

# Mini-drone Quadrotor Altitude Control Using Characteristic Ratio Assignment PD Tuning Approach

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**Abstract**— This paper presents PD tuning approach using characteristic ratio assignment (CRA) method for Parrot rolling spider mini-drone quadrotor altitude control system. The aim of this study is to examine the PD controller design and parameter tuning for achieving six degrees of freedom motion control. The CRA technique has been used for determining the proper controller parameters based on the selection of the characteristic polynomial coefficients of the closed loop system conform to the specified the performance criteria. The altitude control simulations applied to Parrot rolling spider mini-drone quadrotor system so that investigated the performance of the proposed controller design scheme. The results clearly exposed that the control system is fairly stable and out performance, likewise has the effectiveness of controller's parameters adjustability.

**Index Terms**— Parrot rolling spider mini-drone, Quadrotor, Altitude Control, Characteristic Ratio Assignment, PD Tuning

## I. INTRODUCTION

THE quadrotor unmanned aerial vehicles (UAVs) are widespread interest due to the usefulness for many applications for instance, mobile measurement [1], surveying and inspection [2-3], agriculture [4-5], environment monitoring [6-7], disaster rescue [8]. As well as several control theories have been applied to the UAVs motion control for improving their performance and robustness. Fractional order PID was applied for UAV roll control [9]. A Fuzzy complementary Kalman filter (FCKF) data fusion algorithm was introduced to estimate the altitude of the UAV landing mode [10]. The discrete-time sliding mode control (DSMC) was applied to small quadrotor UAV system for evaluating the performance of the position and altitude tracking control [11]. Cascade iterative feedback tuning technique was implemented as the algorithm for of roll and pitch controllers tuning [12].

This paper presents PD tuning approach using characteristic ratio assignment (CRA) technique for Parrot

rolling spider mini-drone quadrotor altitude control system. The aim of this study is to examine the PD controller design and parameter tuning for achieving six degrees of freedom motion control. The CRA technique has been used for determining the proper controller parameters based on the selection of the characteristic polynomial coefficients of the closed loop system conform to the specified the performance criteria [13]. The impact of CRA lies in the simplest adjusting the parameters of PID, PD, PI controller which are able to improve the performance of control system such as decreasing percent overshoot or settling time [14-15].

The altitude control simulations and experiments applied to Parrot rolling spider mini-drone quadrotor system so that investigated the performance of the proposed controller design scheme. The rest of this paper is organized as follows: The Parrot rolling spider mini-drone quadrotor with its mathematical dynamic modeling is described in the section 2; The details of characteristic ratio assignment method applied to PD controller design are demonstrated in the section 3; The simulation included with the results and discussions are illustrated in section 4; while the conclusions are presented in the section 5.

## II. DYNAMIC MODELING



Fig. 1. Parrot Mini-drone.

The Parrot rolling spider mini-drone quadrotor built by a French company consisted of four DC motors, four propellers, electrical board and X type frame. Fig. 1 shows Parrot mini-drone while Fig. 2 shows the quadrotor reference frame and Euler Angles.

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From Fig. 2, the mini-drone quadrotor is controlled by a thrust and a torque created from the angular speed of DC motors which produce specified roll torque, pitch torque, yaw torque and the main thrust so that generate translation and rotation of quadrotor motion.

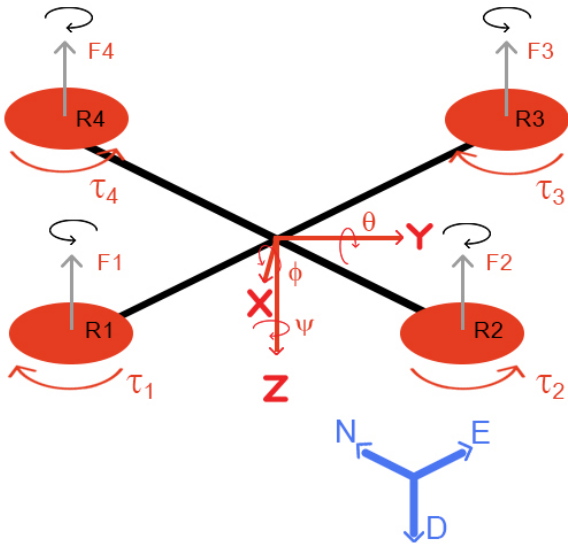


Fig. 2. Quadrotor Reference Frame and Euler Angles.

