the mateial is superconducting or non-superconducting.

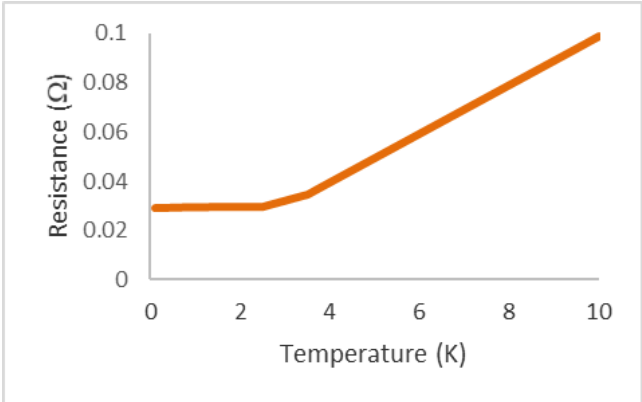

Fig. 4. Example of electrical resistance for electrons in the region of the 'gaps'. This represents the resistance of resistor R_1.

For resistance R_2, when electrons are 'not in gaps', they will be in the path of atoms. These atoms will impede the progress of the conduction electrons and thus contribute to the electrical resistivity (which is the inverse of the elecctrical conductivity) in this region. This electrical resistivity will also be termperature dependent and thus will be modeled by a linearly decreasing function of temperature. Since the resistivity ρ associated with R_2 decreases with decreasing temperature, resistance R_2 will also decrease with temperature.

For superconducting materials, it is presupposed that at the transition temperature ($T = T_C$), resistance R_2 will have a value of zero and remain at zero for temperatures less than T_C ($T < T_C$). Therefore, resistance R_2 will appear as shown in Figure 5. This represents the of resistance in the path of atoms in the model for superconducting materials.

For non-superconducting materials, there is no transition temperature, so it is presupposed that resistance R_2 will have a linearly decreasing response over all temperatures. Therefore, resistance R_2 will appear as shown in Figure 6. This represents the resistance in the path of atoms in the model for non-superconducting materials.

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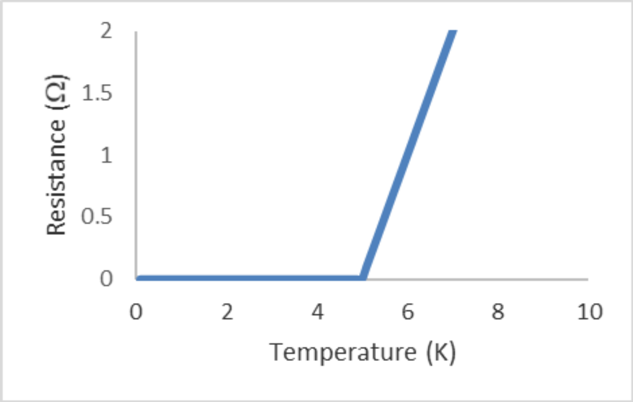


Fig. 5. Example of electrical resistance for electrons in the path of atoms. This represents the resistance of resistor R_2 for superconducting materials.

