

Modelling Study of Dishwasher Hydraulic Filtration System

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Abstract-This paper describes the process of theoretical analysis, experimental test and computer modelling of dishwasher hydraulic filtration system. Analytical equations based on theoretical study and empirical formulas derived from experimental tests are integrated to form a computer model where the closed-loop wash cycle is simulated, critical variables are calculated in the time domain, the effects of core design parameters are evaluated. Simulation results correlate reasonably well with experimental rig test and reference dishwasher performance in broad terms at particle removal and accumulation, filter clogging and cleaning, washing efficiency and water-energy saving etc. It is shown the model can be utilized to enhance the understanding of fundamentals in hydraulic filtration systems, optimize dishwasher design and performance, and reduce product time to market. Modelling could be of great assistance in, and an inseparable part of product development even in traditionally experiment-intensive companies.

Keywords-filter-clogging-and-cleaning, hydraulic-pressure-and-flowrate, particle-removal-and-accumulation, simulation-and-test-correlation

I. INTRODUCTION

In 2011, the author was tasked with studying dishwasher filtration system after joining a large US appliance company one year before. Following the tradition of hands-on work in advanced dishwasher group, the author designed filter test rigs, developed procedures, carried out flow and particle distribution tests, invented and prototyped filter self-cleaning device [1],[2]. However, these tests and analysis work were mostly done in isolation at component or subsystem level. Full system evaluation could not be conducted until a prototype dishwasher had been built, when major design change can be very costly. This is rather different from automotive industry where computer simulation has been widely used hand in hand with vehicle testing, even before the vehicle is built, in new product development. Based on years' experience in simulation, the author proposed to develop an integrated dishwasher hydraulic filtration system model which could assist design analysis. With manager approval, the author started modelling alongside his daily work. Core parameters and critical variables were identified and measured, theoretical equations and empirical formulas were derived and implemented, building blocks were created and integrated. In 2013, the preliminary model became functional when the work was interrupted. The debugging work was resumed years later after the author's retirement, hence without further test verification.

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Despite the shortcoming, the author felt that publishing the modelling work could contribute insight and add value which may still be relevant to the industry and scientific community. This is the purpose for publishing the paper.

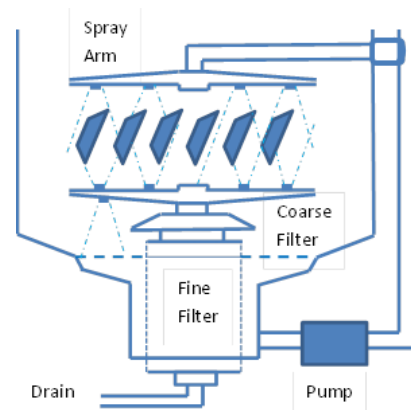


Fig. 1. Diagram of dishwasher hydraulic filtration system

Dishwasher (Fig. 1.) is much smaller than vehicle in size but not at all simpler to model. The complexity mainly comes from the handling of statistical data in a definitive model and handling of food particle along its path of movements. Particles of variable sizes are removed from dishware, carried away by multiple spray arm jets of different rates and pressures. They are then captured by fine and coarse filters, which themselves may be clogged hence have to be cleaned. If the particle accumulation rate on filters is higher than the cleaning rate, the filter clean area will be reduced to a level unable to supply sufficient washing water to the circulation pump. The wash cycle has to be abandoned prematurely, resulting in waste of water and energy. Filter clogging is in fact one of the most challenging issues in dishwasher design and is affected by many parameters. There is generally no equilibrium state in the whole washing cycle where pressures, flow rates, filter clean areas, particle concentrations, accumulations, water head etc. are constantly changing. To capture the essence and improve performance, the model must qualify and quantify the relationships between all critical variables, calculate them at every time step.

II. BUILDING BLOCKS

A preliminary dishwasher hydraulic filtration model should have at least the following building blocks: pump, conduit, orifice flow rates and pressures; particles removal from dishware; filter flow rates; water head and volume; particle concentration; particle accumulation and clearing. Each of the building block is explained as follows.

A. Pump, conduit, orifice flow rates and pressures

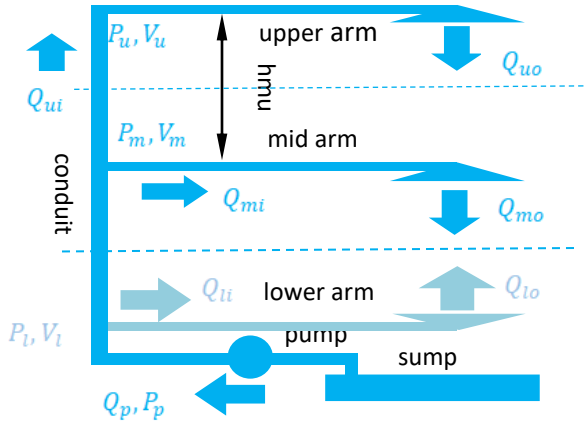


Fig. 2. Pressures and flow rates diagram of dishwasher hydraulic system

Fig. 2. is the schematic diagram of dishwasher hydraulic system where pressurized clean water from the circulation pump travels upwards in the conduit and turns into multiple spray arms jets for dish washing. Hydraulic pressures, fluid volumes and flow rates are illustrated at various positions in the diagram. Continuity equations, orifice and pipe flow rate equations [3] are utilized to describe the dynamic relationship between variables as in a lumped parameter model. For example, the upper spray arm continuity equation is in (1):