The dynamic modeling of mini-drone quadrotor is based on Newton–Euler formulation. Using the analysis of translational equations of motion and rotational equation of motion, the state vector is derived and definite the relation of the parameters as shown below.

$$X = [x \ y \ z \ \phi \ \theta \ \psi \ \dot{x} \ \dot{y} \ \dot{z} \ p \ q \ r]^T \quad (1)$$

$$\dot{X} = f(X, U) \quad (2)$$

$$f_1 = \dot{x} \quad (3)$$

$$f_2 = \dot{y} \quad (4)$$

$$f_3 = \dot{z} \quad (5)$$

$$f_4 = \ddot{x} \quad (6)$$

$$= -\frac{U_1}{m} [\cos(\psi) \sin(\theta) \cos(\phi) + \sin(\psi) \sin(\phi)] \quad (6)$$

$$f_5 = \ddot{y} \quad (7)$$

$$= -\frac{U_1}{m} [\sin(\psi) \sin(\theta) \cos(\phi) - \cos(\psi) \sin(\phi)] \quad (7)$$

$$f_6 = \ddot{z} = g - \frac{U_1}{m} (\cos(\theta) \cos(\phi)) \quad (8)$$

$$f_7 = \dot{\phi} = p + \left( \frac{r \cos(\phi) \sin(\theta)}{\cos(\theta)} + \frac{q \sin(\theta) \sin(\phi)}{\cos(\theta)} \right) \quad (9)$$

$$f_8 = \dot{\theta} = q \cos(\phi) - r \sin(\phi) \quad (10)$$

$$f_9 = \dot{\psi} = (r \cos(\phi) / \cos(\theta) + q \sin(\phi) / \cos(\theta)) \quad (11)$$

$$f_{10} = \dot{p} = (\tau_x + I_y q r - I_z q r) / I_x \quad (12)$$

$$f_{11} = \dot{q} = (\tau_y - I_x p r + I_x p r) / I_y \quad (13)$$

$$f_{12} = \dot{r} = (\tau_z + I_x p q - I_y p q) / I_z \quad (14)$$

where  $x, y, z$  are position of quadrotor in the  $X, Y, Z$  axis.  $\theta, \phi, \psi$  are rotation of quadrotor in roll, pitch and yaw respectively.  $\dot{x}, \dot{y}, \dot{z}$  are acceleration of quadrotor in the  $X, Y, Z$  axis.  $p, q, r$  are angular body rates.  $U_1$  is the input vector from thrust force.  $m$  is a mass of quadrotor.  $g$  is the acceleration of the gravity.  $I_i$  ( $i = x, y, z$ ) is the total moment of inertia of the  $X, Y, Z$  axis of quadrotor,  $J$  is the total moment of inertia of the propeller.

The control input ( $U_i$ ) of quadrotor consists of four inputs as  $[U_1 \ U_2 \ U_3 \ U_4]^T$  which is thrust force, roll torque, pitch torque, yaw torque respectively.

$$U_1 = K_a (\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \quad (15)$$

$$U_2 = K_a (-\Omega_1^2 + \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \quad (16)$$

$$U_3 = K_a (-\Omega_1^2 - \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \quad (17)$$

$$U_4 = K_m (\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \quad (18)$$

where  $\Omega_{1-4}$  is the angular speed of the propeller (1~4).

### III. CHARACTERISTIC RATIO ASSIGNMENT METHOD

The implementation of characteristic ratio assignment method (CRA) is proposed in [13]. The relations of parameters in closed-loop characteristic equation are derived and expressed the advantage for damping ratio and time constant adjustment.

The relation of characteristic equation can be illustrated as follow.

$$p(s) = a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0, \quad \forall a_i > 0 \quad (19)$$

Where, the characteristic ratio is given by,

$$\alpha_1 = \frac{a_1^2}{a_0 a_2}, \quad \alpha_2 = \frac{a_2^2}{a_1 a_3}, \quad \dots, \quad \alpha_{n-1} = \frac{a_{n-1}^2}{a_{n-2} a_n} \quad (20)$$

The time constant is given by

$$\tau = \frac{a_1}{a_0} \quad (21)$$

where  $\tau$  is time constant.  $\alpha$  is characteristic pulsataces.