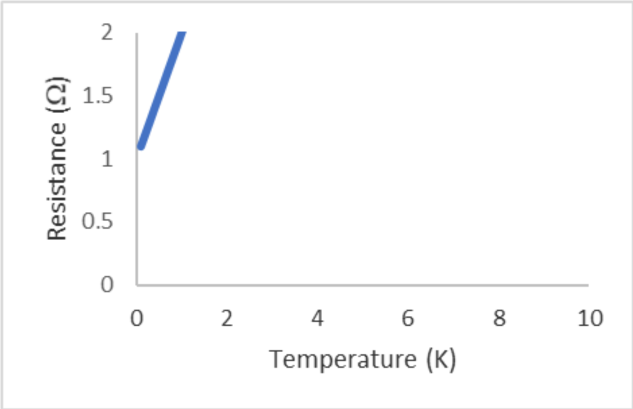
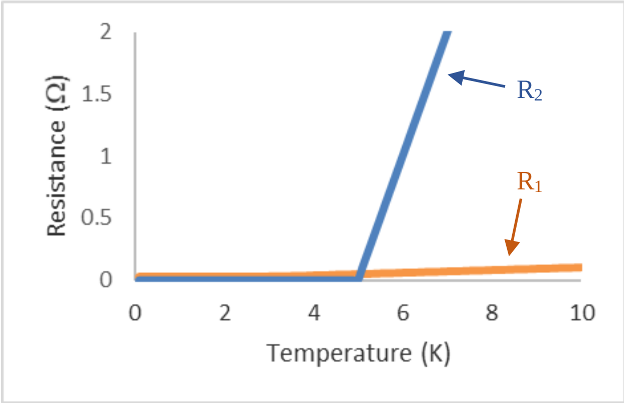
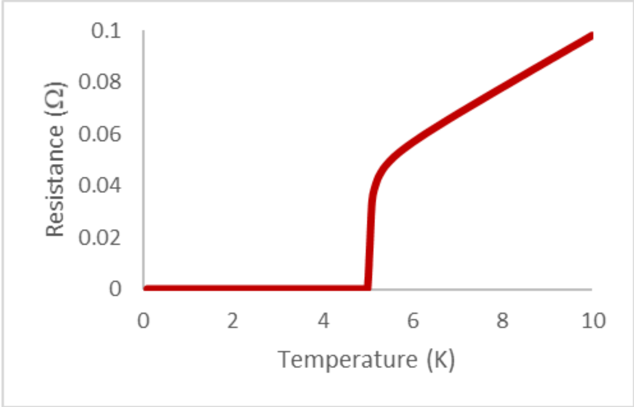


Fig. 6. Example of electrical resistance for electrons in the path of atoms. This represents the resistance of resistor R_2 for non-superconducting materials.



(a)



(b)

Fig. 7. Electrical resistances for a superconducting material. (a) Graph showing the resistance of both R1 and R2 individually. (b) Graph showing the total resistance of the material (i.e., parallel combination of R_1 and R_2).

III. Results

Based on the theoretical model with the parallel resistor concept, this information will be used to demonstrate that this model adequately characterizes the electrical resistance of both superconducting and non-superconducting materials.

Again, resistor R_1 represents the electrical resistance for electrons traveling in the gaps or corridors. Electrons that travel in this region will only directly encounter other electrons and will not directly encounter atoms (with the exception of misaligned atoms or impurities). Resistor R_2 represents the electrical resistance for electrons that are not traveling in the corridors or gaps. Electrons that travel in this region will routinely encounter atoms and electrons and thus will experience large resistance unless proper conditions are met.

When resistances R_1 and R_2 are shown on the same graph, the result is presented in Figure 7a for superconducting materials and Figure 8a for non-superconducting materials. When these resistances are combined in parallel fashion to represent their equivalent value, we obtain the result shown in Figure 7b for superconducting materials and Figure 8b for non-superconducting materials.

For superconductors, the equivalent resistance has a linearly decreasing slope until the temperature approaches the transition temperature T_C and then the resistance abruptly drops to zero. For non-superconductors, the equivalent resistance R resembles R_1 because R_2 is always much larger than R_1 since R_2 never reaches a value of zero. As a result, when the resistance is measured over a wide temperature range, there is no abrupt response because R_2 does not have a transition temperature. Therefore, traveling electrons will never experience zero resistance when they encounter atoms in this material. These responses are typical of the behavior of materials with superconducting and non-superconducting properties [8].

IV. Discussion

A theoretical model has been created to demonstrate and explain why some materials become superconducting at sufficiently low temperatures. (and thus exhibit zero electrical resistance), and why other materials do not. This model produces results that show how electrical resistance changes as a function of temperature. These theoretical results are consistent with experimental results whether the material is classified as a superconductor or not [8].