$$V_u \times \dot{p}_u / B_e = Q_{ui} - Q_{uo} \quad (1)$$

where V_u is the fluid volume (including conduit section above the top dash line), p_u is fluid pressure, B_e is the effective bulk modulus representing fluid compressibility. The orifice flow rate Q_{uo} and conduit flow rate Q_{ui} are calculated in (2) and (3):

$$Q_{uo} = C_u \times A_u \sqrt{p_u / \rho} \quad (2)$$

$$Q_{ui} = (p_m - p_u - p_{hmu}) \times pp_u \quad (3)$$

where C_u is orifice discharge coefficient, A_u is orifice area, ρ is fluid mass density, p_m is the middle spray arm pressure, p_{hmu} is additional pressure due to conduit water head between upper and middle spray arms, $pp_u = \pi D^4 / \mu LC$ represents the dynamic effect of conduit resistance to flow rate, where μ is friction coefficient, L and D are conduit length (h_{mu}) and inner diameter (ID) (assuming circular cross section), and C is a constant. Equations for middle and lower spray arms can be derived similarly. The values of constant parameters in above equations were derived by measuring a known hydraulic system. For example, turn on the circulation pump in the top/middle spray-arm mode. When the spray arm flows are stabilized, p_u and p_m can be measured by pressure sensors at correct locations. As p_u is not changing, \dot{p}_u is 0 in (1), hence $Q_{ui} = Q_{uo}$ can be measured by flow rate sensor. pp_u can then be calculated from (3), where water head p_{hmu} is determined by h_{mu} (Fig. 2). The value of $C_u \times A_u / \sqrt{\rho}$ in (2) can be calculated as p_u and Q_{uo} are known. Once all constant values are known, the equations can be implemented in the model. It

is worth noting that on average the time spent by jet water travelling from the nozzle to the filtration system is a couple of seconds. Time is also spent on pumping clean water from pump chamber to the spray arms. A pure time delay of 2.5 sec. is introduced between flows in different phases to close the hydraulic loop.

B. Particle removal from dishware

Food particle removal rates from dishware is the input to dishwasher filtration system. Increasing spray arm jet pressure and flow rate will increase particle removal rate hence increase the load on, even clog the filtration system. To derive particle removal rate, tests were conducted to capture particle amounts removed from soiled dishware in a reference dishwasher over increasing time period. The soil was specially prepared with twenty common food ingredients, mixed up and evenly applied to dishware, then fan dried for certain amount of time. During test, 1/5 of the soiled dishware was mixed up with clean dishware in randomly selected positions, for the purpose of saving time. At end of each wash period, soiled dishware was removed, two rinse cycles were then applied to collect all left particles in dishwasher, which were filtered with desired coarse and fine filters (1400 and 27 microns in example) and then weighted. As shown in Fig.3., the marked solid polylines were captured fine, coarse and total particle amounts, bold exponential curves were generated by curve fitting in (4), while dotted curves were their differentiations representing particle removal rates in (5):

$$T = T_{max}(1 - e^{(1-Bt)}) \quad (4)$$

$$\dot{T} = r_{in} = T_{max} B e^{(1-Bt)} \quad (5)$$

where T_{max} is the maximum particle quantity, and B is the time constant.

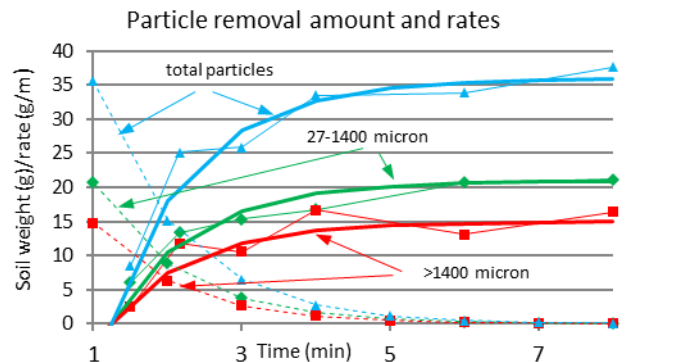


Fig. 3. Examples of particle removal amount and rate from dishware

C. Filter flow rates

Filter flow rate calculation is at the heart of filtration system modeling. Theoretical and empirical formulas [4] were available though not covering all important factors in dishwashing at the same time. A general filter test rig was built to study the joint effect of those factors such as filter open ratio (OR %), hole size (ID in), inclination angle (θ deg), thickness (gauge G), and water surface tension (St dyns/cm)

etc. In the Fig. 4. setting, water was transferred from one box to another at a desired rate for a short period of time. Water going through the fixed area of coarse filter was collected to derive unit filter flow rate (gpm/in²) under gravity. Due to the large number of combined test conditions, MiniTab regression analysis was conducted on some 150 results (including repetitions). First and second order multiple-predictor equations were generated as shown in (6.1&2) with confidence levels of 57.3% and 58.5% respectively. The positive signs indicate unit flow rate increasing with filter open ratio and hole size, while negative signs indicate unit flow rate decreasing with inclination angle and water surface tension. G and G^2 in (6.2) have opposite signs though the value of $-0.0916 G + 0.0023 G^2$ remains negative, indicating that unit flow rate decreases with thickness nonlinearly.