Defining the value of  $\alpha_i$  is conformed to the rule of Lipatov and Sokolov [13] so that retains the system stability which is given by,

$$\sqrt{\alpha_i \alpha_{i+1}} > 1.4656, \quad i = 1, 2, \dots, n-2 \quad (22)$$

$$\alpha_i \geq 1.12374 \alpha_i^*, \quad i = 2, 3, \dots, n-2 \quad (23)$$

$$\alpha_i^* = \frac{1}{\alpha_{i+1}} + \frac{1}{\alpha_i - 1}, \quad \alpha_n = \alpha_0 = \infty \quad (24)$$

Regarding the adjusting a speed response of control system, the CRA method can be adopted by changing a

value of time constant as following.

$$G_k(s) = \frac{k^n a_0}{a_n s^n + k a_{n-1} s^{n-1} + \dots + k^{n-1} a_1 s + k^n a_0} \quad (25)$$

$$\tau = \frac{1}{k} \left( \frac{a_1}{a_0} \right) \quad (26)$$

The time constant is able to change as shown in equation (26) by an adjustment of the constant  $k$ .

To adjust the damping ratio so that improve the maximum overshoot of control system, the CRA method enables controller tuning by the adjustment of one parameter that follows as,

$$G_k(s) = \frac{k^{\frac{1}{2}n^2 - \frac{1}{2}n} a_0}{a_n s^n + k^{n-1} a_{n-1} s^{n-1} + \dots + k^{\frac{1}{2}n^2 - \frac{1}{2}n - 1} a_1 s + k^{\frac{1}{2}n^2 - \frac{1}{2}n} a_0} \quad (27)$$

$$\alpha_i = k \left( \frac{a_i^2}{a_{i-1} a_{i+1}} \right) \quad (28)$$

when  $k > 1$ ; damping ration will be increased then the overshoot of process repose will be decreased.

#### IV. SIMULATION AND RESULTS DISCUSSION

This section presents controller design for altitude motion control that makes quadrotor undergoes translational motion in the z directions.

##### A. Simulation

The simulations are carried out using MATLAB/Simulink models for designing and validating the proposed control technique. Fig. 3 depicts the airframe Simulink block of quadrotor dynamic model discussed in Section II as well as included the flight control system (FCS) block, sensors block, environment block and virtualization block which is used to implement the dynamics and kinematics of the quadrotor discussed in Section II.

The FCS block in Fig. 3 consists of PD altitude controller that is design using CRA algorithm. Regarding the steady-state trimmed-flight conditions, state-space matrices of altitude control loop are shown as

$$A = \begin{bmatrix} 1 & 0 \\ 0.005 & 1 \end{bmatrix} \quad (29)$$

$$B = \begin{bmatrix} 0.07937 \\ 0.001984 \end{bmatrix} \quad (30)$$

$$C = [0 \quad 1] \quad (31)$$

$$D = [0] \quad (32)$$

The 0.005 sec sampling period conform to discrete and continuous transfer function as (33-34) respectively.

$$G_{p,d}(s) = \frac{0.001984z - 0.001587}{z^2 - 2z + 1} \quad (33)$$

$$G_{p,c}(s) = \frac{0.3571s + 15.87}{s^2} \quad (34)$$

To design PD controller in (35) using CRA technique, the diagram of control system is shown as Fig. 4.

$$G_{con.kd}(s) = K_d s + K_p \quad (35)$$

The transfer function in (36) is the closed loop transfer function of plant and PD controller.

$$G_{cl}(s) = \frac{0.3571K_d s^2 + (15.87K_d + 0.3571K_p)s + 15.87K_p}{(0.3571K_d + 1)s^2 + (15.87K_d + 0.3571K_p)s + 15.87K_p}$$

Feed forward compensator in (37) is used for cancellation zero affection.

$$G_{ff}(s) = \frac{15.87K_p}{0.3571K_d s^2 + (15.87K_d + 0.3571K_p)s + 15.87K_p} \quad (37)$$

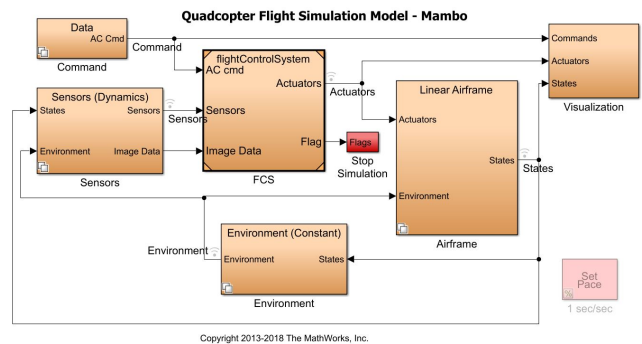


Fig. 3. Quadrotor Flight Simulation System.