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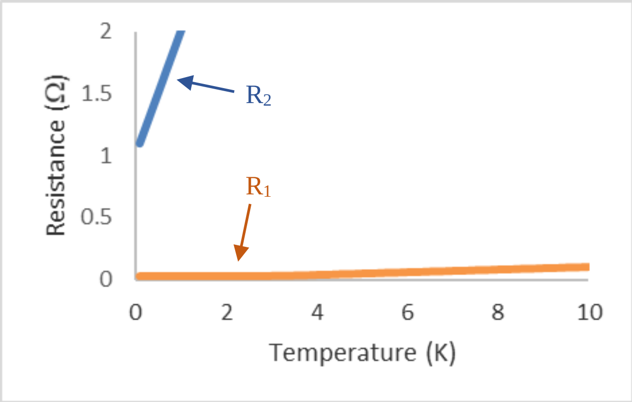
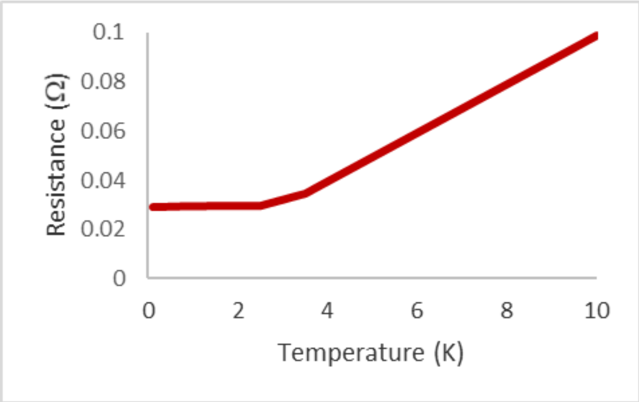


(a)



(b)

Fig. 8. Electrical resistances for a non-superconducting material. (a) Graph showing the resistance of both R1 and R2 individually. (b) Graph showing the total resistance of the material (i.e., parallel combination of R_1 and R_2).

The response of R_1 as shown in Figure 4 will linearly decrease as temperature decreases. This is a reasonable response for superconductors and non-superconductors since the 'gaps' or corridors will widen due to the decreased thermal motion of the atoms. It is widely understood that electrical resistance is directly proportional to length and inversely proportional to cross sectional area, therefore this result is consistent with experimental evidence.

The response of resistor R_2 is also based a linearly decreasing function with respect to temperature. As shown in Figures 5 and 6, these responses are slightly different depending upon whether the material behaves as a superconductor or non-superconductor. Since superconductors have a transition temperature in which the resistance moves from a linear response to a flat response (in which the resistance equals zero), the superconducting resistance response for R_2 is reasonable. Also, since non-superconductors do not exhibit a transition temperature, and basically display a linear response (except when T~0 where impurities and/or lattice defects dominate) the non-superconducting resistance response for R_2 is reasonable.

For superconducting materials, the resistance as shown in Figure 7b is linear for temperatures above the critical temperature, zero when the temperature is below the critical temperature, and has an almost instantaneous transition between these two regions. The total resistance will be similar to the smaller of the two resistance values at a given temperature. Therefore, the resistance at temperatures above the critical temperature will resemble R_1, (since R_1 is much smaller than R_2), and the resistance at temperatures below the critical temperature will resemble R_2, (since R_2 is much smaller than R_1). The resistance at the transition temperature will have a value between the two individual values. Based on the concept of parallel resistors, this model accurately reflects the electrical resistance of superconducting materials.

For materials that do not display superconducting properties, the electrical resistance as shown in Figure 8b exhibits a 'normal' or expected response. The resistance is a linear function of temperature except near T=0 where a constant or residual resistance appears. The purity of the material and/or the number of lattice defects will affect the value of this residual resistance. For non-superconducting materials, the resistance offered by the atoms (i.e., R_2) is always much larger than the resistance in the 'gaps' (i.e., R_1) and therefore only the resistance in the 'gaps' needs to be considered when determining the total resistance.

This drastic change in resistance shown in Figure 7b is based on the ratio of the resistance slopes for $T > T_C$. For example, as shown in Figure 7b, the R_2 / R_1 value of 100 produces an overall resistance that changes from 0.0338Ω to 0. If the resistance ratio changes to 10, the overall resistance changes from 0.00836Ω to 0. If the resistance ratio changes to 1,000, the overall resistance in the transition region changes from 0.0483Ω to 0. The impurity level in the material is another factor that will determine how drastic this resistance change is. If the material has more impurities, it will have a larger resistance change in the transition region. However, since the number of impurities is much smaller than the number of atoms, the impurity level will have very little effect on the resistance for $T < T_C$.

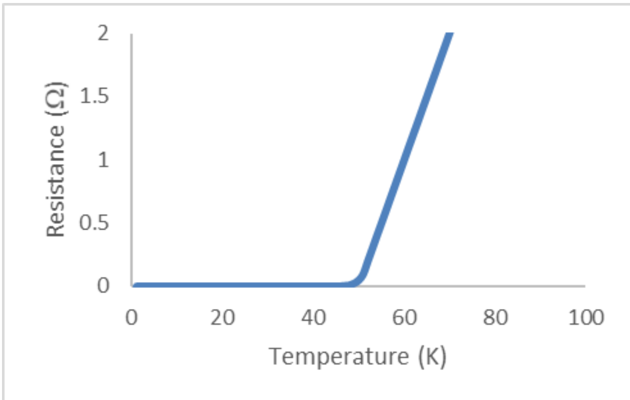


Fig. 9. Electrical resistance for resistor R_2 (resistance that electrons see when traveling in the path of atoms) for type II superconducting materials.