$$Q_{rg} = 0.0947 + 0.00174 OR + 1.23 ID - 0.00733 \theta - 0.00157 St \quad (6.1)$$

$$Q_{rg} = 0.921 + 0.000037 OR^2 + 2.08 ID - 0.00749 \theta - 0.00176 St - 0.0916 G + 0.0023 G^2 \quad (6.2)$$

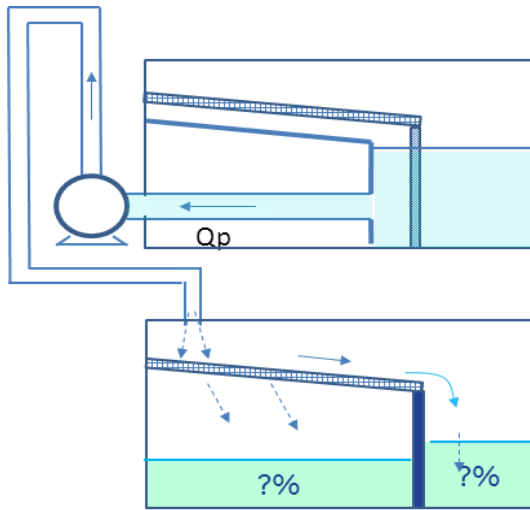


Fig. 4. General purpose dishwasher filtration system test rig (setting I) for unit flow rate test (under gravity)

The unit flow rates of filters under pressure (head) were also measured on test rig (Fig. 5) and in a reference dishwasher, showing flow rates increasing non-linearly with pressure. With the linear leakage flow rate removed, filter unit flow rate could be expressed as $Q = C \times \sqrt{\Delta h} + B$, where C is filter resistance coefficient, Δh is the water head across the filter, B is filter unit flow rate at zero head. In the two empirical formulas below, A_f and A_c are fine and coarse filter surface areas, B is Q_{rg} for coarse filter, 0 for fine filter:

$$Q_f = A_f \times 3.9 \times \sqrt{\Delta h} \quad (7)$$

$$Q_c = A_c \times (1.7 \times \sqrt{\Delta h} + Q_{rg}) \quad (8)$$

It is worth noting that these test results were from the reference dishwasher filters. Different filters will result in different curves though the trend could be similar. A general

formula covering the effect of more filter features should be desirable in the future as it will be more representative and consistent.

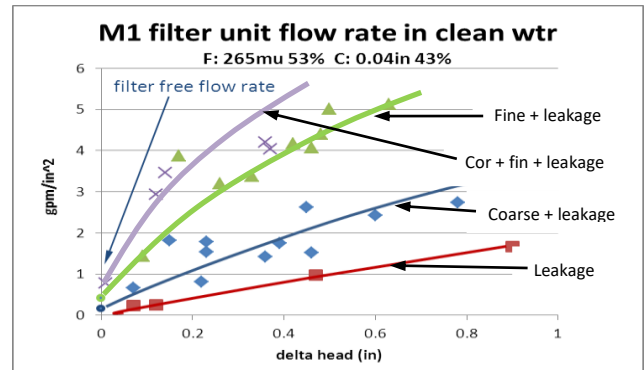
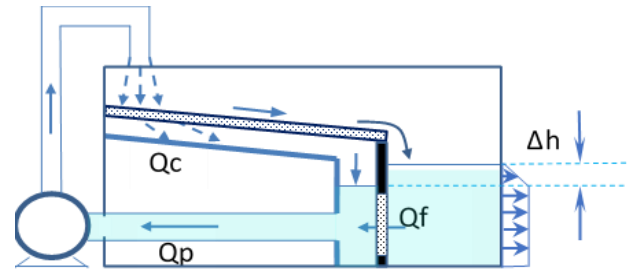


Fig. 5. Filtration system test rig (setting II) and test results of unit filter flow rates under pressure (head)

It is seen in the Fig. 5. diagram that the head across the fine filter is constant Δh when it is submerged completely. If part of the filter surface is above the water level in the left (pump) chamber, the pressure across will be reduced, linearly with elevation, from Δh down to zero when it reaches the water level in the right (sump) chamber, illustrated by the arrow length. As Δh is constantly changing in dishwasher (Fig. 6), the fine filter flow rate Q_{ft} on the top section has to be integrated from h_p , the water level in pump chamber, to h_f , the top of fine filter, or within different boundary conditions (Appendix):

$$Q_{ft} = \int_{h_p}^{h_f} A_f \times 3.9 \times \sqrt{\Delta h} \quad (9)$$

The filter flow rate on the bottom section (Q_{fb}) is simply calculated by (7) where Δh is constant.