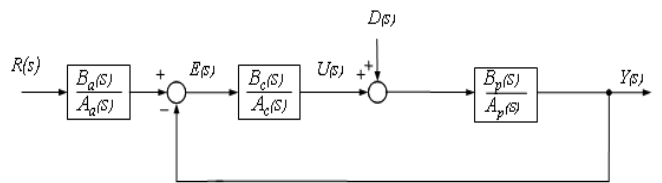


Fig. 4. Block Diagram of Control System.

$$G_{ff}(s)G_{cl}(s) = \frac{15.87K_p}{(0.3571K_d + 1)s^2 + (15.87K_d + 0.3571K_p)s + 15.87K_p} \quad (38)$$

Using CRA techniques, starting with defining time constant ( $\tau$ ) and characteristic ratio ( $\alpha_1$ ).

$$\tau = 0.5, \alpha_1 = 1.8 \quad (39)$$

The parameter of PD controller ( $K_p, K_d$ ) can be calculated for equating characteristic ration and CRA parameters. The values of ( $K_p, K_d$ ) are shown as

$$K_p = \frac{p_0 \alpha_1}{\tau^2 p_0^2 - \tau \alpha_1 p_0 p_1 + \alpha_1 p_1^2} \quad (40)$$

$$K_d = \frac{\alpha_1 - \tau p_1 K_p}{\tau p_0 - \alpha_1 p_1} \quad (41)$$

In case of time constant and percentages of maximum overshoot varying for adjusting the settling time of output response,  $k$  factor is used for calculating the proper values of PD controller ( $K_p, K_d$ ) as shown in Table I and Table II respectively.

TABLE I  
EFFECTS OF PD GAINS ON THE ALTITUDE RESPONSE  
WITH TIME CONSTANT VARYING

$k$	$K_p$	$K_d$	$t_s$ (s)
0.65	0.33	0.19	2.7322
0.8	0.51	0.24	2.6584
1	0.82	0.30	2.5467
1.1	1.01	0.33	2.2132
1.35	1.56	0.41	2.0256

TABLE II  
EFFECTS OF PD GAINS ON THE ALTITUDE RESPONSE  
WITH PERCENTAGE OF MAXIMUM OVERSHOOT VARYING

$k$	$K_p$	$K_d$	% $M_p$
0.65	0.51	0.19	26.5548
0.8	0.64	0.23	12.4563
1	0.82	0.30	4.6845
1.1	0.91	0.33	0.6666
1.35	1.15	0.42	0.0000

TABLE III  
PARAMETERS USED FOR THE QUADROTOR

Parameter	Symbol	Value	Units
Mass	$m$	0.068	kg
Thrust constant	$K_a$	0.0107	$Ns^2$
Torque constant	$K_m$	$0.783e-3$	$Nms^2$
Inertial matrix	$I_{xx}, I_{yy}, I_{zz}$	$\begin{pmatrix} 0.0686e-3 & 0 & 0 \\ 0 & 0.092e-3 & 0 \\ 0 & 0 & 0.1366e-3 \end{pmatrix}$	$kgm^2$
Distance between rotor and center of mass	$d$	0.0624	m

Time constant varying

$$K_p = \frac{k^2 p_0 \alpha_1}{\tau^2 p_0^2 - k \tau \alpha_1 p_0 p_1 + k^2 \alpha_1 p_1^2} \quad (42)$$

$$K_d = \frac{k \alpha_1 - \tau p_1 K_p}{\tau p_0 - k \alpha_1 p_1} \quad (43)$$

Percentages of maximum overshoot varying

$$K_p = \frac{k^2 p_0 \alpha_1}{\tau^2 p_0^2 - k \tau \alpha_1 p_0 p_1 - \tau p_0 p_1 + k \alpha_1 p_1^2 + k \tau p_0 p_1} \quad (44)$$

$$K_d = \frac{k \alpha_1 - \tau p_1 K_p}{\tau p_0 - k \alpha_1 p_1} \quad (45)$$

Table III demonstrates a summary of the physical parameters used for the MIT Parrot mini-drone quadrotor. The parameters are used in MATLAB/Simulink models in Fig. 3. Then simulate the results from designed PD controller so that received the validation of output response as shown in Fig. 5~7