The electrical resistance response as shown in Figure 7b resembles the response for type I superconductors. Research has shown that the electrical resistance for type II superconductors is very similar, but the resistance is less abrupt before it becomes zero [8]. The theoretical model presented in this paper can account for type II material responses by allowing R_2 to have a curved transition instead of a sharp transition at the critical temperature as shown in Figure 5. A graph of the response for R_2 that will produce a type II response is shown in Figure 9.

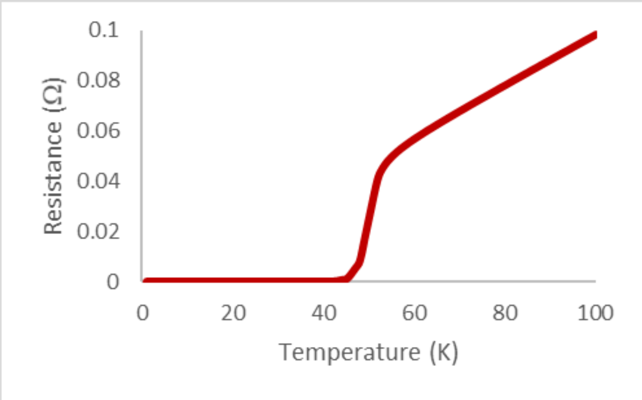


Fig. 10. Graph showing the equivalent electrical resistance of a type II superconducting material after combining resistors R_1 (from Figure 4) and R_2 (from Figure 9) in a parallel manner.

Type II superconductors are composite materials and thus contain different types of atoms. Therefore, the electrical resistance response of type II materials is expected to be more complex than type I materials due to the diversity of atoms that make up type II materials. When the resistance of R_2 as shown in Figure 9 is combined with the resistance of R_1 as shown in Figure 4, the equivalent electrical resistance of type II superconducting materials will resemble the graph shown in Figure 10. This electrical resistance shown in Figure 10 matches the response found in experimental studies [8].

V. Conclusion

The electrical resistance response of superconducting and non-superconducting materials has been duplicated using a theoretical model based on conduction electron flow through these materials along with parallel resistor theory. Since this model is accurate, it explains why superconductors exhibit their electrical resistance behavior and why non-superconductors exhibit their electrical resistance behavior. This model can be used to represent the electrical resistance behavior of type I and type II superconductors.

References

[1] C. Kittel, *Introduction to Solid State Physics,* Hoboken, New Jersey: John Wiley and Sons, 2005, ch. 10.
[2] C. Buzea and K. Robbie, “Assembling the Puzzle of Superconducting Elements: A Review,” *Superconducting Science and Technology*, vol. 18, no.1, Nov. 2004.
[3] C. Probst and J. Wittig, “Chapter 10 Superconductivity: Metals, Alloys, and Compounds,” Handbook on the Physical and Chemistry of Rare Earths, vol. 1, 1978, pp. 749 – 795.
[4] J. Schmalian, "Failed Theories of Superconductivity,” *Modern Physics Letters B,* vol. 24, no. 27, 2010, pp. 2679 – 2691.
[5] J. R. Hull and M. Murakami, “Applications of Bulk High-Temperature Superconductors,” Proceedings of the IEEE, vol. 92, no. 10, pp. 1705 – 1718, Oct. 2004.
[6] A. P. Malozemoff, “Computer Applications of High Temperature Superconductivity,” Physica C: Superconductivity, vol. 153-155, part 3, pp. 1049 – 1054, June 1988.
[7] F. Lacy, “Evaluating the Resistivity-Temperature Relationship for RTDs and Other Conductors,” *IEEE Sensors Journal* vol. 11, no. 5, pp. 1208 – 1213, May 2011.
[8] A. Marouchkine, *Room-Temperature Superconductivity*, Cambridge UK, Cambridge International Science Publishing, 2004, ch. 2, 6.