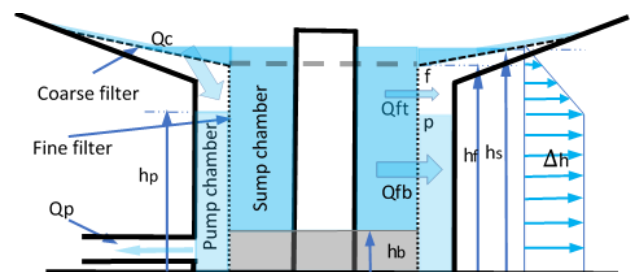


Fig. 6. Fine filter chamber/head/flow rates illustration

D. Water head and volumes

Water head Δh is the difference between water levels h_p and h_s (Fig. 6), which are directly related to water volumes in the pump and sump chambers. In the dishwasher model, the water volumes are constantly modified by the input/output flow rates as:

$$V_p = V_{pi} + \int (Q_c + Q_f - Q_p) \quad (10)$$

$$V_s = V_{si} + \int (Q_{crs} - Q_f) \quad (11)$$

$$Q_{crs} = Q_{arm} - Q_c \quad (12)$$

where V_{pi} and V_{si} are initial water volumes in pump and sump chambers, Q_{crs} is coarse filter crossflow rate, Q_{arm} is the total spray arm flow rate. When upper and middle spray arms are running, $Q_{arm} = Q_{uo} + Q_{mo}$ (time delayed), when the lower spray arm is running, $Q_{arm} = Q_{lo}$ (Fig. 2). Once volumes are known, water levels can be calculated from the volume-height relationship derived from the geometric shape of relevant chambers (not shown). Water levels may also be calculated from lookup tables of water volume/level measured on a reference or known dishwasher.

E. Particle concentration

In dishwasher, fine and coarse particle concentrations are constantly changing with time and location, except for pump chamber where coarse particles concentration is always zero (protecting spray arms). Particle concentration directly affects filter clogging hence must be calculated separately for washing water (off the dish), sump water (before filtering) and pump water (after filtering). It is assumed that particles are evenly distributed in each chamber, and midair e.g. the jet flow, for the reason of simplicity. The relationship between particle concentration c (g/in³), mass change rate r (g/s) and water flow rate Q (in³/s) is expressed as $c = r/Q$ in the model. Assuming that the removal rates of fine and coarse particle from dishware are $r_{f,in}$ and $r_{c,in}$ (5), as the jet water Q_{arm} already contains fine particles of concentration $c_{f,o}$ from previous cycle, the total fine particle concentration is then $r_{f,in}/Q_{arm} + c_{f,o}$, while the coarse particles concentration is $r_{c,in}/Q_{arm}$ in the washing water. Once the washing water reaches the coarse filter, it splits into two parts: through-flow rate Q_c and crossflow rate Q_{crs} ($= Q_{arm} - Q_c$).

Q_c enters the pump chamber and increases its fine particle quantity by rates of $Q_c \times (r_{f,in}/Q_{arm} + c_{f,o})$. In the meantime, the pump flow rate Q_p takes some fine particles away from the pump chamber at rates of $c_{f,o} \times Q_p$ (Fig. 5), where $c_{f,o}$ is previous fine particles concentration. The total fine particle concentration c_f in pump chamber is then:

$$c_f = \int [Q_c \times (r_{f,in}/Q_{arm} + c_{f,o}) - c_{f,o} \times Q_p] / V_{p+} \quad (13)$$

where V_{p+} is pump chamber plus conduit volumes. On the other hand, the crossflow rate Q_{crs} enters the sump chamber and increases the fine particle quantity by rates of $Q_{crs} \times (r_{f,in}/Q_{arm} + c_{f,o})$. In the meantime, the fine filter flow rate Q_f takes some fine particles out of circulation by

depositing them on the fine filter surface at rates of $c_{f,sm} \times Q_f$, where $c_{f,sm}$ is the previous fine particle concentration. The total fine particle concentration $c_{f,sm}$ in sump chamber is then:

$$c_{f,sm} = \int [Q_{crs} \times (r_{f,in}/Q_{arm} + c_{f,o}) - c_{f,sm} \times Q_f] / V_s \quad (14)$$

For coarse particle, rig test results show that its traversing rate along filter surface is about 25% lower than the carrying water - the crossflow rate Q_{crs} , due to surface friction. As the tests were conducted only for a limited number of times at different inclination angles, it is desirable in future to include various materials with different frictions, if available, in the test, to make the formula more representative. For now, a constant of 25% is used in related calculations. The coarse particle mass increase rate in the sump chamber is $75\% \times Q_{crs} \times (r_{c,in}/Q_{arm})$, the concentration $c_{c,sm}$ is then (modified further in (18) with down jet cleaning):

$$c_{c,sm} = \int (75\% \times Q_{crs} \times (r_{c,in}/Q_{arm})) / V_s \quad (15)$$