### B. Results and Discussion

All the time domain Simulink simulations were carried out in 10 sec duration for each model. The responses, using the various  $k$  CRA Parameters, are shown in the Fig. 5-7. The PD gains ( $K_p, K_d$ ) in this study were obtained by adjusting the various valued of  $k$  to obtain the satisfactory time constant and percentage overshoot.

Table I shows the effect of the PD Gains on the altitude response in case of time constant variation. By the value of  $k$  CRA Parameters, the settling time will be decreased when  $k$  is increased. Table II shows the effect of the PD Gains on the altitude response in case of percent overshoot variation. By the value of  $k$  CRA Parameters, the percent overshoot will be decreased when  $k$  is increased.

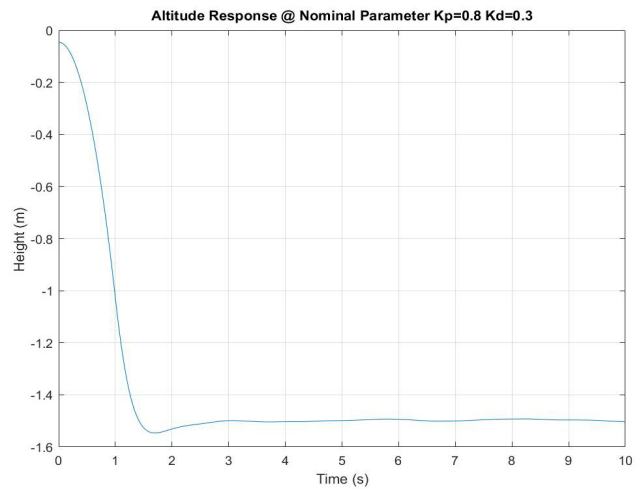


Fig. 5. Altitude Control: Step Response.

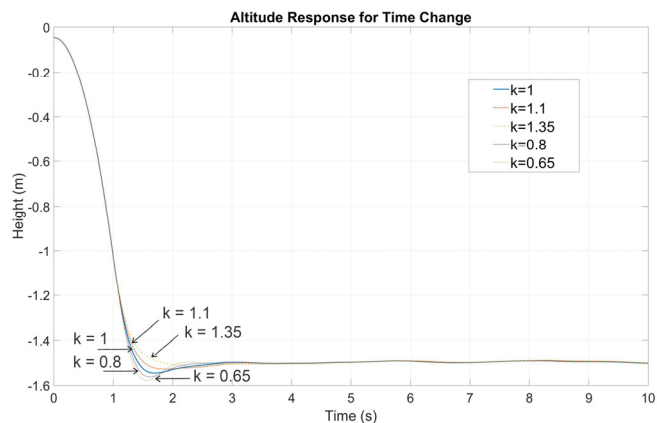


Fig. 6. Effect of  $k$  on the time constant varying.

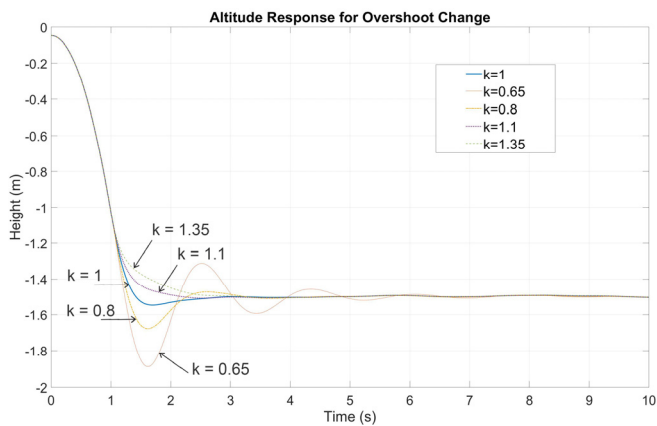


Fig. 7. Effect of  $k$  on the maximum overshoot varying.

## V. CONCLUSION

In this paper, PD tuning approach using characteristic ratio assignment (CRA) method is presented. The proposed controller design scheme is applied to mini-drone quadrotor system to achieve the performance and robustness of control system. The controller design of the proposed system has been derived, and the effective performance has been validated via MATLAB simulation performed with Parrot rolling spider mini-drone quadrotor system. The results demonstrated that the proposed approach is the effective scheme for improving the performance of UAVs altitude motion control.

In future the authors will design non-linear controller using sliding mode control technique and implement in mini-drone. Performance, in terms of efficiency and accuracy, will then be compared of controllers such as traditional PD controller.

## REFERENCES

- [1] R. Vepa, "Nonlinear Control of Robots and Unmanned Aerial Vehicles: An Integrated Approach", CRC Press, 2016.
- [2] H. Freimuth and M. König, "Planning and executing construction inspections with unmanned aerial vehicles", *Automation in Construction*, Vol. 96, 2018, pp. 540-553.
- [3] S. S.C. Congress, A. J. Puppala and C. L. Lundberg, "Total system error analysis of UAV-CRP technology for monitoring transportation infrastructure assets", *Engineering Geology*, Vol. 247, 2018, pp. 104-116.
- [4] X. Wang, H. Sun, Y. Long, L. Zheng, H. Liu and M. Li, "Development of Visualization System for Agricultural UAV Crop Growth Information Collection", *IFAC-PapersOnLine*, Vol. 51, Issue 17, 2018, pp. 631-636.
- [5] L. Deng, Z. Mao, X. Li, Z. Hu, F. Duan and Y. Yan, "UAV-based multispectral remote sensing for precision agriculture: A comparison between different cameras", *ISPRS Journal of Photogrammetry and Remote Sensing*, Vol. 146, 2018, pp. 124-136.
- [6] S. Wang, L. Liu, L. Qu, C. Yu, Y. Sun, F. Gao and J. Dong, "Accurate Ulva prolifera regions extraction of UAV images with superpixel and CNNs for ocean environment monitoring", *Neurocomputing*, 2018.
- [7] E. De Beni, M. Cantarero and A. Messina, "UAVs for volcano monitoring: A new approach applied on an active lava flow on Mt. Etna (Italy), during the 27 February–02 March 2017 eruption", *Journal of Volcanology and Geothermal Research*, Vol. 369, 2019, pp. 250-262.
- [8] M. Półka, S. Ptak and Ł. Kuziora, "The Use of UAV's for Search and Rescue Operations", *Procedia Engineering*, Vol. 192, 2017, pp. 748-752.
- [9] S. Seyedtabaii, "New flat phase margin fractional order PID design: Perturbed UAV roll control study", *Robotics and Autonomous Systems*, Vol. 96, 2017, pp. 58-64.

- [10] Q. Yang and L. Sun, "A fuzzy complementary Kalman filter based on visual and IMU data for UAV landing", *Optik*, Vol. 173, 2018, pp. 279-291.
- [11] J. Jing Xiong and G. Zhang, "Discrete-time sliding mode control for a quadrotor UAV", *Optik*, Vol. 127, Issue 8, 2016, pp. 3718-3722.
- [12] D. A. Tesch, D. Eckhard and W. Cechin Guarienti, "Pitch and Roll control of a Quadcopter using Cascade Iterative Feedback Tuning", *IFAC-PapersOnLine*, Vol. 49, Issue 30, 2016, pp. 30-35.
- [13] Y.C. Kim, L.H. Keel and S.P. Bhattacharyya, "Transient Response Control via Characteristic Ratio Assignment" *IEEE Transactions On Automatic Control*, Vol.48, No.12, 2003, pp.2238-2244.
- [14] J. Chaorai-ngern, A. Numsomran, T. Suesut, T. Trisuwannawat and V. Tipsuwanporn, "PID Controller Design using Characteristic Ratio Assignment Method for Coupled-Tank Process," *International Conference on Computational Intelligence for Modelling, Control and Automation and International Conference on Intelligent Agents, Web Technologies and Internet Commerce (CIMCA-IAWTIC'06)*, Vienna, 2005, pp. 590-594.
- [15] A. Numsomran, T. Wongkhum, T. Suksri, P. Nilas and J. Chaoraingern, "Design of decoupled controller for TITO system using characteristic ratio assignment," *2007 International Conference on Control, Automation and Systems*, Seoul, 2007, pp. 957-962.