F. Particle accumulation and cleaning

With fewer coarse particles entering the sump due to surface friction, more accumulate on coarse filter surface at rates of $(Q_c + Q_{crs} \times 25\%) \times (r_{c,in}/Q_{arm})$. The clogged area can be calculated by:

$$A_{c,cum} = f_{M2A} \times \int (Q_c + Q_{crs} \times 25\%) \times (r_{c,in}/Q_{arm}) \quad (16)$$

where $f_{M2A} = 0.816$ (in²/g) is the coarse particle mass to area conversion factor found by test. To prevent coarse filter from clogging, lower spray arm down-jet is often used to clean accumulated particles. The effectiveness may depend on the jet orientation, coverage, flow rate and pressure etc. With limited test data, the cleaning rate in the model is assumed as $\sqrt{P_l} \times Q_l \times C_{cln}$ where P_l and Q_l are lower spray arm pressure and flow rate, C_{cln} is a scaling factor found by test or estimation. When the lower spray arm is operating, the clean coarse filter surface area is then:

$$A_c = A_{c0} - A_{c,cum} + f_{M2A} \times \int \sqrt{P_l} \times Q_l \times C_{cln} \quad (17)$$

where A_{c0} is initial coarse filter surface area. The coarse particle concentration in the sump chamber is then:

$$c_{c,sm} = \int (75\% \times Q_{crs} \times r_{c,in}/Q_{arm} + \sqrt{P_l} \times Q_l \times C_{cln}) / V_s \quad (18)$$

While fine particles are more likely to accumulate on fine filter surface, the coarse particles are less so due to size and weight. However, in (19) which calculates fine filter surface clogging, a factor f is introduced to study the potential contribution of coarse particle on fine filter clogging if needed:

$$A_{f,cum} = f_{M2A} \times \int Q_f \times (c_{f,sm} + f \times c_{c,sm}) \quad (19)$$

III. MODEL STRUCTURE

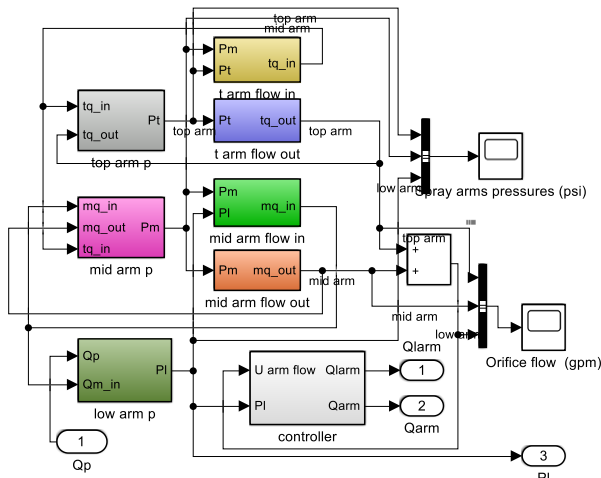


Fig. 7.1. Dishwasher hydraulic subsystem block diagram

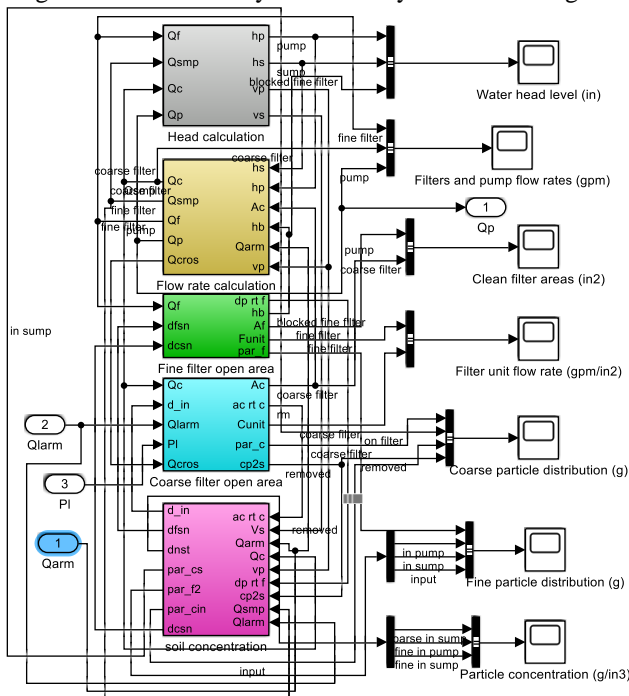


Fig. 7.2. Dishwasher filtration subsystem block diagrams

Dishwasher computer model is assembled with multiple building blocks in Simulink (Fig. 7). At the top level, the hydraulic and filtration blocks are simply connected through pump pressure, pump and spray arm flow rates. The filtration block further contains five sub-blocks which calculate water heads, filter flow rates, filters clean surface areas and particle concentrations. The hydraulic block contains eight sub-blocks which calculate conduit and spray arms pressures and flow rates, and a controller determining the operation order and time for spray arms. Design parameter can be changed offline, thirty system variables are calculated and displayed in the time domain for validation, correlation and optimization purposes. Simulation can be terminated either when washing time reaches the target time, or the circulation pump chamber is nearly empty due to filter clogging.

IV. RESULTS ANALYSIS

Nine scopes in Fig. 8. show the overall simulation results of the dishwasher pre-wash cycle. From left to right and top to bottom, they illustrate the time histories of water heads, filter and pump flow rates, clear filter areas, filter unit flow rates, coarse particle distributions, fine particle distributions, particle concentrations, spray arm pressures and orifice flow rates. The cycle starts with low spray arm washing for 180 sec. (low-mid), followed by upper/middle arms washing till 240.87 sec. when the operation is abandoned, due to near zero clean water level in the pump chamber (top-left). With fine filter completely clogged (mid-left), the coarse filter flow rate is not enough to match the pump requirement (top-mid), causing water head level to decrease in pump chamber, and increase in sump chamber.

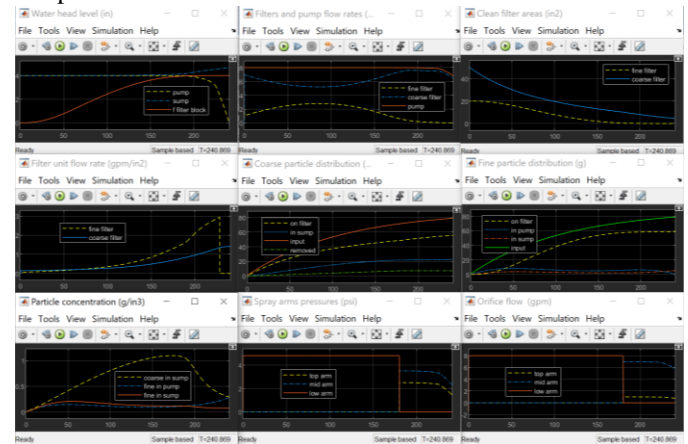


Fig. 8. Pre-wash simulation results with coarse particle clogging fine filter factor 0.05, down jet clean factor 0.005

The fine filter unit flow rate is roughly twice that of coarse filter most of the time though its contribution to the flow (top-mid) is still lower, as fine filter total surface area is less than half of the coarse filter (top-right). In the coarse particle distribution scope (mid-mid), the 'input' is particles washed down from dishware, most of which were accumulated on coarse filter surface, more than those entering the 'sump' chamber, even with some 'removed' or cleared by the down jets. The fine particle distributions (mid-right) displays the same trend, i.e. more particles accumulated on filter surface than in the chamber, though the pump chamber has more fine particles than the sump due to continuous supply of fine particles through coarse filter flow rate. Both fine and coarse particle concentrations (bottom-left) decrease in the sump chamber toward the end when water volume is significantly increased. Finally, the spray arm pressures and flow rates (bottom-mid/right) are nearly constant except for the very end when they are all reduced due to the drop of supply water level in the pump chamber. This is based on test results showing up to 15% pump rate reduction when the water level is close to the pump inlet.

It was found that both coarse filter inclination and spray arm down-jet can help clean the clogged coarse filter surface hence prolong the washing time. However, simulation results show somewhat difference in the two cases due to different

mechanism. With increased inclination angle, the coarse filter unit through-flow rate is reduced, and the unit crossflow rate is increased, leading to more coarse and fine particles down into the sump. The increase in particle concentration in the sump may clog the fine filter surface quicker, particularly if coarse particles also contribute part of this. Opposite to this, the cleaning effect will increase coarse filter clear surface area hence the overall coarse filter flow may increase despite the reduced unit flow rate.

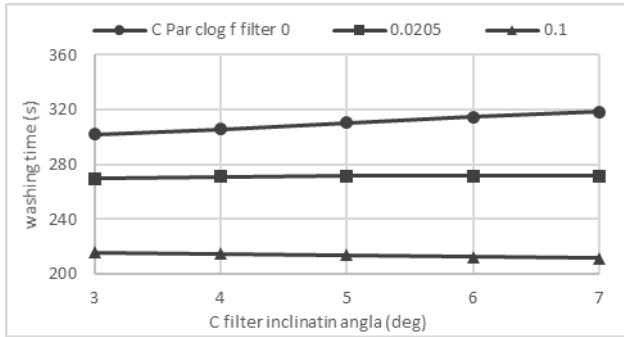


Fig. 9. Effect of coarse filter inclination angle and coarse particle clogging on fine filter

Results in Fig. 9. illustrate the combined effect of multiple factors on washing time. Without coarse particle clogging effect on fine filter, the washing time will increase continuously. With more than 2% coarse particle clogging fine filter, the washing time will decrease with inclination angle.

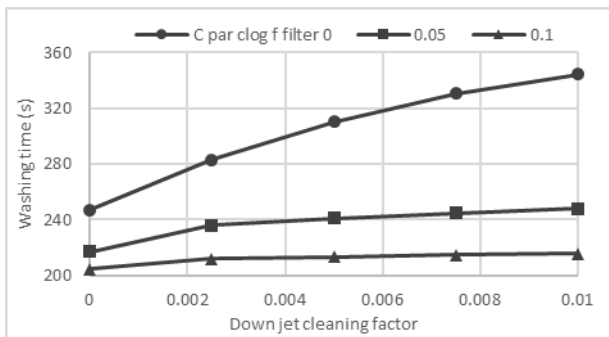


Fig. 10. Effect of down jet cleaning and coarse particle clogging on fine filter

Unlike coarse filter inclination, the down-jet flow brings down only accumulated coarse particles from coarse filter surface hence does not increase the fine particle concentration in the sump water. Results in Fig. 10. show that increasing down-jet cleaning factor will always improve washing time, at higher rates if coarse particle does not clog fine filter.

Detailed results are shown in Fig. 11. where the down jet cleaning factor is increased from 0.005 to 0.035. Compared with results in Fig. 8., the washing time is significantly increased from 240 to 446 sec. When the lower arm and down jet are operating (0-180 s. and over 360 s.), the cleaned particles increase much quicker, resulting in the lowest particles accumulation on coarse filter (mid-mid), and the largest clean surface area (top-right).

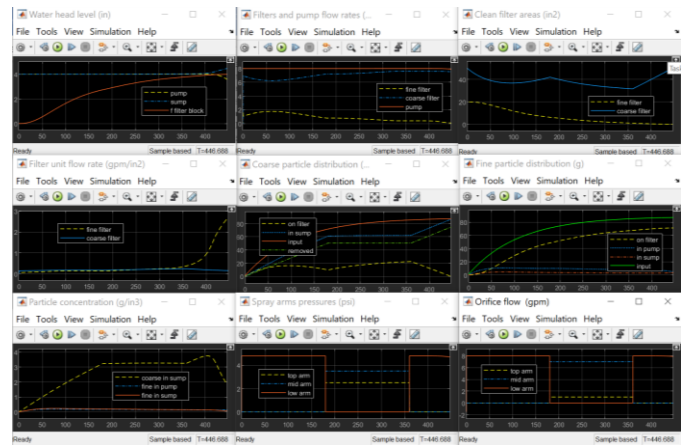


Fig. 11. Pre-wash simulation results with no coarse particle clogging on fine filter, down jet clean factor 0.035

At end of simulation, the fine filter is close to complete clogging while the coarse filter surface is completely clean (top-right). During reference dishwasher tests with inclined (5°) coarse filter and improved down jet cleaning, a large amount of food particle dough was found trapped at bottom of the sump, making final drainage a very difficult task. Some manufacturers did make effort to take food particles off the fine filter, e.g. inclined fine filter surface, and cage to catch large particles hence effectively improved the filtration and drain system efficiency. If more particles are removed in pre-wash, fewer wash cycles will be needed later to achieve the same washing quality, more water/energy saving, and/or better quality can be achieved.

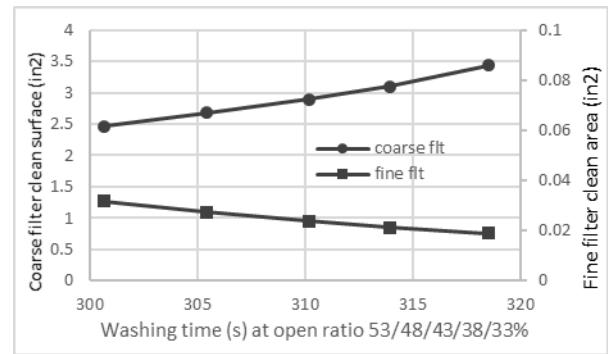


Fig. 12. Effect of coarse filter OR

The effect of coarse filter OR on washing time is shown in Fig. 12. With OR reduced from 53% to 33%, the coarse filter through-flow rate is reduced, and crossflow rate increased. More particles are brought into sump quickly, resulting in better coarse filter cleaning but worsening fine filter clogging. With the coarse filter clean surface area increased and fine filter clean surface area decreased, the overall washing time is still improved. Fig. 13. shows that additional clean water volume in pump chamber (13) could postpone the clogging of filtration system, as also shown by reference dishwasher test though the downside would be the usage of additional water and energy every cycle.

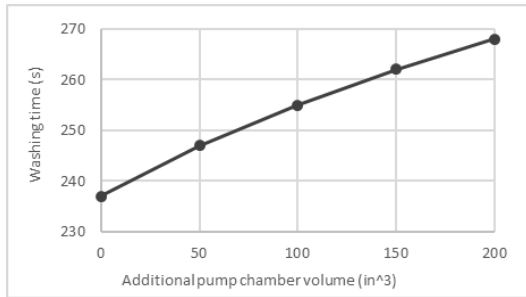


Fig. 13. Effect of additional clean water volume (in³)

It is worth noting that a representative simulation model should not only be based on and correlated to test results, but also be able to validate the test results and methods. One example is the discrepancy found in the old conduit pressure measurement used for model calibration. It showed low pressure at low conduit position and high pressure at high position. The explanation was that the small cross section at higher conduit position restricted the flow hence the pressure was higher. However, Bernoulli equation indicates that the fluid speeds up at small cross section hence pressure goes down. There is also added pressure at the lower position due to water head, hence the pressure at lower position is even higher. Further investigation and cross examination revealed that pressure sensors had been placed at random (vertical) positions on test rigs hence measurements had been contaminated by water head errors. A mystery of inconstant pressure measurement was finally solved, and a common misunderstanding was corrected.

V. CONCLUSION

A preliminary dishwasher simulation model is built based on experimental test and theoretical analysis. Building blocks of hydraulic, filtration and peripheral systems are created and connected via core variables to form a close-loop full-system model, where virtual dishwashing is conducted, the effect of critical parameters is evaluated in the time domain. The simulation results correlate well with the experimental rig test data and reference dishwasher performance. It shows that a preliminary model can be useful even with assumptions and simplifications, as it provides a good starting point for further improvement and enhancement. In the model, certain assumptions and extracted formula may need further verification, while some test methods could be more robust and inclusive. For example, the particle input test could utilize dishware soiled according to industrial process, and the test procedure could capture the joint effects of flow rates and pressures etc. Once more sophisticated test results and analytical findings become implemented, the model can be utilized to evaluate dishwasher design and optimize performance in detail for a very long time.

It is widely recognised that computer modelling process should include three steps: understand fundamentals, verify test results and optimize performances. Computer modelling could be of great assistance with, and an inseparable part of product development, even in traditionally experimental intensive companies. Simulation and experiments should work

hand in hand not only to speed up new product time to market and improve performance in terms of particle handling, filter cleaning, water-energy saving etc., but also to increase the theoretical knowledge, technical skill and practical experience of the workforce.

APPENDIX

Fine filter flow rate calculation

Refer to Fig. 6. and (9), the calculation of fine filter flow rate includes two parts:

the bottom part from the top of clogged fine filter surface h_b to pump chamber water level h_p where Δh is constant $h_s - h_p$, the flow rate Q_{fb} through area $cir \times (h_p - h_b)$ is calculated by $cir \times (h_p - h_b) \times 3.9\sqrt{h_s - h_p}$ where cir is the circumference of fine filter, h_s is sump chamber water level, and $h_p > h_b$;

the top part from h_p to the top of fine filter h_f where Δh decreases linearly from $(h_s - h_p)$ to $(h_s - h_f)$, the flow rate Q_{ft} is calculated by integration (between h_f and h_p if $h_p > h_b$, or between h_f and h_b if $h_p < h_b$):

$$dQ_{ft} = (cir \times dh) \times 3.9\sqrt{h_s - h} \quad (A1)$$

$$Q_{ft} = 3.9 \times cir \times \int \sqrt{h_s - h} \times dh \quad (A2)$$

According to Math Handbook, if $f(h) = \sqrt{ah + b}$,

$$\int f(h)dh = \frac{2}{3a}\sqrt{(ah + b)^3} \quad (A3)$$

As $a = -1$, $b = h_s$,

$$\int f(h)dh = -\frac{2}{3}\sqrt{(h_s - h)^3} \quad (A4)$$

If $h_s > h_f$ and $h_p > h_b$, the upper and lower limits of the definite integration are h_f and h_p :

$$\int_{h_p}^{h_f} f(h)dh = -\frac{2}{3}\left(\sqrt{(h_s - h_f)^3} - \sqrt{(h_s - h_p)^3}\right) \quad (A5)$$

$$Q_{ft} = \frac{2}{3} \times 3.9 \times cir \times \left(\sqrt{(h_s - h_p)^3} - \sqrt{(h_s - h_f)^3}\right) \quad (A6)$$

If $h_s > h_f$ and $h_p < h_b$, the upper and lower limits are h_f and h_b :

$$Q_{ft} = \frac{2}{3} \times 3.9 \times cir \times \left(\sqrt{(h_s - h_b)^3} - \sqrt{(h_s - h_f)^3}\right) \quad (A7)$$

If $h_s < h_f$, $h_p > h_b$, the upper and lower limits are h_s and h_p :

$$Q_{ft} = -\frac{2}{3} \times 3.9 \times cir \times \left(\sqrt{(h_s - h_s)^3} - \sqrt{(h_s - h_p)^3}\right) \\ = \frac{2}{3} \times 3.9 \times cir \times \sqrt{(h_s - h_p)^3} \quad (A8)$$

If $h_s < h_f$ and $h_p < h_b$, the upper and lower limits are h_s and h_b :

$$Q_{ft} = \frac{2}{3} \times 3.9 \times cir \times \sqrt{(h_s - h_b)^3} \quad (A9)$$

If both h_p and $h_s > h_f$, $Q_{ft} = 0$.

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